What is Philosophical Logic?

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Proverbs

1. He who knows not and knows not he knows not: he is a fool - shun him.
2. He who knows not and knows he knows not: he is simple - teach him.
3. He who knows and knows not he knows: he is asleep - wake him.
4. He who knows and knows he knows: he is wise - follow him.. . . Arabian Proverb.

To attain knowledge add things every day.
To attain wisdom delete things every day.
. . . . . . Lao Tzu (604 BC - 531 BC) Chinese Taoist Philosopher

Lofti Zadeh

Classical logic is like a person who comes to a party dressed in a black suit, a white, starched shirt, a black tie, shiny shoes, and so forth.
And Fuzzy Logic is like a person dressed informally, in jeans, tee shirt, and sneakers. In the past, this informal dress won't have been acceptable. Today, it's the other way.
Lotfi A. Zadeh Communications of the Association for Computing Machinery(ACM), Volume 27, 1984

What is Logic?

- Logic is the study of right reason or valid inferences, and attending fallacies, formal, informal.
- Logic is a way to think so that we can come to correct conclusions by understanding implications and mistakes people often make in thinking.
- Logic is the science of valid processes of reasoning. In Mathematical Logic, we investigate these processes by mathematical methods.

Formal Logic

1. Formal logic in the sense that the validity of inferences depends on the form and not on the matter or meaning.
2. Formal logic as a formal science by opposition to an empirical science.
3. Formal logic in the sense of a formalized theory, to be understood in relation with the formalist program promoted by Hilbert, Curry and others.
4. Formal logic as symbolic logic, a science using symbols rather than ordinary language.
5. Formal logic as mathematical logic, logic developed by the use of mathematical concepts or/and the logic of mathematics.

classical logic (propositional logic/first-order logic) is formal in these 5 senses.

http://wwwa.unine.ch/unilog/jyb/form-bonn.pdf

What is Philosophical Logic?

Philosophical Logic

1. Wikipedia: Philosophical logic refers to those areas of philosophy in which recognized methods of logic have traditionally been used to solve or advance the discussion of philosophical problems (Meaning, truth, Identity, Paradoxes).
2. Alternative Logic, Deviant Logic, Non-Classical Logic.
3. Extensions, deviations of First order logic (Propositional and predicate logic).

What is Philosophical Logic?

- Philosophical logic is philosophy that is logic and logic that is philosophy. It is where logic and philosophy come together become one.
- Logic is the theory of consequence relations and valid inferences.
- A part of Logic dealing with what classical logic leaves out.
- Non-classical Logic, Non-standard logics, Deviant Logics
- Limitations of classical Logic: Explanation of conditionals,
- Classical Logic (First order Logic) is created for the purposes of mathematical reasoning,
- Philosophical logic develops formal systems and structures to be applied to the analysis of concepts and arguments that are central to philosophical inquiry.

Philosophical concepts and Various Logics

- Necessity and possibility: Modal Logic
- Knowledge and Belief: Epistemic Logic
- Obligation: Deontic Logic
- Time: Temporal Logic
- Existence: Free logic
- Reasoning: Non monotonic Logic and Probabilistic Logic.

Philosophical Concerns

Extensions and Alternatives

- Intutionistic Logic: Particular perspectives on nature and judgment of truth.
- Many valued logic: Logics that avoid conclusions of fatalism and determinism.
- Vagueness: Fuzzy logic
- Philosophical concerns on logic itself: Relevant logic (critique of classical consequence relation).
- Fuzzy Logic

Syllabus

- Crash Course on Classical Logic: Basic concepts of Propositional Logic
- Normal Modal Logic: Kripke semantics
- Conditional Logic
- Epistemic Logic, Puzzles in Epistemic Logic, Logical Omniscience problem
- Many- Valued Logic, Degrees of Truth, Fuzzy logic in handling Sorties Paradox

Evaluation

Mid semester exam: 30%
End Semester Exam 40%
Assignment: 20%
Attendance: 10%

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Other Books

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Example 1

If I am wealthy, then I am happy. I am happy. Therefore, I am wealthy.

Example

There was a robbery in which a lot of goods were stolen. The robber (s) left in a truck. It is known that :

1. Nobody else could have been involved other than A, B and C.
2. C never commits a crime without A's participation.
3. B does not know how to drive.

Is A innocent or guilty?

Example

There was a robbery in which a lot of goods were stolen. The robber (s) left in a truck. It is known that :

1. Nobody else could have been involved other than A, B and C. $(A \vee B \vee C)$.
2. C never commits a crime without A' s participation.$(C \rightarrow A)$
3. B does not know how to drive $(B \rightarrow [(B \wedge A) \vee (B \wedge C)]$.

Is A innocent or guilty?

A is Guilty

Example

During a murder investigation, you have gathered the following clues:

1. if the knife is in the store room, then we saw it when we cleared the store room;
2. the murder was committed at the basement or inside the apartment;
3. if the murder was committed at the basement, then the knife is in the yellow dust bin;
4. we did not see a knife when we cleared the store room;
5. if the murder was committed outside the building, then we are unable to find the knife;
6. if the murder was committed inside the apartment, then the knife is in the store room.

Where is knife?

Example: Analysis

First, we assigned symbols to the above clues:

1. s : the knife is in the store room;
2. c : we saw the knife when we clear the store room;
3. b : the murder was committed at the basement;
4. a : murder was committed inside the apartment;
5. y : the knife is in the yellow dust bin;
6. o : the murder was committed outside the building;
7. u : we are unable to find the knife;

Complicated Example

The following example is due to Lewis Carroll. Prove that it is a valid argument.

1. All the dated letters in this room are written on blue paper.
2. None of them are in black ink, except those that are written in the third person.
3. I have not filed any of those that I can read.
4. None of those that are written on one sheet are undated.
5. All of those that are not crossed out are in black ink.
6. All of those that are written by Brown begin with Dear Sir.
7. All of those that are written on blue paper are filed.
8. None of those that are written on more than one sheet are crossed out.
9. None of those that begin with Dear sir are written in the third person.
10. Therefore, I cannot read any of Browns letters.

Example

Let

1. p be the letter is dated,
2. q be the letter is written on blue paper,
3. r be the letter is written in black ink,
4. s be the letter is written in the third person,
5. t be the letter is filed,
6. u be I can read the letter,
7. v be the letter is written on one sheet,
8. w be the letter is crossed out,
9. x be the letter is written by Brown,
10. y be the letter begins with Dear Sir

Knights and Knaves

Suppose, A and B say the following:

1. A: All of us are knaves.
2. B: Exactly one of us is a knave.

Can it be determined what B is? Can it be determined what C is?

Lady or Tiger:

There is a lady or Tiger; both Tigers, or Both Ladies.
One of the sign boards is true and the other is false.

1. **Room A:** In this Room there is a Lady, and in the other room there is a Tiger
2. **Room B:** In one of these rooms there is a Lady and in one of these room there is a tiger.

Three functions of Language:

Logical Function

1. When the language is used to convey the information.
2. Sentences uttered can be spoken as either true or false.

There are two doors in this room.
On September 1939, Adolf Hitler's army invaded Poland.

Expressive

Indicative of Emotions and feelings.
Example: The dirty cockroach.

Evocative

Language is employed to evoke response in others.
Example: Help! save me! Pardon me!
What are we doing here?

Propositions

Propositions

1. Noun: A statement or assertion that expresses a judgment or opinion
2. A proposition or statement is a sentence which is either true or false.
3. If a proposition is true, then we say its truth value is true, and if a proposition is false, we say its truth value is false.
4. The sun is shining. Mayawati is the current CM of UP.
5. A propositional variable represents an arbitrary proposition. We usually represent propositional variables with uppercase letters.
6. **Stanford Encyclopedia of Philosophy:** The primary bearers of truth-value, the objects of belief and other "propositional attitude" (i.e., what is believed, doubted, etc.), the referents of that-clauses, and the meanings of sentences.

Form and Content of an argument

Form

The form of an argument is its logical structure or the manner in which the premises offer support for the conclusion.
Since the form describes the relationship between the premises and the conclusion, it cannot be true or false
Note: only propositions can be true or false.
Example: If elephants can fly, then rocks can float in water. Elephants can fly. Therefore, rocks can float in water.

Content

Content

The content of an argument is the group of actual propositions that comprise the argument.
It is with respect to *content* alone that one may consider truth and falsehood
Example: Kheer is better than nothing. Nothing is better than eternal happiness. Therefore Kheer is better than eternal happiness.

Example 2

All knowledge is power.
All power corrupts.
Therefore, all knowledge corrupts.

Syntax of Propositional Logic:

1. $\mathcal{L}$ is a language of propositional logic
2. The alphabet of $\mathcal{L}$ is composed of a finite or countably infinite set of propositions (n-ary relation symbols): A, B, C, . . .
3. The following connectives: $\neg$,$\wedge$, $\vee$,$\rightarrow$ $\leftrightarrow$
4. The punctuation symbols: (,)
5. Note: No meaning is attached to the symbols of the alphabet (this is the role of the semantics

Propositional Logic: Syntax

1. The set of formulas of L is the minimal set such that:
   1. Each proposition is a formula
   2. If F and G are formulas, then $(\neg F)$, $(F \wedge G)$, $(F \vee G)$, $(F \rightarrow G)$, and $(F \leftrightarrow G)$ are formulas
2. Parentheses are omitted whenever possible using the precedence: $\neg > \wedge > \vee > \rightarrow > \leftrightarrow$
3. If A is a proposition, then A and $\neg A$ are called literals
4. A is called a positive literal; $\neg A$ is called a negative literal
5. If L is a literal, $\bar{L}$ is the complementary literal defined as $\neg A$ if $L = A$, or A if $L = \neg A$.

Some Definitions

Definition (Sentence in L_1)

1. All sentence letters are sentences of L_1.
2. If ϕ and ψ are the sentences of L_1, then $\neg\phi$, $\phi \rightarrow \psi$, $\phi \wedge \psi$, $\phi \vee \psi$, $\phi \leftrightarrow \psi$ are the sentences.
3. Nothing else is a sentence of L_1.

Some Rules

1. 4+ 5 X 3 is written as 4+(5 x3) and is a short form for (4+(5 x3)).
2. $P \wedge (Q \rightarrow \neg P_1)$ for $(P \wedge (Q \rightarrow \neg P_1))$
3. $\wedge$, $\vee$ have stronger binding than $\rightarrow$, $\leftrightarrow$. so $P \rightarrow Q \vee R$ is written as $P \rightarrow (Q \vee R)$.
4. These are conventions not rules of the language.

Convention

We can omit the use of parenthesis by assigning decreasing ranks to the propositional connectives as follows: $\leftrightarrow$, $\rightarrow$, $\wedge$, $\vee$,$\neg$. The connective with greater rank always reaches further.
First preference is given to $\neg$ and then $\vee$ etc.

Example

1. $p \rightarrow q \wedge r \vee s$ is written as $p \rightarrow [q \wedge (r \vee s)]$.
2. $p \rightarrow \neg p \vee \neg q \wedge p \leftrightarrow q$ is written as ??
3. $p \vee \neg(q \wedge r) \leftrightarrow p$???

Propositional Logic: Syntax

Sub formula

Informally, a sub formula is the notion of a formula that is included in a formula

1. Let $T(F)$ be defined as the smallest set of formulas such that $F \in \mathcal{T}(F)$
2. if $\neg G \in \mathcal{T}(F)$ then $G \in \mathcal{T}(F)$
3. if $G \wedge H$, $G \vee H$, $G \rightarrow H$, or $G \leftrightarrow H$ belongs to $T(F)$, then $H, G \in \mathcal{T}(F)$

Semantics of Propositional Logic

1. Semantics is introduced to assign meaning to formulas
2. In propositional logic, each formula can be either true or false
3. It is a two-valued logic; other logics introduce additional truth values
4. An interpretation I is a total mapping from the set of propositions to the truth values
5. Any interpretation can be conveniently represented as the set of true propositions

Semantics of Propositional Logic

1. An interpretation can be extended to the set of formulas
2. $I \models F$ means I makes F true
3. $\models A$ iff $I(A) =$ true (A is a proposition)
4. $I \models \neg F$ iff $\quad I \not\models F$
5. $I \models F \wedge G \quad$ iff $I \models F$ and $I \models G$
6. $I \models F \vee G \quad$ iff $I \models F$ or $I \models G$.
7. $I \models F \rightarrow G \quad$ iff $I \not\models F$ or$I \models G$
8. $I \models F \leftrightarrow G$ iff $I \models F \rightarrow G$ and $I \models G \rightarrow F$.

Problem

Example

Example: $I1 = \{A, C\}$, $I2 = \{C, D\}$, $F = (A \vee B) \wedge (C \vee D)$ then $I1 \models F$ but$I2 \not\models F$

Semantics

1. If $I \models F$, then we say I is a model for F. This notion can be extended to sets of formulas
2. F is valid or a tautology iff for all interpretations I it is true that $I \models F$. In this case one can also write$\models F$
3. F is satisfiable iff there exist an interpretation I such that $I \models F$
4. F is falsifiable iff there exist an interpretation I such that $I \not\models F$
5. F is unsatisfiable iff for all interpretation I it is true that $I \not\models F$
6. F is contingent iff it is both satisfiable and falsifiable.

Example

1. $A \vee \neg A$ is valid; $A \vee B$ is both satisfiable and falsifiable;
2. $A \wedge \neg A$ is unsatisfiable
3. Any formula $F \wedge \neg F$ is called contradiction and often written $\perp$;
4. $F \vee \neg F$ is called excluded middle and often written as $\top$.

Propositional Logic: Semantics

Logical consequence

A set of formulas F logically entails a formula G (or G is a logical consequence of F) if every model of F is also a model of G. It is written $F \models G$.

Examples

1. $\{A, A \rightarrow B\} \models B$, $\{A, A \rightarrow B\} \models B \vee C$, $\{A, A \rightarrow B\} \not\models C$
2. In order to systematically determine whether a formula follows from a set of formulas, truth tabling can be used.
3. All possible combinations of truth values for every proposition should be considered

Logical Consequence

Two formulas$F$ and G are logically (semantically) equivalent iff both
$F \models G$ and $G \models F$
It is written $F \equiv G$.

Normal Forms

Some named equivalences:

1. $(F \wedge F) = F$ $\wedge$ idempotency.
2. $(F \vee F) = F$ $\vee$ idempotency
3. $(F \wedge G) = (G \wedge F)$ $\wedge$ commutativity.
4. $(F \vee G) = (G \vee F)$ $\vee$ commutativity
5. $(F \wedge (G \wedge H)) = ((F \wedge G) \wedge H)$ $\wedge$-associativity
6. $(F \vee (G \vee H)) = ((F \vee G) \vee H)$ $\vee$ associativity

Some Equivalences

1. $((F \wedge G) \vee F) = F$ absorption
2. $((F \vee G) \wedge F) = F$ absorption
3. $(F \wedge (G \vee H)) = ((F \wedge G) \vee (F \wedge H))$ distributivity
4. $(F \vee (G \wedge H)) = ((F \vee G) \wedge (F \vee H))$ distributivity
5. $\neg(\neg(F)) = F$ double negation
6. $(\neg(F \wedge G)) = (\neg F \vee \neg G)$ de Morgans law
7. $(\neg(F \vee G)) = (\neg F \wedge \neg G)$ de Morgans law
8. $(F \leftrightarrow G) = (F \rightarrow G) \wedge (G \rightarrow F)$ equivalence
9. $(F \rightarrow G) = (\neg F \vee G)$ material implication
10. $(F \rightarrow G) = (\neg G \rightarrow \neg F)$ contraposition

Normal Forms

The Substitution Theorem and the given equivalences can be used to introduce so-called normal forms.
Whenever we replace a subformula G of F by a formula H, the resulting formula is indicated $F[G \mid H]$

Substitution Theorem

If G is a subformula of F, and $G \equiv H$ then $F = F[G \mid H]$

A set of connectives is said to be functionally complete iff any propositional formula can be transformed into a semantically equivalent one which only contains connectives in the set
Example: $\{\neg, \wedge\}$ is functionally complete

Normal Forms

1. A formula is in negation normal form iff it is built only by literals, conjunctions and disjunctions
2. A formula is in conjunctive normal form (CNF) iff it has the form $C1 \wedge C2 \wedge ... \wedge Cn$ where each Ci is a disjunction of literals
3. A formula is in disjunctive normal form (DNF) iff it has the form $D1 \vee D2 \vee ... \vee Dn$ where each Di is a conjunction of literals
4. $C1 \wedge C2 \wedge ... \wedge Cn = C1 \wedge C2 \wedge ... \wedge Cn \wedge \top$, so we can say that $\top$ is in CNF taking $n = 0$
5. $D1 \vee D2 \vee ... \vee Dn = D1 \vee D2 \vee ... \vee Dn \vee \bot$ so we can say that $\bot$ is in DNF taking $n = 0$
6. The Cis are called clauses; the Dis are dual clauses; set notation is generally used.

Logical Calculas

Having defined a logic, we are now interested in knowing whether the logical consequences can be mechanically computed

1. A calculus consists of a set of axioms and a set of inference rules that produce the logical consequences in a logic
2. These elements define a derivability relation between a set of formulas F and a formula G. We have $F \vdash G$
3. If G can be obtained from F by applying only inference rules and axioms.
4. Ideally, the derivability relation should be sound (i.e., if F ‘ G then $F \models G$) and complete (i.e., if $F \models G$ then $F \vdash G$)
5. If a formula F can be derived in a theory F using the axioms and inference rules of a calculus, then we say that F is a theorem

Validity:

1. An argument is logically (or formally) valid if and only if there is no interpretation under which the premisses are all true and the conclusion is false.
2. Interpretations for the language L_1 is also called Structure.
3. An L_1 structure is an assignment of exactly one truth value (T or F) to every sentence letter of the language L_1.
4. Let Γ be a set of sentences of L_1 abd ϕ be a sentence of L_1, The argument with all sentences in Γ as premisses and ϕ as conclusion is valid (or $\Gamma \models \phi$ (for short) if and only if there is no L_1 structure in which all sentences in Γ are true and ϕ is false

Validity: Example

An argument is logically (or formally) valid if and only if there is no interpretation under which the premisses are all true and the conclusion is false.

Example (Validity)

1. $P \wedge Q) \models Q$.
2. $\{P \rightarrow \neg Q, Q\} \models \neg P$

Types of Sentences

1. A sentence ϕ of L_1 is logically true (tautology) if and only if ϕ is true in all L_1 structures.
2. A sentence ϕ of L_1 is a contradiction if and only if ϕ is not true in any L_1 structure.
3. A sentence ϕ and a sentence ψ, are logically equivalent if and only if ϕ and ψ, are true in exactly the same L_1 structures.