



Introduction to Neural Networks

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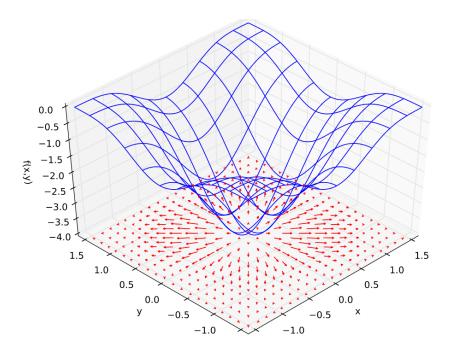


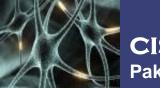


Basics

- Gradient
 - Generalization of the slope to multidimensional functions

•
$$\nabla f(\mathbf{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}$$





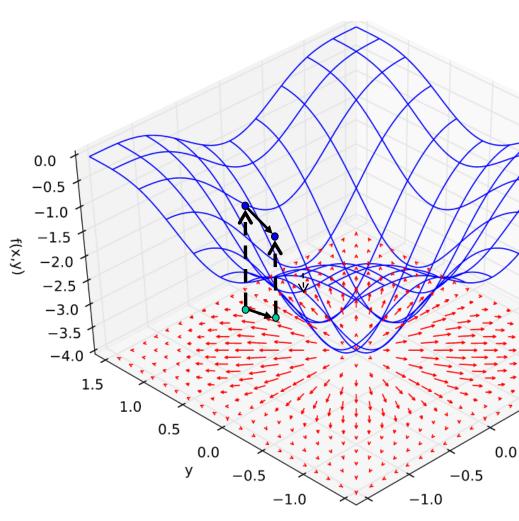


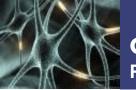
Basics

- Gradient Descent
 - A method for optimization
 - To find the minima of a function, take a step in the direction opposite to the gradient

 $\boldsymbol{x_i} = \boldsymbol{x_{i-1}} + \alpha \nabla \boldsymbol{f}(\boldsymbol{x})$

Local minimas?

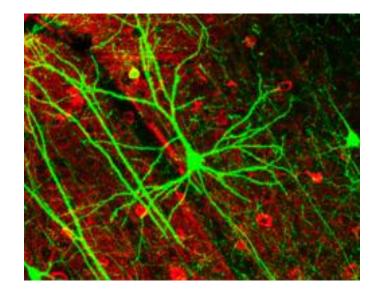






The Human Brain: Neurons and Nerve Cells

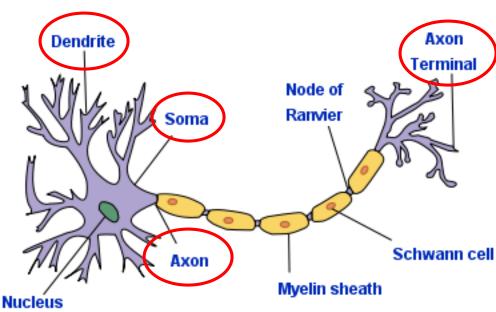
- Most complex organ in the human body
- Contains some 10¹⁰ neurons, which are capable of electrical and chemical communication with tens of thousands of other nerve cells
- Nerve cells in turn rely on some quadrillion (10¹⁵) synaptic connections for their communications.



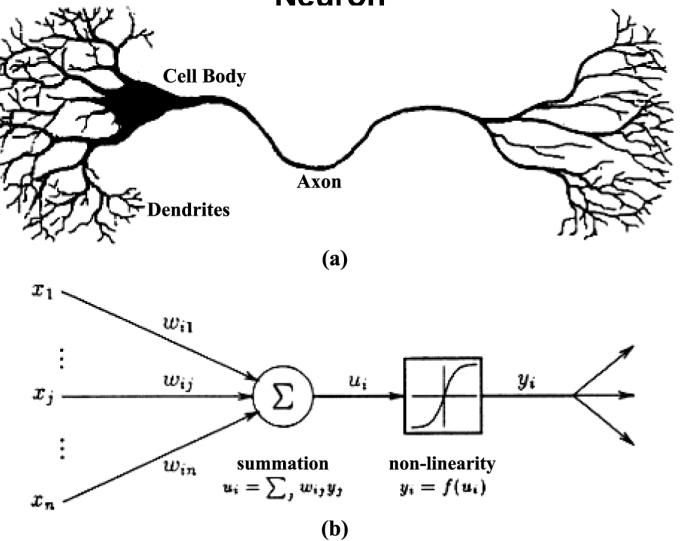


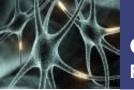
Functioning of the Biological Neuron

- Electrically Excitable Cells
- Process and Transmit Information
- Major Parts
 - Soma (3-18um)
 - Cell Body
 - Dendrites
 - Receive Inputs from other Neurons
 - Axon
 - Transmit Output to other Neurons



Structural Mathematical Model for the Biological Neuron







Concepts of Linear Separability

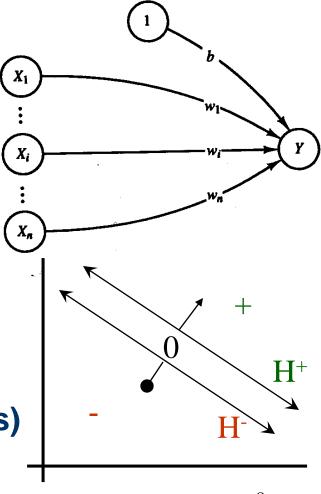
- Find a line that separates
 - **(0,0),(0,1),(1,0)**
 - (1,1)

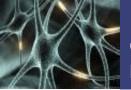




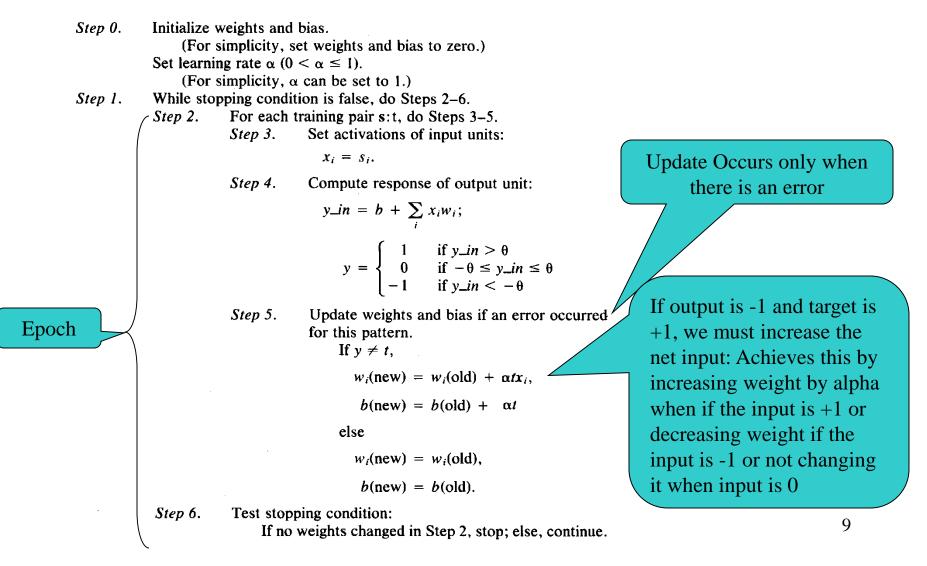
Perceptron

- Given:
 - Training data and labels $y_{in} = w_{1}X_{1} + w_{2}X_{2} + \dots + w_{n}X_{n} + b = W^{T}X + b$ $W = \begin{bmatrix} w_{1} \\ w_{2} \\ \vdots \\ w_{n} \end{bmatrix}, X = \begin{bmatrix} X_{1} \\ X_{2} \\ \vdots \\ X_{n} \end{bmatrix}$ $Y = \begin{cases} +1 & W^{T}X + b > \theta \\ 0 & -\theta \le W^{T}X + b \le \theta \\ -1 & W^{T}X + b \le -\theta \end{cases}$
 - Thus there are two hyperplanes)
 - H⁺: W^TX+b=θ
 - H⁻: W^TX+b=-θ





Learning Algorithm







Example: AND Gate, θ =0.2, α =1

x ₁	x ₂	1	У _{net}	у	Т	dw ₁	dw ₂	db	$w_1 = w_1 + dw_1$	w ₂ = w ₂ +dw ₂	b=b+db
									0	0	0
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1	2	1	-1	-1	0	-1	0	1	0
0	1	1	1	1	-1	0	-1	-1	0	0	-1
0	0	1	-1	-1	-1	0	0	-1	0	0	-1
1	1	1	-1	-1	1	1	1	1	1	1	0
1	0	1	1	1	-1	-1	0	-1	0	1	-1
0	1	1	0	0	-1	0	-1	-1	0	0	-2
0	0	1	-2	-1	-1	0	0	-1	0	0	-2
1	1	1	-2	-1	1	1	1	1	1	1	-1
1	0	1	0	0	-1	-1	0	-1	0	1	-2
0	1	1	-1	-1	-1	0	-1	-1	0	1	-2
0	0	1	-2	-1	-1	0	0	-1	0	1	-2
1	1	1	-1	-1	1	1	1	1	1	2	-1
1	0	1	0	0	-1	-1	0	-1	0	2	-2
0	1	1	0	0	-1	0	-1	-1	0	1	-3
0	0	1	-3	-1	-1	0	0	-1	0	1	-3
1	1	1	-2	-1	1	1	1	1	1	2	-2
1	0	1	-1	-1	-1	-1	0	-1	1	2	-2
0	1	1	0	0	-1	0	-1	-1	1	1	-3
0	0	1	-3	-1	-1	0	0	-1	1	1	-3



x ₁	x ₂	1	y _{net}	у	Т	dw ₁	dw ₂	db	$w_1 = w_1 + dw_1$	$w_2 = w_2 + dw_2$	b=b+db
									1	1	-3
1	1	1	-1	-1	1	1	1	1	2	2	-2
1	0	1	0	0	-1	-1	0	-1	1	2	-3
0	1	1	-1	-1	-1	0	-1	-1	1	2	-3
0	0	1	-3	-1	-1	0	0	-1	1	2	-3
1	1	1	0	0	1	1	1	1	2	3	-2
1	0	1	0	0	-1	-1	0	-1	1	3	-3
0	1	1	0	0	-1	0	-1	-1	1	2	-4
0	0	1	-4	-1	-1	0	0	-1	1	2	-4
1	1	1	-1	-1	1	1	1	1	2	3	-3
1	0	1	-1	-1	-1	-1	0	-1	2	3	-3
0	1	1	0	0	-1	0	-1	-1	2	2	-4
0	0	1	-4	-1	-1	0	0	-1	2	2	-4
1	1	1	0	0	1	1	1	1	3	3	-3
1	0	1	0	0	-1	-1	0	-1	2	3	-4
0	1	1	-1	-1	-1	0	-1	-1	2	3	-4
0	0	1	-4	-1	-1	0	0	-1	2	3	-4
1	1	1	1	1	1	1	1	1	2	3	-4
1	0	1	-2	-1	-1	-1	0	-1	2	3	-4
0	1	1	-1	-1	-1	0	-1	-1	2	3	-4
0	0	1	-4	-1	-1	0	0	-1	2	3	-4





Videos!





Observations on Perceptron

- Learning rate impacts the speed of learning
- Perceptron was unable to learn the XOR problem
- Perceptron learning rule convergence theorem
 - If the data is linearly separable, you can always use a perceptron algorithm to find a separating hyperplane



Feed Forward Backpropagation Neural Networks aka Multi-layer Perceptrons

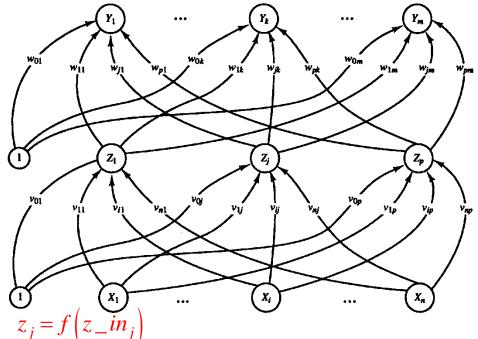
- Very General Nature
 - Applicable to a variety of practical problems
 - NETtalk
 - Signature Classification
 - Disease Classification
 - Hand Written Character Recognition
 - Combat Outcome Predication
 - Earthquake Prediction
 - Etc...
- Objective:
 - To achieve a balance between Memorization and Generalization
 - Memorization: Ability to respond correctly to the input patterns used for training
 - Generalization: Ability to give reasonable responses to input that is similar but not identical to that used in training





Architecture

- Consists of multiple layers
- Layers of units other than the input and output are called hidden units
- Unidirectional weight connections and biases
- Activation functions
 - Use of sigmoid functions
 - Nonlinear Operation: Ability to solve practical problems
 - Differentiable: Makes theoretical assessment easier
 - Derivative can be expressed in terms of functions themselves: Computational Efficiency
 - Activation function is the same for all neurons in the same layer
 - Input layer just passes on the signal without processing (linear operation)



$$z_{-}in_{j} = \sum_{i=0}^{n} x_{i}v_{ij}, \qquad x_{0}$$

=1, j=1...p

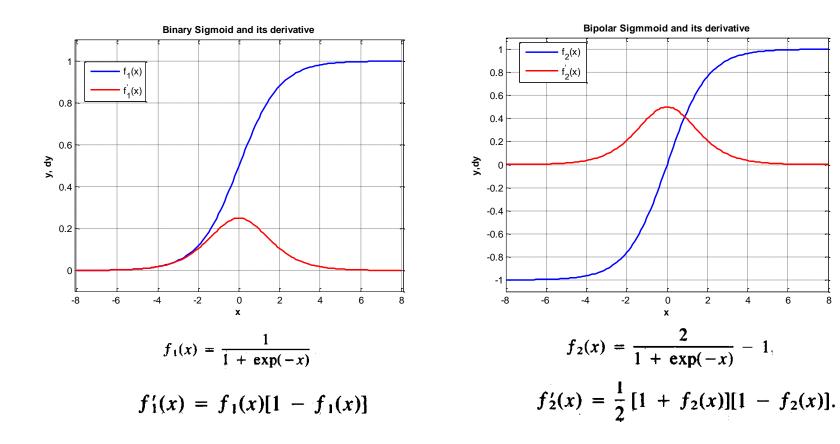
$$y_k = f(y_in_k)$$

 $y_in_k = \sum_{j=0}^p z_j w_{jk}, \qquad z_0 = 1, \qquad k = 1...m$
15





Architecture: Activation functions



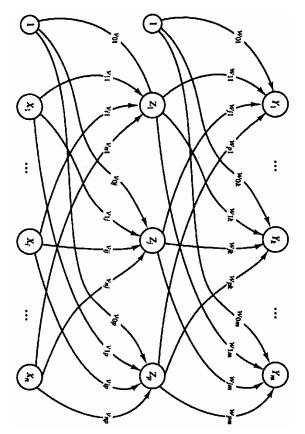
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Training

- During training we are presented with input patterns and their targets
- At the output layer we can compute the error between the targets and actual output and use it to compute weight updates through the Delta Rule
- But the Error cannot be calculated at the hidden input as their targets are not known
- Therefore we propagate the error at the output units to the hidden units to find the required weight changes (Backpropagation)
- 3 Stages
 - Feed-forward of the input training pattern
 - Calculation and Backpropagation of the associated error
 - Weight Adjustment
- Based on minimization of SSE (Sum of Square Errors)

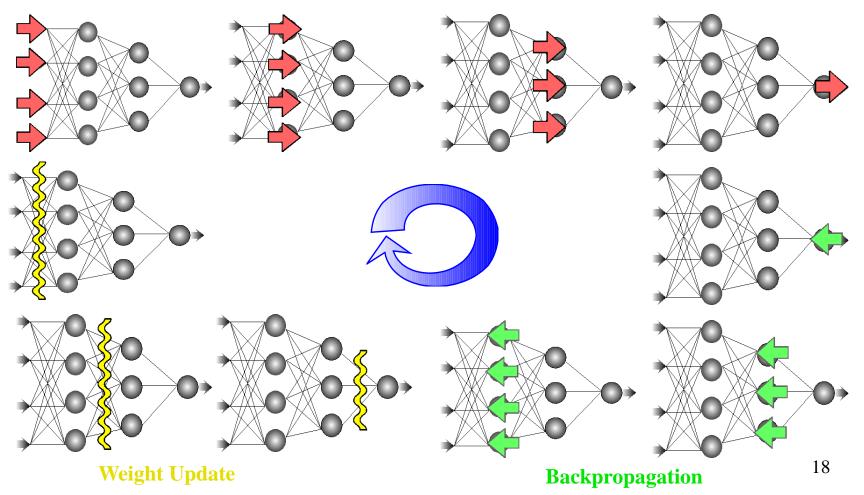






Backpropagation training cycle

Feed forward







Z,

х.

Zi

Change in W_{ik} affects only Y_{k}

Proof for the Learning Rule

$$E = .5 \sum_{k} [t_k - y_k]^2.$$

By use of the chain rule, we have

$$\frac{\partial E}{\partial w_{JK}} = \frac{\partial}{\partial w_{JK}} \cdot 5 \sum_{k} [t_{k} - y_{k}]^{2}$$

$$= \frac{\partial}{\partial w_{JK}} \cdot 5[t_{K} - f(y_{inK})]^{2}$$

$$= -[t_{K} - y_{K}] \frac{\partial}{\partial w_{JK}} f(y_{inK})$$

$$= -[t_{K} - y_{K}] f'(y_{inK}) \frac{\partial}{\partial w_{JK}} (y_{inK})$$

$$= -[t_{K} - y_{K}] f'(y_{inK}) \frac{\partial}{\partial w_{JK}} (y_{inK})$$

It is convenient to define δ_K :





Proof for the Learning Rule...

For weights on connections to the hidden unit Z_J : $\frac{\partial E}{\partial v_{kl}} = -\sum_{k} \left[t_k - y_k \right] \frac{\partial}{\partial v_{kl}} y_k \quad \Leftarrow$ WOr Wo. $= -\sum_{k} [t_{k} - y_{k}] f'(y_{ink}) \frac{\partial}{\partial v_{kl}} y_{ink}$ $= -\sum_{k} \delta_{k} \frac{\partial}{\partial v_{k}} y_{k} i n_{k}$ Zp $= -\sum \delta_k w_{Jk} \frac{\partial}{\partial v_{Jk}} z_J$ $= -\sum_{k} \delta_k w_{Jk} f'(z_{inJ})[x_I].$ Define: $\delta_J = \sum_k \delta_k w_{Jk} f'(z_i n_J)$ $\Delta v_{ij} = -\alpha \frac{\partial E}{\partial u_{ij}}$ Change in v_{ii} affects all Y_{1.m} Change in v_{ij} affects only z_i $= \alpha f'(z_{in_j}) x_i \sum_k \delta_k w_{jk},$ **Use of Gradient Descent Minimization** $= \alpha \delta_i x_i$. 20



Training Algorithm

Step 0. Initialize weights.

(Set to small random values).

- Step 1. While stopping condition is false, do Steps 2–9.
 - Step 2. For each training pair, do Steps 3–8. Feedforward:
 - Step 3. Each input unit $(X_i, i = 1, ..., n)$ receives input signal x_i and broadcasts this signal to all units in the layer above (the hidden units).

Step 4.

Each hidden unit $(Z_j, j = 1, ..., p)$ sums its weighted input signals,

$$z_{inj} = v_{0j} + \sum_{i=1}^{n} x_i v_{ij},$$

applies its activation function to compute its output signal,

$$z_j = f(\underline{z_in_j}),$$

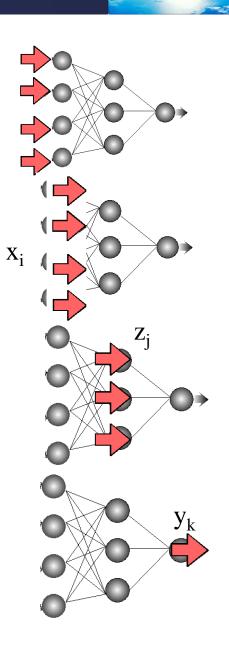
and sends this signal to all units in the layer above (output units).

Step 5. Each output unit $(Y_k, k = 1, ..., m)$ sums its weighted input signals,

$$y_{-in_k} = w_{0k} + \sum_{j=1}^{p} z_j w_{jk}$$

and applies its activation function to compute its output signal,

$$y_k = f(y_in_k).$$







Training Algorithm...

Backpropagation of error:

Step 6. Each output unit $(Y_k, k = 1, ..., m)$ receives a target pattern corresponding to the input training pattern, computes its error information term,

 $\delta_k = (t_k - y_k)f'(y_in_k),$

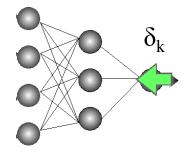
calculates its weight correction term (used to update w_{jk} later),

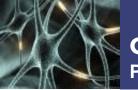
 $\Delta w_{jk} = \alpha \delta_k z_j,$

calculates its bias correction term (used to update w_{0k} later),

 $\Delta w_{0k} = \alpha \delta_k,$

and sends δ_k to units in the layer below.







Training Algorithm...

Step 7. Each hidden unit $(Z_j, j = 1, ..., p)$ sums its delta inputs (from units in the layer above),

$$\delta_{in_{j}} = \sum_{k=1}^{m} \delta_{k} w_{jk},$$

multiplies by the derivative of its activation function to calculate its error information term,

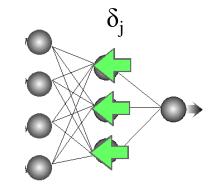
 $\delta_j = \delta_{in_j} f'(z_{in_j}),$

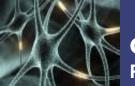
calculates its weight correction term (used to update v_{ij} later),

$$\Delta v_{ij} = \alpha \delta_j x_i,$$

and calculates its bias correction term (used to update v_{0j} later),

$$\Delta v_{0j} = \alpha \delta_j.$$







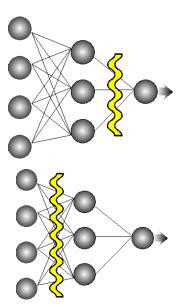
Training Algorithm...

Update weights and biases: Step 8. Each output unit $(Y_k, k = 1, ..., m)$ updates its bias and weights (j = 0, ..., p): $w_{jk}(\text{new}) = w_{jk}(\text{old}) + \Delta w_{jk}$. Each hidden unit $(Z_j, j = 1, ..., p)$ updates

its bias and weights (i = 0, ..., n):

 $v_{ij}(\text{new}) = v_{ij}(\text{old}) + \Delta v_{ij}.$

Step 9. Test stopping condition.

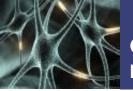






Effect of Learning Rate

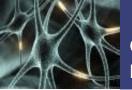
- Controls the change in synaptic weights
- The smaller the learning rate the smoother the trajectory in the weight space
 - Slower rate of learning
- If learning rate is made too large (for speedy convergence) the network may become unstable (oscillatory)





Stopping Criterion

- The Backpropagation (BP) algorithm cannot be shown to converge
 - No well defined criterion for stopping its operation
- Criterion Used
 - BP is considered to have converged when the Euclidean norm of the gradient vector reaches a sufficiently small threshold
 - Ideally Gradient is zero at the minima
 - Drawbacks
 - For successful trials, learning times may be large
 - Calculation of the gradient is required
 - BP is considered to have converged when the absolute rate of change in the average squared error per epoch is sufficiently small
 - May result in premature termination of the learning process



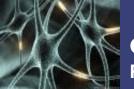


Application Procedure

Involves only the feed-forward phase (fast!)

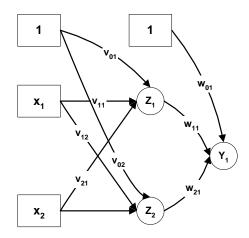
Step 0.Initialize weights (from training algorithm).Step 1.For each input vector, do Steps 2-4.
Step 2.Step 2.For $i = 1, \ldots, n$: set activation of input unit
 x_i ;Step 3.For $j = 1, \ldots, p$:
 $z_{-inj} = v_{0j} + \sum_{i=1}^{n} x_i v_{ij}$;
 $z_j = f(z_{-inj}).$ Step 4.For $k = 1, \ldots, m$:
 $y_{-in_k} = w_{0k} + \sum_{j=1}^{p} z_j w_{jk}$;

$$y_k = f(y_{in_k}).$$





- Initial weights
 - $v_{01} = -0.8$ $v_{11} = 0.5$ $v_{12} = 0.9$ $v_{02} = +0.1$ $v_{21} = 0.4$ $v_{22} = 1$ $w_{01} = -0.3$ $w_{11} = -1.2$ $w_{21} = 1.1$



$$z_{j} = f(z_{i}n_{j})$$

$$z_{i}n_{j} = \sum_{i=0}^{2} x_{i}v_{ij}, \quad x_{0} = 1, \quad j = 1...2$$

$$y_{k} = f(y_{i}n_{k})$$

$$y_{i}n_{k} = \sum_{j=0}^{2} z_{j}w_{jk}, \quad z_{0} = 1, \quad k = 1$$





- We consider the input pattern as (1,1) with target = 0 and α=0.1
- Feed Forward

$$z_{1} = sigmoid (V_{1}^{T}X) = 1/\left[1 + e^{-(1 \cdot 0.5 + 1 \cdot 0.4 - 1 \cdot 0.8)}\right] = 0.5250$$

$$z_{2} = sigmoid (V_{2}^{T}X) = 1/\left[1 + e^{-(1 \cdot 0.9 + 1 \cdot 1.0 + 1 \cdot 0.1)}\right] = 0.8808$$

$$y_{1} = sigmoid (W^{T}Z) = 1/\left[1 + e^{-(-0.5250 \cdot 1.2 + 0.8808 \cdot 1.1 - 1 \cdot 0.3)}\right] = 0.5097$$

Backpropagation

$$\begin{split} e &= t - y_1 = 0 - 0.5097 = -0.5097 \\ \delta_{k=1} &= y_1 \left(1 - y_1 \right) e = 0.5097 \cdot (1 - 0.5097) \cdot (-0.5097) = -0.1274 \\ \Delta w_{21} &= \alpha \cdot z_2 \cdot \delta_{k=1} = 0.1 \cdot 0.8808 \cdot (-0.1274) = -0.0112 \\ \Delta w_{11} &= \alpha \cdot z_1 \cdot \delta_{k=1} = 0.1 \cdot 0.5250 \cdot (-0.1274) = -0.0067 \\ \Delta w_{01} &= \alpha \cdot (1) \cdot \delta_{k=1} = 0.1 \cdot (+1) \cdot (-0.1274) = -0.0127 \end{split}$$

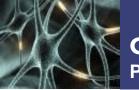




Backpropagation...

$$\begin{split} &\delta_{j=1} = z_1(1-z_1) \cdot \delta_1 \cdot w_{11} = 0.5250 \cdot (1-0.5250) \cdot (-0.1274) \cdot (-1.2) = 0.0381 \\ &\delta_{j=2} = z_2(1-z_2) \cdot \delta_1 \cdot w_{21} = 0.8808 \cdot (1-0.8808) \cdot (-0.1274) \cdot 1.1 = -0.0147 \end{split}$$

$$\begin{split} \Delta v_{11} &= \alpha \cdot x_1 \cdot \delta_{j=1} = 0.1 \cdot 1 \cdot 0.0381 = 0.0038\\ \Delta v_{21} &= \alpha \cdot x_2 \cdot \delta_{j=1} = 0.1 \cdot 1 \cdot 0.0381 = 0.0038\\ \Delta v_{01} &= \alpha \cdot (+1) \cdot \delta_{j=1} = 0.1 \cdot (+1) \cdot 0.0381 = +0.0038\\ \Delta v_{12} &= \alpha \cdot x_1 \cdot \delta_{j=2} = 0.1 \cdot 1 \cdot (-0.0147) = -0.0015\\ \Delta v_{22} &= \alpha \cdot x_2 \cdot \delta_{j=2} = 0.1 \cdot 1 \cdot (-0.0147) = -0.0015\\ \Delta v_{02} &= \alpha \cdot (+1) \cdot \delta_{j=2} = 0.1 \cdot (+1) \cdot (-0.0147) = -0.0015 \end{split}$$



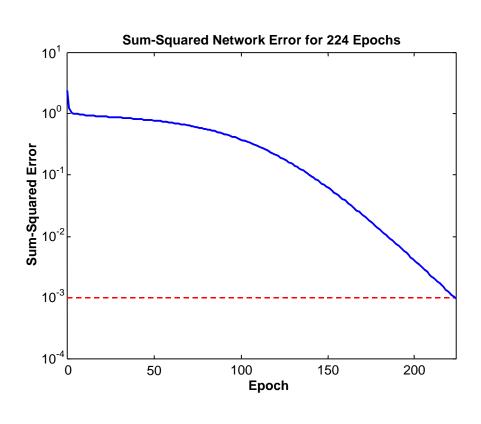


Weight Update

 $v_{01} = -0.7962$ $v_{11} = 0.5038$ $v_{12} = 0.8985$ $v_{02} = +0.0985$ $v_{21} = 0.4038$ $v_{22} = 0.9985$ $w_{01} = -0.3127$ $w_{11} = -1.2067$ $w_{21} = 1.0888$

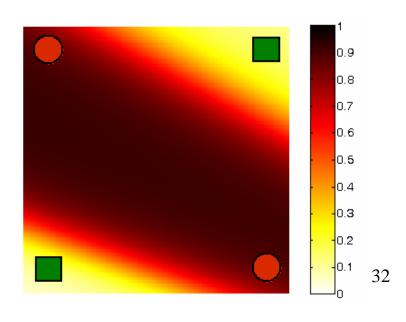
The training process is repeated until the sum of squared errors is less than 0.001.





See Video!

Ing	uts	Desired output	Actual output	Enter	Sum of squared
x_1	<i>x</i> ₂			е	errors
1	1	0	0.0155	-0.0155	0.0010
0	1	1	0.9849	0.0151	
1	0	1	0.9849	0.0151	
0	0	0	0.0175	-0.0175	







Number of hidden layers...

- Theoretically, One hidden layer is sufficient for a Backpropagation net to approximate any continuous mapping from the input patterns to the output pattern to an arbitrary degree of accuracy
- However two hidden layers may make training easier in some situations





Selecting parameters

- Architecture
 - Number of layers
 - Number of neurons in each layer
- Activation Function
- Learning rate
- Stopping criterion



Implementation

```
from keras.models import Sequential
1
   from keras.layers import Dense
\mathbf{2}
   import numpy
3
   seed = 7
4
   numpy.random.seed(seed)
5
   # Load the dataset
6
   dataset = numpy.loadtxt("pima-indians-diabetes.csv", delimiter=",")
7
   X = dataset[:,0:8]
8
  Y = dataset[:.8]
9
   # Define and Compile
10
   model = Sequential() # The network is not recurrent and has a sequence of layers
11
   model.add(Dense(12, input_dim=8, init='uniform', activation='relu'))
                                                                               # Number of layers,
12
   model.add(Dense(8, init='uniform', activation='relu'))
                                                                              neurons, activations &
13
                                                                                  weight init.
   model.add(Dense(1, init='uniform', activation='sigmoid'))
14
   model.compile(loss='binary_crossentropy', optimizer='adam', metrics=['accuracy'])
15
   # Fit the model
16
                                                                   # Loss function and optimization
   model.fit(X, Y, nb_epoch=150, batch_size=10)
17
   # Evaluate the model
18
   scores = model.evaluate(X, Y)
19
   print("%s: %.2f%%" % (model.metrics_names[1], scores[1]*100))
20
```

Doing all this in Keras

• Layers

```
model = Sequential()
model.add(Dense(32, input_shape=(500,)))
model.add(Dense(10, activation='softmax'))
model.compile(optimizer='rmsprop',
loss='categorical_crossentropy', metrics=['accuracy'])
```

Useful attributes of Model

model.layers: is a flattened list of the layers comprising the model graph.
model.inputs: is the list of input tensors
model.outputs: is the list of output tensors.

 Activations
 from keras.layers import Activation, Dense model.add(Dense(64))
 model.add(Activation('tanh'))

model.add(Dense(64, activation='tanh'))

- Available Activation
 - Softmax
 - Elu
 - Softplus
 - Softsign
 - Relu
 - Tanh
 - Sigmoid
 - Hard Sigmoid
 - Linear

• Losses

model.compile(loss='mean_squared_error', optimizer='sgd')

from keras import losses model.compile(loss=losses.mean_squared_error, optimizer='sgd')

- Available
 - Mean Squared Error
 - Mean Absolute Error
 - Mean Absolute Percentage Error
 - Mean Squared Logarithmic Error
 - Squared Hinge
 - Hinge
 - Categorical Cross Entropy
 - Sparse categorical crossentropy
 - Binary Crossentropy
 - Kullback Leibler Divergence
 - Posison
 - Cosine Proximity

- Metrics
 - Used to evaluate model performance

- Available
 - Binary Accuracy
 - Categorical Accuracy
 - Sparse Categorical Accuracy
 - Top K Categorical Accuracy
 - Custom

• Optimizers

```
from keras import optimizers
model = Sequential()
model.add(Dense(64, init='uniform', input_shape=(10,)) model.add(Activation('tanh'))
model.add(Activation('softmax'))
sgd = optimizers.SGD(lr=0.01, decay=1e-6, momentum=0.9, nesterov=True)
model.compile(loss='mean_squared_error', optimizer=sgd)
```

- Available
 - SGD
 - RMSprop
 - Adagrad
 - AdaDelta
 - Adam
 - Adamax
 - Nadam

Initializers

```
model.add(Dense(64,
       kernel_initializer='random_uniform',
       bias_initializer='zeros'))
```

Regularization

-L1 and L2

- Drop-Out
- Batch Normalization

- Data Augmentation
 - Noise Layer
 - ImageDataGenerator

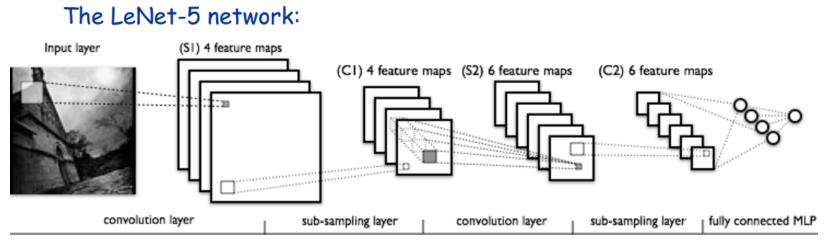
Class Exercise!

- Requires Keras based computers
- Solve the XOR using a single hidden layer BPNN with sigmoid activations
 - See what is the effect of different parameters on the convergence characteristics of the neural network





Applications



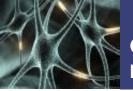
Important ideas:

Extract local features (local receptive fields) and merge them later to create global features

Local features that are useful in one region are likely to be useful elsewhere - weight sharing

http://yann.lecun.com/exdb/lenet/

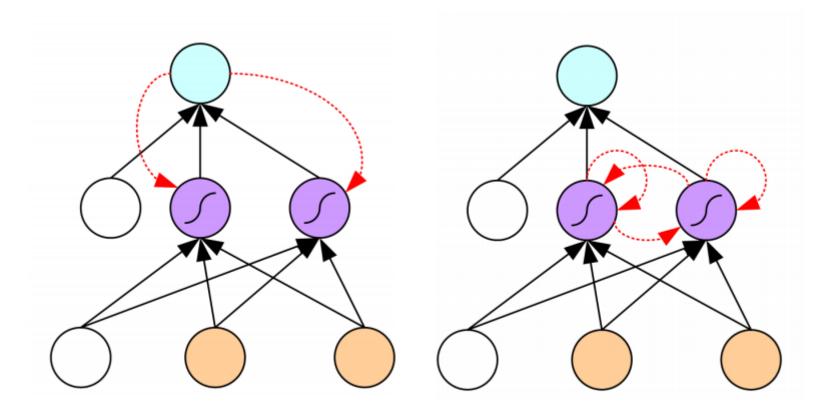
Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, november 1998.



CIS530: ARTIFICIAL INTELLIGENCE Pakistan Institute of Engineering and Applied Sciences (PIEAS).



Recurrent Networks



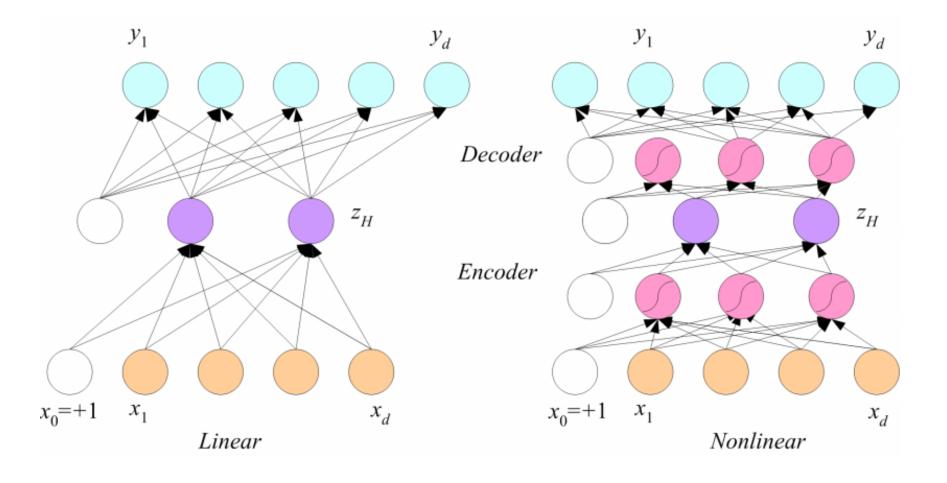
The cycles allow the network to exhibit dynamic temporal behavior

RNNs can use their internal memory to process arbitrary sequences of inputs.





Dimensionality Reduction

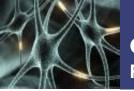






Deep learning

NNs can be used for feature learning





Reading

- Fundamentals of Neural Networks (Laurene Faucett)
 - Perceptron: Chapter 2
 - MLP: Chapter 6
- Neural Networks a comprehensive foundation (1999)