HW 3

Due Friday, September 22, 3pm

Section 1.5: 6, (14), 16, (22), 24, 26, 34;
Section 1.7: 2, 16, 18, (20), 22, (28), (30), 34, (36), (38);
Section 1.8: 6, 10, 12, 17, 22.

Section 1.5

Exercises 6, (14), 16, 22, 24, 26, 3 4, 38

1.5.6. Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 0 \\ x_1 + 4x_2 - 8x_3 &= 0 \\ -3x_1 - 7x_2 + 9x_3 &= 0 \end{aligned}$$

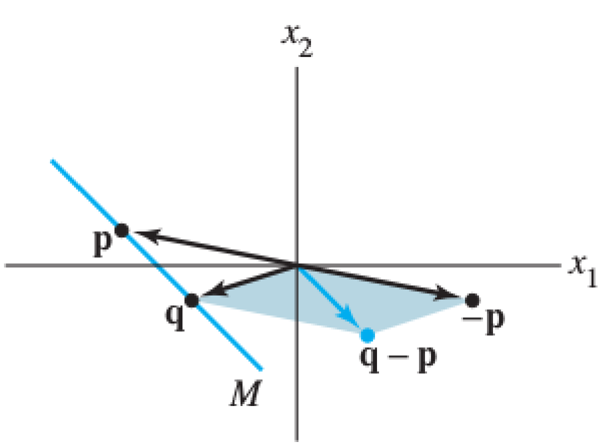
1.5.16. Describe the solutions of the following system in parametric vector form, and provide a geometric comparison with the solution set in Exercise 6.

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6 \end{aligned}$$

1.5.22. (Recommended, not required.) Find a parametric equation of the line M through the vectors

$$\mathbf{p} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{q} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

[Hint: M is parallel to the vector $\mathbf{qp}$. See the figure below.]



The line through **p** and **q**.

1.5.24. Mark each statement True or False. Justify each answer on the basis of a careful reading of the text.

a. If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero.

b. The equation $\mathbf{x} = x_2\mathbf{u} + x_3\mathbf{v}$, with x_2 and x_3 free (and neither $\mathbf{u}$ nor $\mathbf{v}$ a multiple of the other), describes a plane through the origin.

c. The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

d. The effect of adding $\mathbf{p}$ to a vector is to move the vector in a direction parallel to $\mathbf{p}$.

1.5.26. Suppose $A\mathbf{x} = \mathbf{b}$ has a solution. Explain why the solution is unique precisely when $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

1.5.34. Given $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, find one nontrivial solution of $A\mathbf{x} = \mathbf{0}$ by inspection.

[Hint: you should not have to reduce the matrix using elementary row operations; you can just “see” an answer. This is what “by inspection” means.]

Section 1.7

Exercises 2, 16, 18, (20), 22, (28) , (30), 34, (36), (38)

1.7.2. Determine if the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

1.7.16. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

1.7.18. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

1.7.20. (Recommended, not required.) Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.7.22. Mark each statement True or False. Justify each answer on the basis of a careful reading of the text.

a. Two vectors are linearly dependent if and only if they lie on a line through the origin.

b. If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

c. If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in $\mathrm{Span}\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.

d. If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector.

1.7.28. (Recommended, not required.) How many pivot columns must a 5×7 matrix have if its columns span $\mathbb{R}^5$? Why?

1.7.30. (Recommended, not required.)

a. Fill in the blank in the following statement: “If A is an $m \times n$ matrix, then the columns of A are linearly independent if and only if A has ___ pivot columns.”"

b. Explain why the statement in **a** is true.

Each statement in Exercises 34–38 is either true (in all cases) or false (for at least one example). If false, construct a specific example to show that the statement is not always true. (Such an example is called a counterexample to the statement.) If a statement is true, give a justification. (One specific example cannot explain why a statement is always true.)

1.7.34. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are in $\mathbb{R}^4$ and $\mathbf{v}_3 = \mathbf{0}$ then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

1.7.36. (Recommended, not required.) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are in $\mathbb{R}^4$ and $\mathbf{v}_3$ is *not* a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

1.7.38. (Recommended, not required.) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly independent vectors in $\mathbb{R}^4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly independent.

Section 1.8

Exercises 6, 10, 12, 17, 22

1.8.6. Let

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -4 & 5 \\ 0 & 1 & 1 \\ -3 & 5 & -4 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 1 \\ 9 \\ 3 \\ -6 \end{bmatrix}$$

Let T be defined by $T(\mathbf{x}) = A\mathbf{x}$. Find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

1.8.10. Let $A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$. Find all $\mathbf{x}$ in $\mathbb{R}^4$ that are mapped into the zero vector by the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

1.8.12. Let $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$, and let A be the matrix in Exercise 10. Is $\mathbf{b}$ in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Why or why not?

1.8.17. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ into $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and maps $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$. Use the fact that T is linear to find the images under T of $3\mathbf{u}$, $2\mathbf{v}$, and $3\mathbf{u} + 2\mathbf{v}$.

1.8.22. Mark each statement True or False. Justify each answer.

a. Every matrix transformation is a linear transformation.

b. The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.

c. If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ in the range of T?”

d. A linear transformation preserves the operations of vector addition and scalar multiplication.

e. The superposition principle is a physical description of a linear transformation.
