PROBLEM SET

Assignment 5

Math 4330, Spring 2017

April 21, 2017

- Write all of your answers on separate sheets of paper. You can keep the question sheet.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This problem et has 2 problems. There are **200 points total**.

Good luck!

Problem 1. In this problem, you will solve the two dimensional Poisson problem

$$-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = f(x, y)$$

on the unit square with zero boundary conditions. Use a double indexed variables $u_{i,j}$ to represent the approximation to $u(x_i, y_j)$ Pad the array with zero columns on the left and right and zero rows at the top and bottom. Thus, if you have an $N \times N$ grid of points in the interior of the square, array entries u_{ij} with indices in the range from 1 to N will correspond to the grid points. If either i or j is 0 or $N + 1$, u_{ij} will be zero.

Reduce the problem to solving linear equations for the u_{ij}. Write a program to solve the equations by iteration, using the forward Richardson method.

Your program doesn't have to store the coefficent matrix A. To compute Au, use loop(s) to apply the stencil at each interior point. This will be similar to one of the example programs.

How many interactions does it take to solve the problem with a tolerance of 10^{-7} for the relative residual with grid sizes 10×10, 100×100 and 1000×1000. Can you solve it with $10^4 \times 10^4$ in a usable amount of time?

(See the example program. If the question is not clear, ask me.)

Problem 2. Consider a scalar Initial Value Problem

$$y'(t) = f(t, y(t)),$$
$$y(t_0) = y_0.$$

Write a program to approximate the solution using Eulers method. Divide the interval $a = t_0 \leq t \leq b$ into N subintervals.

A case where we can find a formula for the solution is

$$y'(t) = \frac{t - y(t)}{2}$$
$$y(0) = y_0$$

In this case have your program draw the graphs of the approximate solution and and the exact solution over the interval $[0, 1]$ with a suitable initial conditon for a few values of N.

You'll be adding other methods later.
