UP201: Introductory Physics III

IISc Bangalore

Semester I, 2017–2018

PROBLEM SET 7, FINISH BY: DEC. 9, 2017

Revision Problem Set

7/1. **Thermodynamics, Zeroth Law:**

- Revise the idea of temperature and equilibrium.
- Problems:

(a) A sheet of metal has the shape of an equilateral triangle of side a at temperature T_0. If the temperature is changed to $T_1 > T_0$, what will be the the length of the side of the triangle? You may assume that the coefficient of linear expansion of the metal is a temperature insensitive value α.

(b) A block of plastic (density ρ_p at room temperature T_r) of volume V floats in a beaker of water (density ρ_w at room temperature T_r).

i. How much water is displaced by the plastic block?

ii. Suppose that the temperature of the system (plastic block and water) is changed to $2T_r$. Assuming that the materials involved have coefficients of linear expansion α_p (plastic) and α_w (water) such that $\alpha_p = 2\alpha_w$, discuss if the volume of displaced water changes when the temperature changes to $2T_r$. If you conclude that the volume of the displaced water does change, then find the new value.

(c) The faculty hall of IISc can be approximated by a cuboid of volume V =25 meters $\times$ 25 meters $\times$ 15 meters. The pressure inside the hall if atmospheric pressure $P_a(= 10^5 N/m^2)$ and temperature is room temperature T_r $(= 25°C)$. Obtain a symbolic expression the number of gas molecules in the hall, stating any assumptions that you make. After you have obtained a symbolic expression for the above, obtain *an estimate* of its numerical value. (You may use the Boltzmann constant $k_B = 1.4 \times 10^{-23} m^2 kg\, s^{-2} K^{-1}$)

7/2. **Thermodynamics, First Law:**

- Revise the first law of thermodynamics
- Problems:

(a) An ideal gas is expanded from volume V_0 to $2V_0$ along the curve $PV^2 = K$ where $K > 0$ is a given constant. Find, the work done *on* the gas if the process is carried out in a nice reversible manner. Find the heat added *to* the gas in this process.

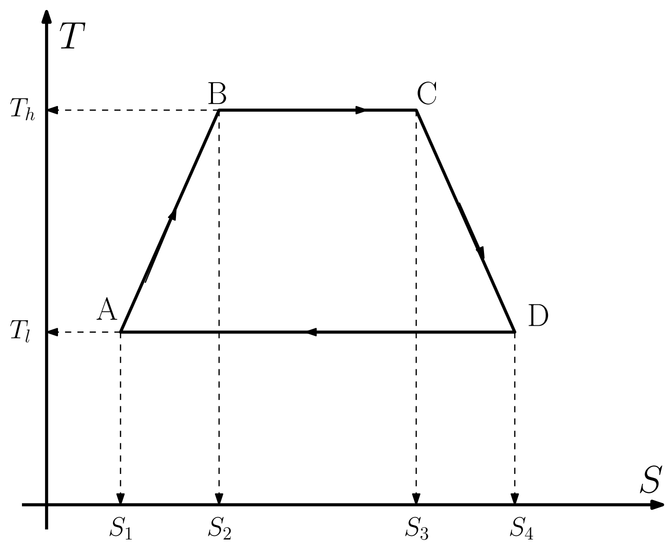
(b) A rod of length L, cross-section area A is made of three equal sections. The leftmost section is a metal with thermal conductivity κ_m, the middle section is a semi-metal of thermal conductivity κ_s and the right most section is made of an insulator with thermal conductivity κ_i. If the left end of the rod is kept at a temperature T_L and the right end at T_R, find the temperatures at the interface betweeen the metal and semi-metal, and at the interface between the semi-metal and the insulator.

7/3. **Thermodynamics, Second Law:**

- Revise the second law of thermodynamics.
- Problems:

(a) For problem 7/2a, find the change in entropy in the process.

(b) One mole of an ideal gas is taken through a cycle $A \to B \to C \to D \to A$ (shown in the $T-S$ diagram). The quantities T_h, T_l, S_1, S_2, S_3 and S_4 are known.



i. Find the heat added to and work done on the gas in each segment in terms of the known quantities.

ii. Find the efficiency of the engine.

7/4. **Kinetic Theory:**

- Revise the kinetic theory of gases.
- Problems:

(a) Referring to problem 7/1c, find the number of molecules hitting the floor surface of the faculty hall per unit area per unit time.

7/5. **Quantum Mechanics, "Old Theory":**

- Revise semiclassical quantization ideas.
- Problems:

(a) Consider the potential

$$V(x) = \begin{cases} \infty, & |x| \geq 2a \\ |x| - a, & a \leq |x| \leq 2a \\ 0, & |x| < a \end{cases}$$

in which a particle of mass m moves. Find the quantized levels of this system using the Bohr-Sommerfeld theory.

7/6. **Quantum Mechanics:**

- Revise the ideas of state, wavefunction, experiment, uncertainty, Schrödinger equation.
- Problems:
  (a) Consider a particle of mass m in a one dimensional box of length L.
    i. Find the energy eigenvalues and eigenstates (you must normalize the eigenstates) and label them E_n and ϕ_n (you must find expressions for E_n and $\phi_n(x)$).
    ii. The particle is in a state $\psi = \frac{3}{5}\phi_1 + \frac{4}{5}\phi_2$.
      A. What is the probability of finding the particle within dx of $x = L/2$ in a position measurement?
      B. What is the expectation value of the position? What is the uncertainty in the position?
    iii. If the particle is in the state ψ at time $t = 0$, what is the state of the particle at an arbitrary time t?
    iv. Obtain the same quantities as in part 7/6(a)ii at time t.

7/7. **Relativity:**

- Revise Lorentz transformations, time dilation, length contraction, proper time, proper velocity, relativistic momentum and energy. Revise the collision thought experiment of Einstein.
- Problems:
  (a) A laser pointer emits a photon of wavelength λ along the x axis. For the observer holding the pointer, what is p^μ of the photon?
  (b) What is p^μ for an electron moving with a (Galielain) velicity w?
  (c) Read section 40.3 of SJ and completely work out the physics of Compton shift.

7/8. **General Relativity:**

- Read section 39.10 of SJ.
- Problems:
  (a) One of key achievements of Newton was to show that the gravitational field of a sphere of radius R of mass M and uniform density is same as that of a *point mass* M when one is "outside" the sphere.
    i. Consider the mass M to be a point mass. Suppose a "test" particle is at a distance r from M. Find the escape velocity of the test particle. Note a nice result – escape velocity does not depend on the mass of the test particle.
    ii. Consider, again, the mass M to be a point mass. At what value of r (call it R_S) does the escape velocity become the speed of light c? For a given M, R_S is called the Schwartzchild radius, and depends only on M, G and c (the last two are "fundamental" constants).
    iii. Now consider M to be a sphere with such high a density that $R < R_S$! Argue that even light cannot escape from the surface of such a compact massive object! This is your first "view" of a black hole.
    iv. Compute numerical values R_S for the earth and the Sun and compare them with their respective radii.

7/9. **: Astrophysics and Cosmology**

- Briefly review the notes posted on the website.
- Problems:
  (a) Here is a bit of an upscale problem: Let us try to make a model of a white dwarf in one spatial dimension. Consider a "star" of mass M occupying a region between $-L/2 \le x \le L/2$ with a uniform density $\rho = M/L$. Note that M is fixed, and we want to find what is the "best" L for this mass M.
    i. We know that gravitational potential goes as $1/r$ in 3 dimensions. What will be the natural form of gravitational potential in 1 dimension? Pay careful attention to the dimensions of the gravitational constant G in one dimension (which could be different from 3D). We will, of course, retain mass to have the same units. (*Hint:* Think of a Gauss law for gravitation.)
    ii. Obtain an expression for the total gravitational potential energy of the star. Your answer will depend on M, L and G (the 1D gravitational constant).
    iii. Now assume that the star is made of identical fermions each of mass m. (Total number of fermions, therefore, is equal to $N = M/m$.) Find an expression for the ground state energy. You answer will depend on N, L, m and $\hbar$.
    iv. For a given M, what is the equilibrium value of L?

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