Review Problems

1. Given a coin which lands heads with probability p, what is the expected number of coin flips until you see heads?

2. Given a list of n distinct numbers, the i-th number is called a *record* if it is larger than the first $i - 1$ numbers. The first number is by default a record. Compute the expected number of records in a random permutation of the list of numbers.

3. Consider a complete ternary tree with height d where every leaf is labeled 0 or 1. Starting at the bottom level, every node computes and is labeled with the majority of the labels on its children.

   We are interested in computing the label of the root. Consider a randomized divide and conquer algorithm that starts at the root. It picks two uniformly random children of the root and recursively computes their labels. If the labels are the same, it returns that bit as the label of the root. Otherwise, it recursively computes the label of the third child and returns that one.

   (a) What is the maximum number of steps that the algorithm takes to run, in the worst case over all internal randomness of the algorithm? That is, how many steps does the algorithm take if it must compute the label of all three children of every node it encounters?

   (b) What is the *expected* number of steps that the algorithm takes to run, in the worst case over all possible labelings of the leaves?