Efficient mining of complex networks

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1. Using what we learnt for graph mining

2. Web spam and Clustering
   - Web spam
     - Web spam Indices

1. Using what we learnt for graph mining

2. Web spam and Clustering
   - Web spam
     - Web spam Indices
   - Clustering

3. Computational challenges
   - Computational model

Using tools in practice ...

Data intensive graph mining

We have a data collection in the form of a large graph

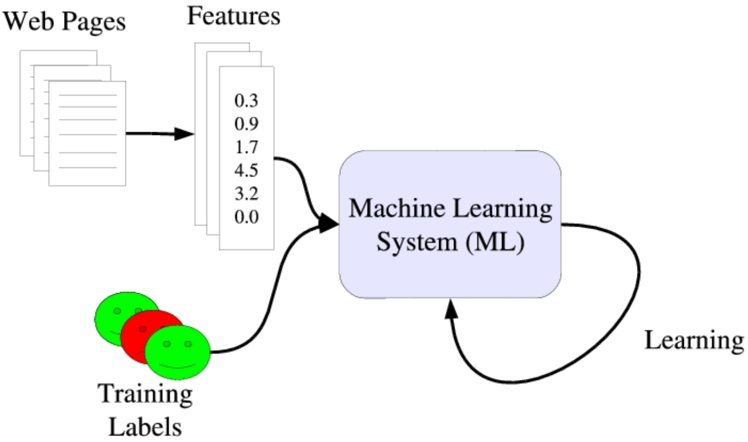
We have a mining task

- Document ranking
- Cyber-community detection
- Web spam detection
- Profiling of users accessing a search engine/on line store
  - Finding "typical" queries/items
  - Suggesting topics/items of potential interest to users who submitted/purchased a given query/item
- Detecting hot spots in epidemic spreading
- Topic distillation over hyperlinked document collections
- Detection of network bottlenecks

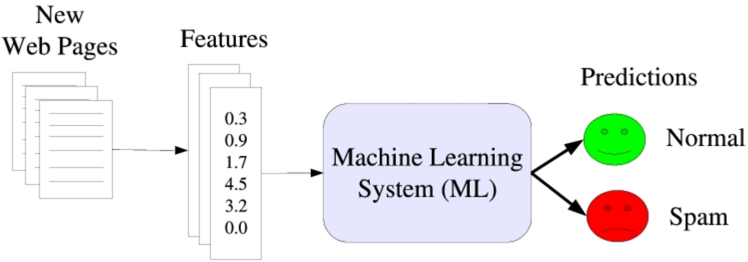
Mapping IR applications to indices

- Many data collections in the form of large scale graphs (e.g., Web crawls, query graphs)
- Many IR applications entail the computation of local indices on a per vertex basis
- Example: Pagerank ranking index
  - Requires a massive graph/matrix computation
  - Result is an index vector (Pagerank) with one component per Web page
- Different IR applications require computation of different indices

Automatic classifiers (e.g.: Web spam)



Automatic classifiers (cont.)



Challenges

Machine Learning Challenges:

- Learning with inter dependent variables (graph)

Information Retrieval Challenges:

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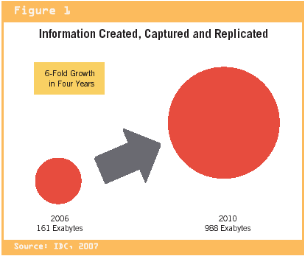
Data size



- Indexable Web estimated to have more than 11.5 billion pages [Gulli and Signorini, 2005]
- As of now: roughly 100 times more (?)
- Facebook has about 1.5 billion users
- Amazon's unique monthly visitors: about 183 millions

General problem

- We are given a (typically large or huge) graph $G = (V, E)$
- Vertices may represent Web pages, people etc.
- Arcs (or edges) represent relationships. E.g., hyperlinks, email exchanges, social ties, interaction etc.
- Goal: compute, for every vertex, some index depending on the application and whose value depends on graph topology

Challenges

- Polynomial solutions may not suffice ...
- Graphs may be too large to fit in main memory
- Solutions must be scalable, both in memory and computational costs

This lecture

- Consider two exemplar applications
- See how techniques can be applied to these cases
  - Partial view, but gives flavour of techniques involved

Our motivating examples

- Web spam detection
  - Boost the Pagerank score of target Web pages
  - Uses content and/or link based techniques
  - We focus on link based spam
- Local clustering in massive graphs
  - Can unveil important aspects of the network's social structure (e.g., identify dense regions, communities etc.)

What is on the Web?

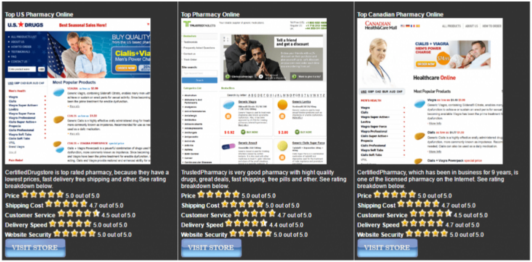
Information

What is on the Web?

Information + Porn

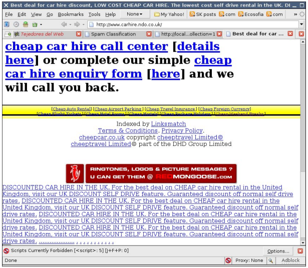
What is on the Web?

Information + Porn + On-line casinos + Free movies + Cheap software + Buy a MBA diploma + Prescription -free drugs + V!-4-gra + Get rich now now now!!!



Forms of Web spamming

Typical Web Spam



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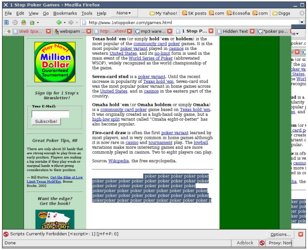

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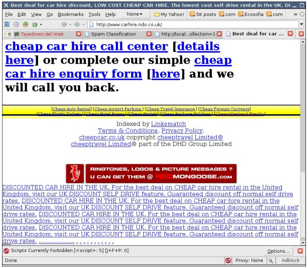

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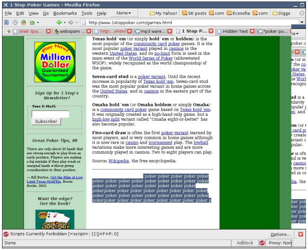
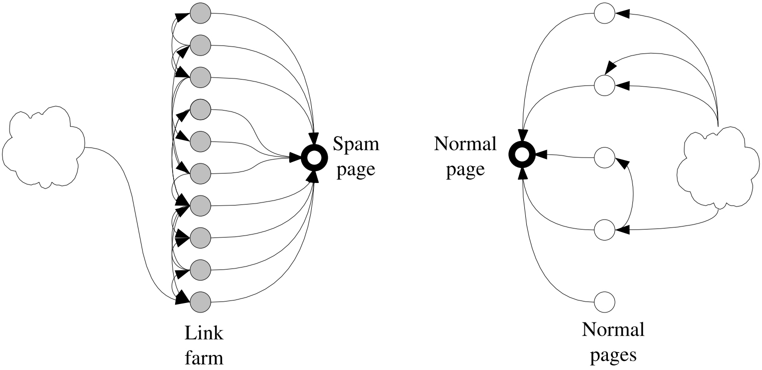


Many others...

Adversarial relationship

Every undeserved gain in ranking for a spammer, is a loss of precision for the search engine.

Topological spam: link farms



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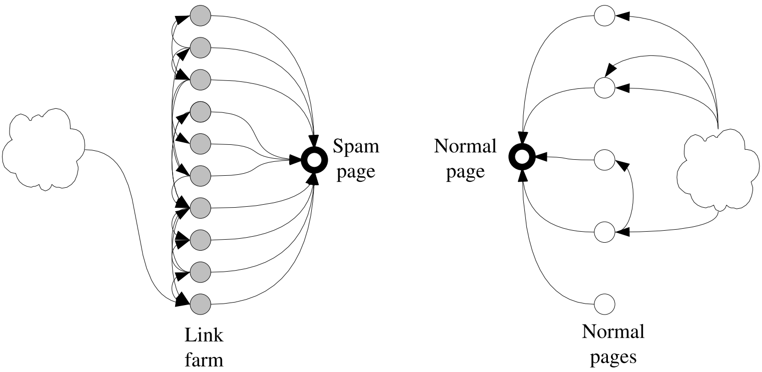

Single-level farms can be detected by searching groups of nodes sharing their out-links [Gibson et al., 2005]

Motivation

[Fetterly et al., 2004] hypothesized that studying the distribution of statistics about pages could be a good way of detecting spam pages:

"**in a number of these distributions, outlier values are associated with web spam**"

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Research goal

Statistical analysis of link-based spam

Spam indices [Becchetti et al., 2007]

U.K. collection

18.5 million pages downloaded from the .UK domain in 2002

5,344 hosts manually classified (6% of the hosts)

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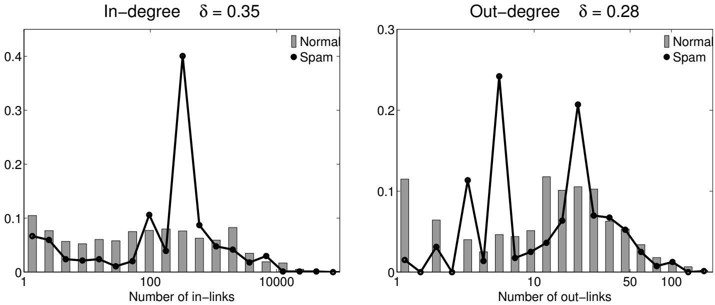
Classified entire hosts:

- ☑ A few hosts are mixed: spam and non-spam pages
- ☒ More coverage: sample covers 32% of the pages

Degree



($\delta =$ max. difference in C.D.F. plot)

PageRank

Let $\mathbf{P}_{N \times N}$ be the normalized adjacency matrix of a graph

- Row-normalized
- No “sinks”

Definition (PageRank)

Stationary state of:

$$\alpha \mathbf{P} + \frac{(1-\alpha)}{N} \mathbf{1}_{N \times N}$$

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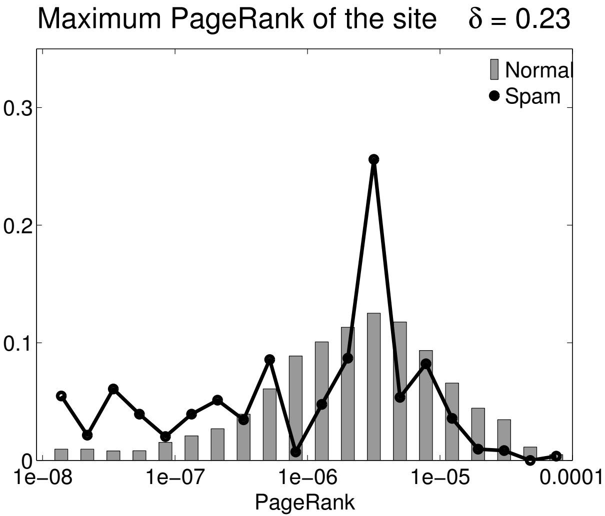
Definition (PageRank)

Stationary state of:

$$\alpha \mathbf{P} + \frac{(1-\alpha)}{N} \mathbf{1}_{N \times N}$$

- Follow links with probability α
  - Every link chosen with prob. $1/deg$.
- Random jump with probability $1-\alpha$

Maximum PageRank in the Host



TrustRank

TrustRank [Gyöngyi et al., 2004]

A node with high PageRank, but far away from a core set of "trusted nodes" is suspicious

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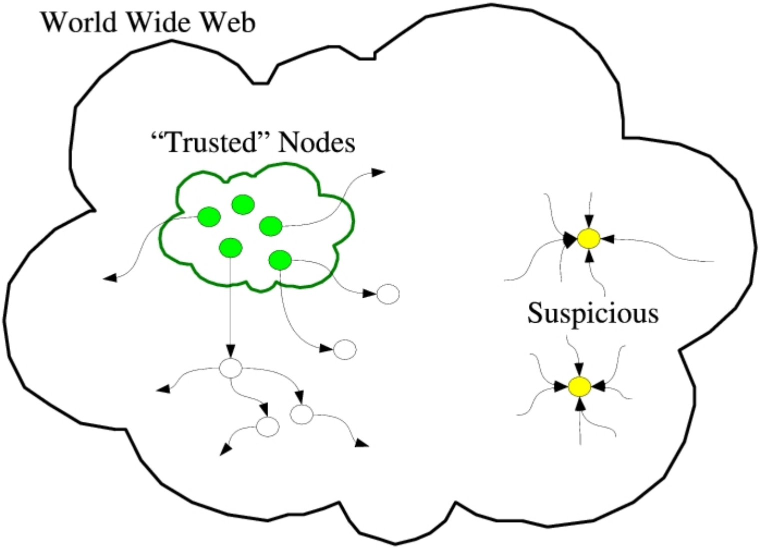
A node with high PageRank, but far away from a core set of "trusted nodes" is suspicious

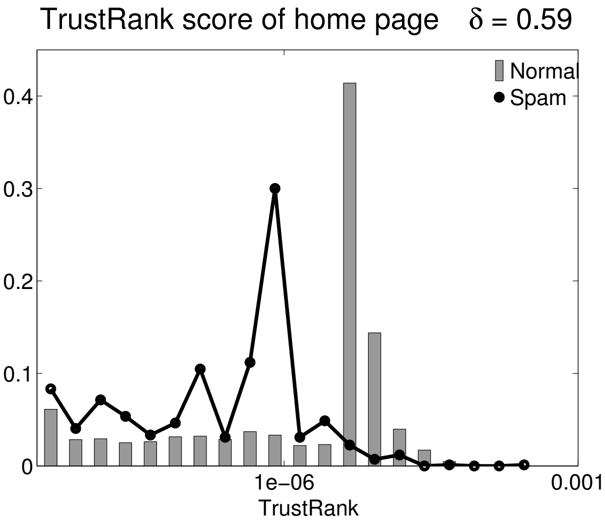
Start from a set of trusted nodes, then do a random walk, returning to the set of trusted nodes with probability $1 - \alpha$ at each step

ℹ Trusted nodes: data from `http://www.dmoz.org/`

TrustRank Idea

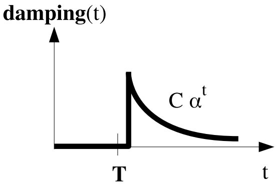


TrustRank score



Truncated PageRank

Reduce the direct contribution of the first levels of links:



$$\text{damping}(t) = \begin{cases} 0 & t \leq T \\ C\alpha^t & t > T \end{cases}$$

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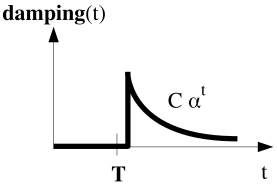
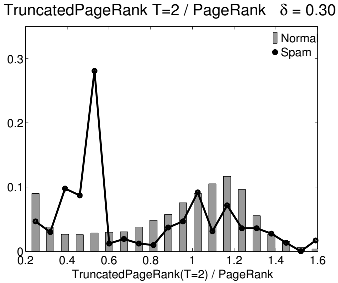
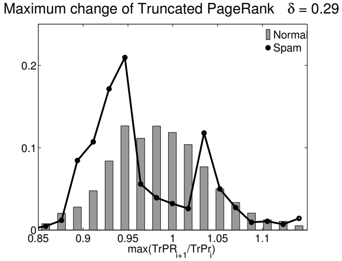


$$\text{damping}(t) = \begin{cases} 0 & t \le T \\ C\alpha^t & t > T \end{cases}$$

- ☑ No extra reading of the graph after PageRank
- ☑ Idea: most of spammers' rank due to pages that are few links away

Truncated PageRank(T=2) / PageRank

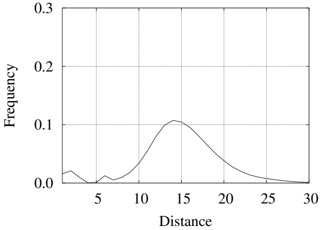




Idea: count "supporters" at different distances

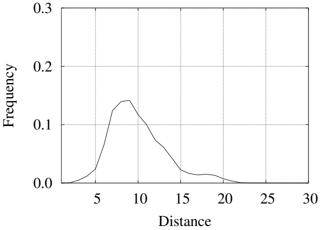
Number of *distinct* nodes at a given distance:

.UK 18 mill. nodes



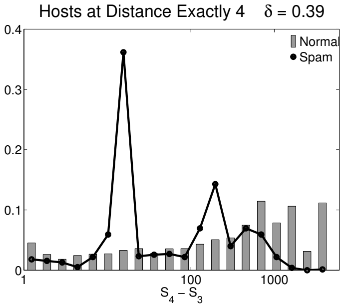
Average distance
14.9 clicks

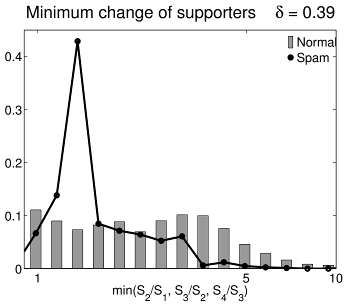
.EU.INT 860,000 nodes



Average distance
10.0 clicks

Supporters and their change





Clustering coefficient

- Compute triangle count for all vertices
- Local clustering coefficient and related statistics

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Motivation

- Analysis of social or biological networks [Newman, 2003]
- Thematic relationships in the Web [Eckmann and Moses, 2002]
- Common interests [Buchsbaum et al., 2003]

Web spam: [Fetterly et al., 2004] hypothesized that studying the distribution of statistics about pages could be a good way of detecting spam pages:

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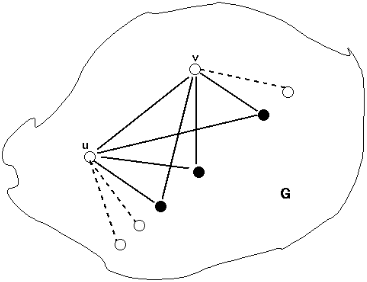
Local Clustering Coefficient



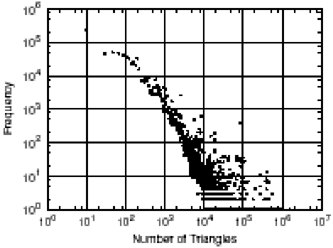
- $S(u) : \{v : (u, v) \in E\}$, $d(u) = |S(u)|$
- $T(u)$: No. triangles to which u belongs

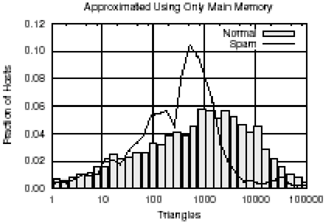
Clustering Coefficient

$CC_1 = \frac{2\sum_u T(u)}{\sum_u d(u)(d(u)-1)}$ (Alternative definition)

$\mathbf{CC_2} = \frac{\mathbf{1}}{|\mathbf{V}|} \sum_{\mathbf{u} \in \mathbf{V}} \frac{\mathbf{2T(u)}}{\mathbf{d(u)(d(u)-1)}}$

Distribution of triangles/clustering coefficient





- Distribution of number of triangles follows power law [Eckmann and Moses, 2002]
- Distributions of number of triangles/clustering coefficient in normal/spam pages
- Allows also to discriminate content quality in Yahoo! Answers [Becchetti et al., 2008]

Semi-streaming [Feigenbaum et al., 2004]

- Graph stored in secondary memory as adjacency or edge list
- **No random access possible**
- $O(N \log N)$ bits available in main memory
  - Limited amount of information per vertex
  - ✗ Not enough to store links in main memory
- Limited (constant or $O(\log N)$) number of passes
- ☑ No previous knowledge about graph
- Compute index for all vertices concurrently

More specifically:

We can store in main memory a (small) constant number of size N vectors with components of size $O(\log N)$ bits

Some previous work

- Computation of approximate matchings and distances [Feigenbaum et al., 2004, Feigenbaum et al., 2005]
- Lower bounds for neighbourhoods problems [Buchsbaum et al., 2003]
- Tradeoffs between number of passes and space for shortest path problems [Demetrescu et al., 2006]

Related: Streaming [Muthukrishnan, 2005]

- Stream of items accessed sequentially
- Maintain statistics (e.g., most frequent elements, histograms etc.)
- $O(\log \mathit{Space})$ overall, $O(\log \mathit{Time})$/item

General algorithm we consider

Require: N: number of nodes, d: distance, k: bits

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for node : 1 ... N, bit: 1 ... k do
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end for
for distance : 1 ... d **do** {Iteration step}
    INIT(Aux)
    for src : 1 ... N **do** {Follow links in the graph}
        for all links from src to dest **do**
            Aux[src] $\leftarrow$ Combine(Aux[dest], V[src,$\cdot$])
        end for
    end for
    V $\leftarrow$ Aux
end for

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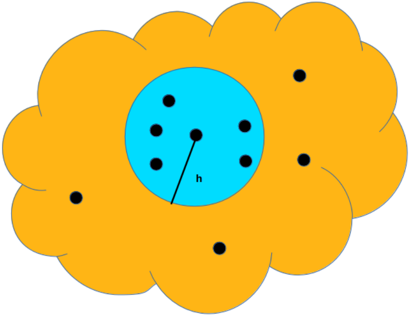
        end for
    end for
    V ← Aux
end for
for node: 1...N do {Estimation}
    Index[node] ← ESTIMATE( V[node,·] )
end for
return Index
```

Counting the number of supporters



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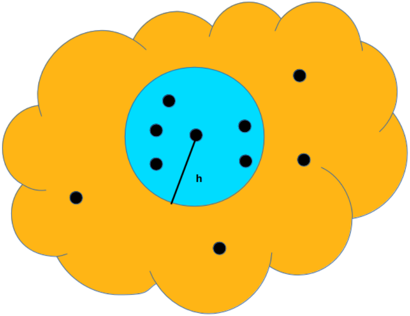

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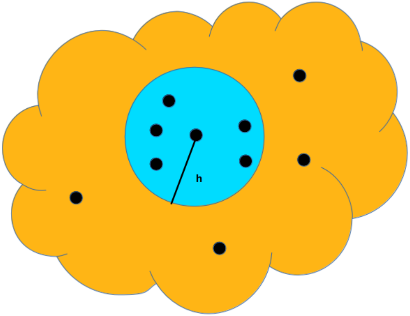

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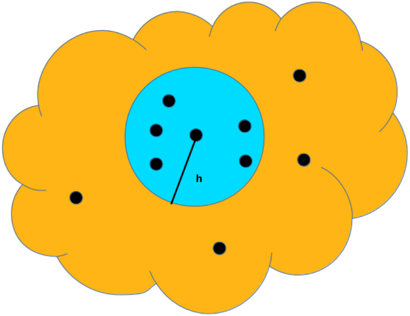

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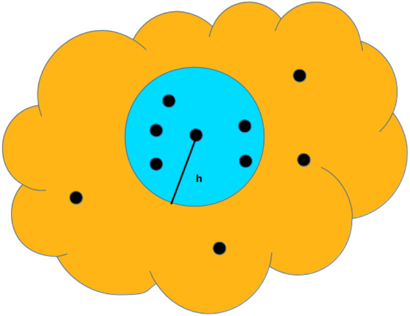

- For every node v, count the number of nodes within h hops
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- Let $N(x, h) = \#$ nodes within h hops of x

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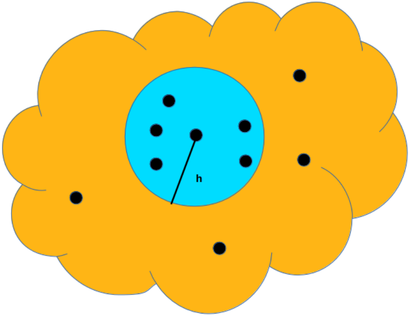

- For every node v, count the number of nodes within h hops
- Do this for different values of h
- Count each supporter *only once*
- Let $N(x, h) = \#$ nodes within h hops of x
- *Can we directly apply the general algorithm seen before?*

A detour: back to distinct counting

Composing two sketches

- Assume two sets S_1 and S_2

A detour: back to distinct counting

Composing two sketches

- Assume two sets S_1 and S_2
- Assume $\mathbf{sk}(S_1) = (R_1(S_1), \ldots, R_k(S_1))$ and $\mathbf{sk}(S_2) = (R_1(S_2), \ldots, R_k(S_2))$ are corresponding sketches

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What is $\mathbf{sk}(S_1 \cup S_2)$?

A detour: back to distinct counting

Composing two sketches

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What is $\mathbf{sk}(S_1 \cup S_2)$?

Composability ...

$$\mathbf{sk}(S_1 \cup S_2) = (\max\{R_1(S_1), R_1(S_2)\}, \ldots, \max\{R_k(S_1), R_k(S_2)\})$$

Let $\text{Combine}(\mathbf{sk}(S_1), \mathbf{sk}(S_2)) = (\max\{R_1(S_1), R_1(S_2)\}, \ldots, \max\{R_k(S_1), R_k(S_2)\})$

Supporters: Probabilistic counting

- Want to estimate $N(Target, 2)$

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- Want to estimate $N(Target, 2)$
- View $N(Target, 2)$ as set

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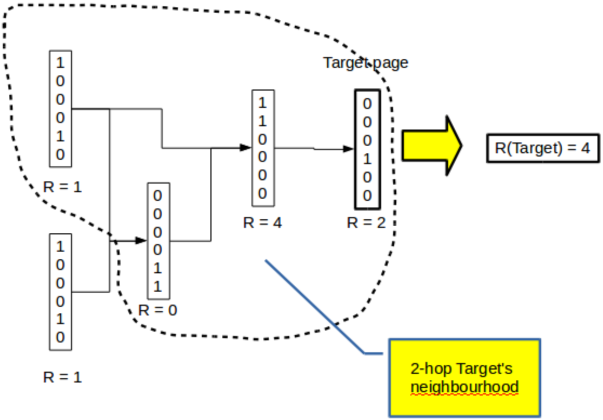
- Want to estimate $N(Target, 2)$
- View $N(Target, 2)$ as set
- Use distinct counting algorithm of [Alon et al., 1999]



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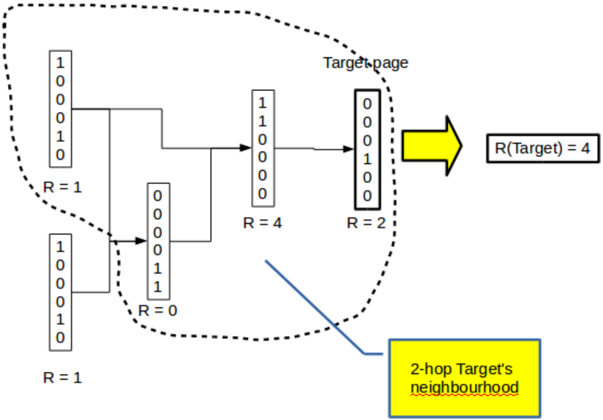

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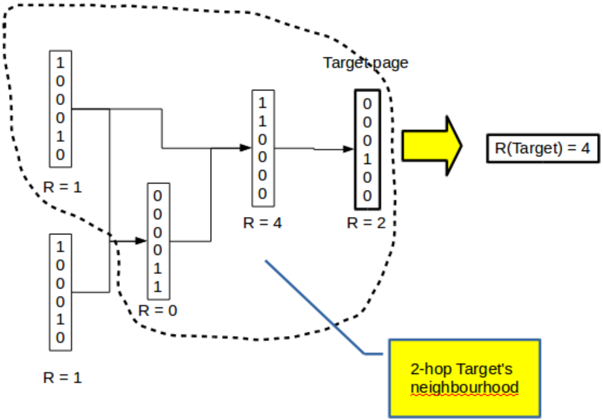

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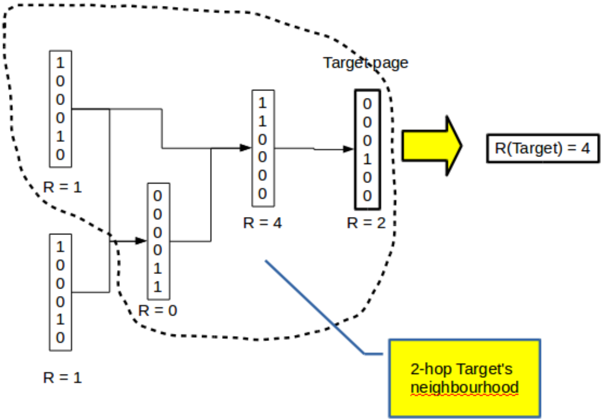

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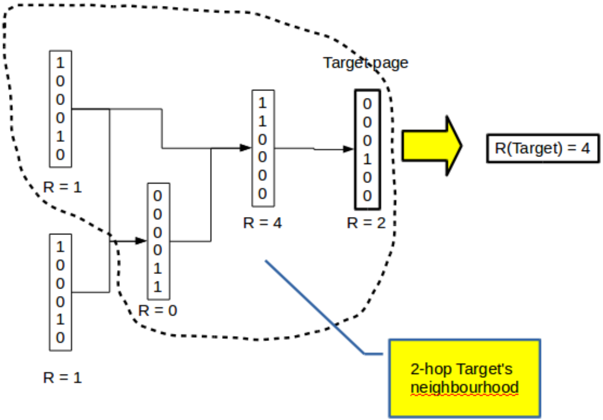

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ANF-like algorithm

Require: N: number of nodes, d: distance, k: bits

1: **for** node : 1 . . . N, bit: 1 . . . k **do**
2: INIT(node,bit) {Initialize node sketches}
3: **end for**

ANF-like algorithm

Require: N: number of nodes, d: distance, k: bits

for node : 1 ... N, bit: 1 ... k **do**
    INIT(node,bit) {Initialize node sketches}
end for
for distance : 1 ... d **do** {Iteration step}
    Aux $\leftarrow \mathbf{0}_k$
    for src : 1 ... N **do** {Follow links in the graph}
        for all links from src to dest **do**
            Aux[src] $\leftarrow$ Combine(Aux[dest], V[src,$\cdot$])
        end for
    end for
    V $\leftarrow$ Aux
end for

ANF-like algorithm

Require: N: number of nodes, d: distance, k: bits

```
for node : 1 ... N, bit: 1 ... k do
    INIT(node,bit) {Initialize node sketches}
end for
for distance : 1...d do {Iteration step}
    Aux ← 0_k
    for src : 1 ... N do {Follow links in the graph}
        for all links from src to dest do
            Aux[src] ← Combine(Aux[dest], V[src,·])
        end for
    end for
    V ← Aux
end for
for node: 1...N do {Estimate supporters}
    Supporters[node] ← ESTIMATE( V[node,·] )
end for
return Supporters
```

Our estimator

- The estimator proposed in [Alon et al., 1999]

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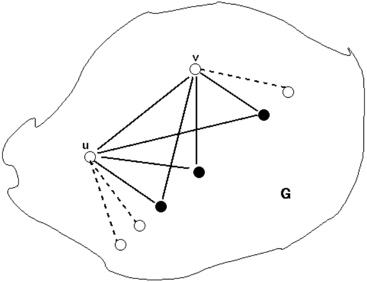
Tuning

For a given value of k, s and t allow to trade off between accuracy and probability

Local Clustering Coefficient

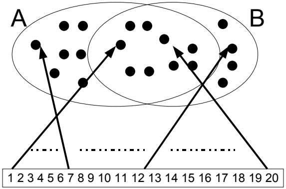


- $S(u) : \{v : (u, v) \in E\}$, $d(u) = |S(u)|$

Number of Triangles and Clustering Coefficient

- Estimate local clustering coefficient concurrently for all vertices
- Semi-streaming model
- Need to pass over the graph as few times as possible
- *Key step*: estimate size of neighbourhood intersection

Estimating Set Intersection: intuition



- Assume items of the universe initially numbered
- Any of the possible $n!$ permutations chosen u.a.r.
- Items reordered accordingly
- $\mathbf{P}[\min \pi(A) = \min \pi(B)] = J(A, B) = \frac{|A \cap B|}{|A \cup B|}$

Estimating Set Intersection: basic technique

Approach assumes family of *minwise independent* permutations
[Broder, 1998, Broder, 2000, Broder et al., 1997]

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In practice...

- Exponential space ($\Omega(n)$ bits) needed to represent minwise families [Broder et al., 1998]
- $\pi(x) = ((ax + b) \mod p) \mod n$, with a and b chosen u.a.r., p a large prime [Bohman et al., 2000]

Triangles: Ideal algorithm

If we new $J(S(u), S(v))$:

$$T_{uv} = |S(u) \cap S(v)| = \frac{J}{J+1}(|S(u)| + |S(v)|)$$

- m independent trials
- Z_{uv}: # times that $\min \pi(S(u)) = \min \pi(S(v))$

Our estimator:

$$\overline{T}_{uv} = \frac{Z_{uv}}{Z_{uv} + m}(|S(u)| + |S(v)|)$$

We use a more efficient modified alg in practice

High probability bound

$$\mathbf{P}\left[|\overline{T}_{uv} - T_{uv}| > \epsilon T_{uv}\right] \leq$$
$$\leq Ce^{-\frac{\epsilon^2}{3} mJ(S(u),S(v))}.$$

for a suitable constant C

General Algorithm

$\mathbf{Z} = \mathbf{0}$
for i: 1 ... m **do** {Independent trials}
  for u : 1 ... $|V|$ **do** {Assign labels}
    $l_i(u) = \text{hash}_i(u)$ {Minwise linear permutation}
  end for

General Algorithm

```
Z = 0
for i: 1 ... m do {Independent trials}
    for u : 1 ... |V| do {Assign labels}
        l_i(u) = hash_i(u) {Minwise linear permutation}
    end for
    for u : 1 ... |V| do {Compute fingerprints}
        F_i(u) = min_{v∈S(u)} l_i(u)
    end for{1 scan of G}
```

General Algorithm

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    for u : 1 ... $|V|$ **do** {Compute fingerprints}
        $F_i(u) = \min_{v \in S(u)} l_i(u)$
    end for{1 scan of G}
    for u : 1 ... $|V|$ **do** {Update counters}
        for $v \in S(u)$ **do**
            if $F_i(u) == F_i(v)$ **then** {Minima are equal}
                $Z_{uv} = Z_{uv} + 1$ {Z_{uv}'s stored on disk}
            end if
        end for
    end for
end for

Estimating Triangles/cont.

- $\overline{T}_{uv} = \frac{Z_{uv}}{Z_{uv}+m}(d(u)+d(v))$ is our estimate of $|S(u) \cap S(v)|$
- $\overline{T}(u) = \frac{1}{2}\sum_{v \in S(v)} \overline{T}_{uv}$ is our estimate of $T(u)$
- In practice, $m = O(\log N)$

Implementation

- The Z_{uv}'s must be stored on disk (size of $\mathbf{Z}$ same order as adjacency list)

- *For every i, updating Z_{uv} requires access to disk*
- *Computing counters most expensive operation*

Using the adjacency matrix ([Tsourakakis, 2008])

Let $\mathbf{A}$ denote the adjacency matrix of an *undirected* graph

- Consider $\mathbf{A}^3$

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As a consequence...

$Trace(\mathbf{A^3}) = 6$ (# triangles)
Reason: triangle (i, j, k) contributes twice to $\mathbf{A}^3_{ii}$, $\mathbf{A}^3_{jj}$ and $\mathbf{A}^3_{kk}$

Spectra and triangles

Recall that **A** is symmetric ...
hence it can be diagonalized:

$$\mathbf{A} = \sum_{i=1}^{n} \lambda_i \mathbf{u_i} \mathbf{u_i}^T,$$

where $(\lambda_i, \mathbf{u_i})$ is the i-th eigenpair

As a consequence...

$$\mathbf{A}^3 = \sum_{i=1}^{n} \lambda_i^3 \mathbf{u_i} \mathbf{u_i}^T$$

Theorem ([Tsourakakis, 2008])

Let $\Delta(G) = \#$ triangles and $\mathbf{A}$ the adjacency matrix of G.
Let $\Delta_i(G) = \#$ triangles in which i is involved. We have:

$$\Delta(G) = \frac{1}{6}\sum_{i=1}^{n}\lambda_i^3$$

$$\Delta_i(G) = \frac{1}{2}\sum_{j=1}^{n}\lambda_i^3\mathbf{u_j}(\mathbf{i})^2,$$

with $\mathbf{u_i}(j)$ the j-th component of $\mathbf{u_i}$

Proof sketch

- First claim follows since trace of a matrix $= \sum$ eigenvalues
- Second claim follows from expression of $\mathbf{A}_{ii}^3$ in spectral decomposition

In practice ...

Graphs we are interested in normally obey power laws
Same applies to distribution of triangles

Implications

- Most triangles incident to relatively small fraction of nodes
- Enough to sum over the first k entries of $\mathbf{A}^3$'s diagonal - k relatively small
- Corresponds to computing the first k eigenvectors of $\mathbf{A}$

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