HW 2

Due Friday, September 15, 3pm

Section 1.3

Exercises 2, 6, 8, 10, 14, (19), (21), 24, 26

(Numbers in parentheses are recommended exercises for which you need not submit solutions.)

1.3.2. Compute $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-2\mathbf{v}$, where $\mathbf{u}=\begin{bmatrix}3\\2\end{bmatrix}$ and $\mathbf{v}=\begin{bmatrix}2\\-1\end{bmatrix}$.

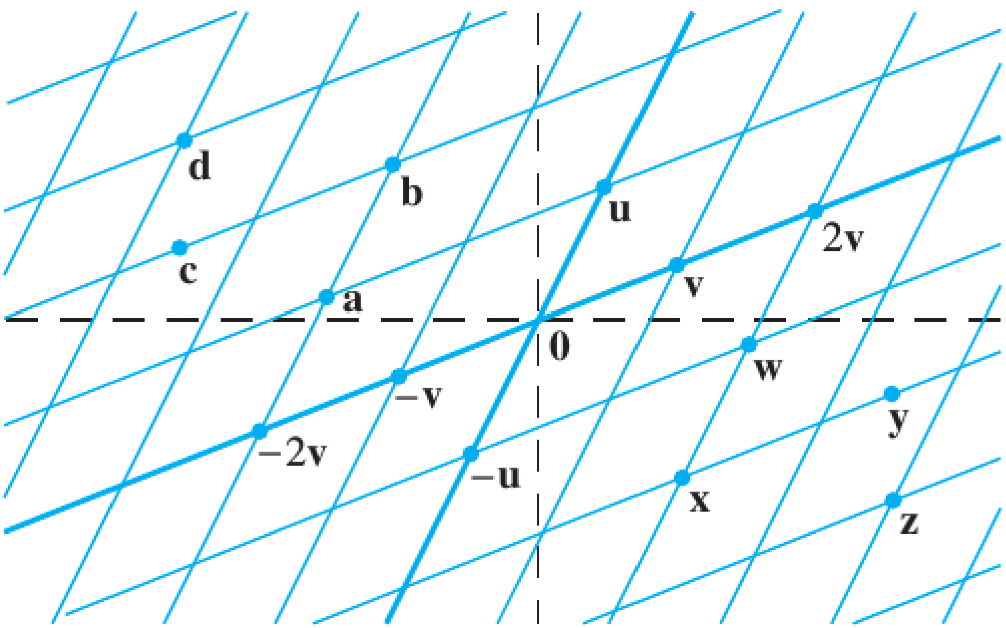
1.3.6. Write a system of equations that is equivalent to the vector equation,

$$x_1\begin{bmatrix}-2\\3\end{bmatrix}+x_2\begin{bmatrix}8\\5\end{bmatrix}+x_3\begin{bmatrix}1\\-6\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

1.3.8. Answer parts (a) and (b).

(a) Using the accompanying figure, write each of the vectors $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

(b) Is every vector in $\mathbb{R}^2$ a linear combination of $\mathbf{u}$ and $\mathbf{v}$?



1.3.10. Write a vector equation that is equivalent to the following system of equations:

$$
\begin{aligned}
4x_1 + x_2 + 3x_3 &= 9 \\
x_1 - 7x_2 - 2x_3 &= 2 \\
8x_1 + 6x_2 - 5x_3 &= 15
\end{aligned}
$$

1.3.14. Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix A, where

$$
A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}.
$$

(1.3.19) Give a geometric description of $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ for the vectors

$$
\mathbf{v}_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix} \text{ and } \mathbf{v}_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}.
$$

(1.3.21) Let $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ for all h and k.

1.3.24. Mark each statement True or False. Justify each answer.

a. Any list of five real numbers is a vector in $\mathbb{R}^5$.

b. The vector $\mathbf{u}$ results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector $\mathbf{v}$.

c. The weights $c_1, \ldots, c_p$ in a linear combination $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p$ cannot all be zero.

d. When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains the line passing through $\mathbf{u}$ and the origin.

e. Asking whether the linear system corresponding to an augmented matrix $[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{b}]$ has a solution amounts to asking whether $\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

1.3.26.

Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A. Answer (a) and (b).

(a) Is $\mathbf{b}$ in W?

(b) Show that the third column of A is in W.

Section 1.4

Exercises 2, 10, (13), 14, 18, 20, 2 2, (27), (29), (31), 32

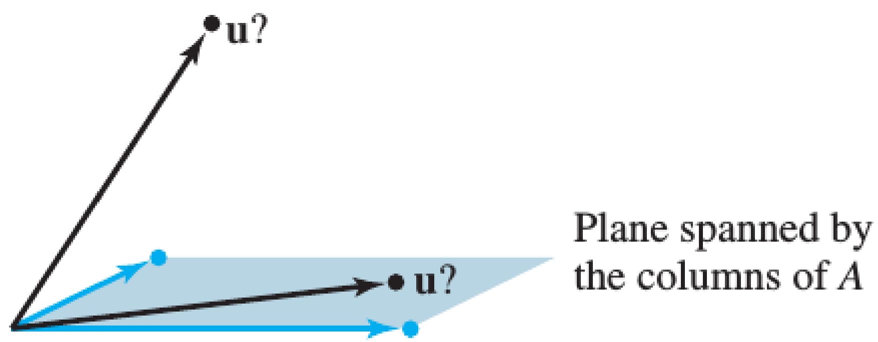
(Numbers in parentheses are recommended exercises for which you need not submit solutions.)

1.4.2. Compute the product $\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$. If the product is undefined, explain why.

1.4.10. Write the following system first as a vector equation and then as a matrix equation:

$$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 - 3x_2 &= 2 \end{aligned}$$

(1.4.13) Let $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is $\mathbf{u}$ in the plane spanned by the columns of A? (See the figure below.) Why or why not?



Where is **u**?

1.4.14. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the plane spanned by the columns of A? (See the figure above.) Why or why not?

Exercises 18 and 20 refer to the matrix B below. Make appropriate calculations that justify your answers and mention an appropriate theorem.

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

1.4.18. Do the columns of B span $\mathbb{R}^4$? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for each $\mathbf{y}$ in $\mathbb{R}^4$?

1.4.20. Can every vector in $\mathbb{R}^4$ be written as a linear combination of the columns of the matrix B above? Do the columns of B span $\mathbb{R}^3$?

1.4.22. Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}.$

Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^3$? Why or why not?

(1.4.27) Let $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$, and $\mathbf{v}$ be vectors in $\mathbb{R}^5$, and let x_1, x_2, and x_3 denote scalars. Write the vector equation

$$x_1\mathbf{q}_1 + x_2\mathbf{q}_2 + x_3\mathbf{q}_3 = \mathbf{v}$$

as a matrix equation. Identify any symbols you choose to use.

(1.4.29) Construct a 3×3 matrix, not in echelon form, whose columns span $\mathbb{R}^3$. Show that the matrix you construct has the desired property.

(1.4.31) Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^3$. Generalize your argument to the case of an arbitrary A with more rows than columns.

1.4.32. Could a set of three vectors in $\mathbb{R}_4$ span all of $\mathbb{R}_4$? Explain. What about n vectors in $\mathbb{R}_m$ when n is less than m?