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# EXAM

Final Exam

Math 3350–015, Spring 2012

May 10, 2012

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- Write all of your answers on blank paper. Do not write on the exam questions handout. When finished, write your name and the section number on the first page of your answers. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g.,  $\sqrt{2}$ , not 1.414).
- Unless otherwise instructed, you can use a calculator to do partial fractions decompositions (indeed, you should) and integrals. Be clear about where you are using the calculator.
- This exam has 9 problems. There are **490 points total**.

Good luck!

70 pts.

**Problem 1.** In each part, find the general solution of the differential equation, or solve the given initial value problem. You must show the steps in solving the equation by one of the methods given in class, you can't just write down the answer.

A.

$$\frac{dy}{dx} = \frac{1-y}{x}$$

B.

$$\frac{dy}{dx} + \frac{2}{x}y = 6x^3, \quad y(1) = -1$$

C.

$$\frac{dy}{dx} + 2y = e^{2x}y^2$$

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60 pts.

**Problem 2.** Newton's law of cooling says that the time rate of change  $dT/dt$  of the temperature  $T$  of a body is proportional to the difference between  $T$  and the temperature  $T_M$  of the surrounding medium (the temperature of the surrounding medium is assumed to stay constant).

A soda can at a temperature of  $35^\circ$  is placed in a room that is at  $75^\circ$ . After 10 minutes the can has warmed to a temperature of  $45^\circ$ .

- A. Find the differential equation for the temperature  $T$  of the can and solve it to find  $T$  as a function of time. Give an exact value for  $k$ .
  - B. What is the temperature of the can after 15 minutes? Give an exact answer and a numerical answer accurate to two decimal places.
  - C. At what time will the temperature of the coffee be  $74^\circ$ ? Give an exact answer and a numerical answer that is accurate to two decimal places.
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100 pts.

**Problem 3.** In each part, find the general solution or solve the given initial value problem.

A.

$$y'' + y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

B.

$$y'' + 6y' + 9y = 0$$

C.

$$y'' + 6y' + 13y = 0$$

D.

$$x^2 y'' + 7xy' + 9y = 0$$

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40 pts.

**Problem 4.** Use the method of Undetermined Coefficients (either version) to find the general solution

A.

$$y'' + y' - 2y = x^2 + x$$

B.

$$y'' + y' - 6y = e^{2x}$$

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40 pts.

**Problem 5.** Find the general solution **by the method of variation of parameters**. No credit for doing it by a different method.

$$y'' - 4y' + 4y = xe^{2x}$$

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60 pts.

**Problem 6.** In each part, find the inverse Laplace transform. Use a calculator to find the partial fractions decompositions!

A.

$$\frac{5s^4 - 12s^3 + 17s^2 - 29s + 4}{s(s-1)^2(s^2+4)}$$

B. Hint: complete the square in the denominator.

$$\frac{s+4}{s^2-4s+13}$$

C.

$$e^{-2s} \left[ \frac{1}{s^2} + \frac{s+1}{s^2+4} \right]$$

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40 pts.

**Problem 7.** Find the Laplace Transform of the function

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ t^2, & 1 < t < 2 \\ 0, & 2 < t < \infty. \end{cases}$$

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40 pts.

**Problem 8.** This problem is about convolutions.

A. Find the convolution  $t * t^2$  directly from the definition (without using Laplace transforms). Work the integral by hand!

B. Use Laplace transforms to compute the convolution  $e^{3t} * \cos(2t)$ .

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40 pts.

**Problem 9.** Solve the following initial value problems **by the method of Laplace Transforms**.

A.

$$y'' - 5y' + 6y = e^{2t} \quad y(0) = 2, \quad y'(0) = 1$$

B.

$$y'' + y = \mathcal{U}(t-2), \quad y(0) = 1, \quad y'(0) = 0.$$

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