CS 577: Introduction to Algorithms	Homework 10
Out: 04/24/17	**Due: 05/01/17**

Ground Rules

- Review problems will be discussed at the following week's review.
- Self-graded problems will be discussed at the following week's review, and are to be self-graded by you during the review. Only students present at the review session and turning in their graded solutions in person will earn points.
- Graded problems are to be turned in on the due date at the beginning of the lecture.
- Self-graded problems should be done individually.
- Graded problems should be done in pairs.
- Please follow page limits for all self-graded and graded problems.
- Write your name(s) clearly on your submissions, and turn in each problem on **a separate sheet of paper**.

Review Problems

1. In the decision version of the Vertex Cover problem, we are given a graph $G = (V, E)$ and a target k, and asked to decide whether G contains a vertex cover of size k. This problem is NP-complete. In the search version of the problem, we are asked to find a vertex cover of size k. Give a polynomial-time reduction from the search version of Vertex Cover to its decision version.

2. In the SquaredSum problem, you are given n positive integers $x_1, \cdots, x_n$, and numbers k and B. You want to know whether it is possible to partition the numbers $\{x_i\}$ into k sets $S_1, \cdots, S_k$ so that the squared sums of the sets add up to at most B:

$$\sum_{i=1}^{k} \left(\sum_{x_j \in S_i} x_j \right)^2 \leq B$$

   Prove that SquaredSum is NP-Complete.

   Hint: Reduce subset sum to this problem.

Self-graded Problems

3. **(Page limit: 1 sheet; 2 sides)** Consider the following problem, called BoxDepth: Given a set of n axis-aligned rectangles in the plane, how big is the largest subset of these rectangles that contain a common point?

   (a) Describe a mapping-time reduction from BoxDepth to MaxClique.

   (b) Briefly describe and analyze a polynomial-time algorithm for BoxDepth. *Hint: $O(n^3)$ time should be easy, but $O(n \log n)$ time is possible.*

   (c) Why don't these two results imply that P=NP?

Graded Problems

4. **(Page limit: 1 sheet; 2 sides)** A subset S of vertices in an undirected graph G is called triangle-free if, for every triple of vertices $u, v, w \in S$, at least one of the three edges uv, uw, vw is absent from G. The input to the decision problem *Large Triangle-Free Subset* is a graph G and a target k; the given instance is a "Yes" instance if G contains a triangle-free subset of vertices of size at least k, and is a "No" instance otherwise.

   Give a mapping reduction from Independent Set to LTFS.

5. **(Page limit: 1 sheet; 2 sides)** Integer Linear Programming (ILP) is an optimization problem. The instance for the decision version of this problem consists of n variables, denoted $x_1, \cdots, x_n$, m linear constraints of the form $\sum_i a_{ij} x_i \geq b_j$, a linear objective $\sum_i c_i x_i$, and a target value k. Here, the a_{ij}s, b_js, and c_is are constants. The instance is a "Yes" instance if the variables x_i can be assigned integral values such that the constraints are all satisfied and the objective achieves a value at most k.

   Give a mapping reduction from Vertex Cover to ILP.

6. **(not to be turned in)** Please fill out the online course evaluation form following the link you received in an email from AEFIS on Monday, Apr 24.