EXAM

Exam 1, Version 2

Math 2360, Fall 2016

Oct 2ff, 2016

- Write all of your answers on separate sheets of paper. Do not write on the exam handout. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **265 points total**.

Good luck!

50 pts. **Problem 1**. In each part, determine if the matrix operation is possible. If it is undefined, say "undefined," otherwise give the result of the operation.

A.

$$3\begin{bmatrix}-1\\3\end{bmatrix}$$

B.

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\0 & 1\\7 & -1\end{bmatrix}$$

C.

$$\begin{bmatrix}2 & 2\\5 & -2\end{bmatrix}-\begin{bmatrix}-5 & 3 & 4\\-6 & -2 & 5\end{bmatrix}$$

D.

$$\begin{bmatrix}2 & 1 & 0\\5 & -1 & -5\end{bmatrix}+\begin{bmatrix}1 & 0 & 2\\3 & 4 & -5\end{bmatrix}$$

E.

$$\begin{bmatrix}3 & -1\end{bmatrix}\begin{bmatrix}2 & 0 & -2 & 1\\-2 & -1 & 4 & 3\end{bmatrix}$$

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50 pts. **Problem 2**. In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in reduced row echelon form. Determine if the system is consistent and, if it is consistent, find all solutions.

A.

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & -1\\0 & 1 & 0 & 2\\0 & 0 & 1 & 3\\0 & 0 & 0 & 5\end{array}\right]$$

B.

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & 5\\0 & 1 & 0 & 7\\0 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

C.

$$\left[\begin{array}{ccccc|c}1 & 0 & 0 & -1 & 4 & 2\\0 & 1 & 0 & -2 & -1 & 1\\0 & 0 & 1 & 3 & 1 & 5\\0 & 0 & 0 & 0 & 0 & 0\end{array}\right]$$

40 pts.

Problem 3. In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in row echelon form. (Not *reduced* row echelon form!) Determine if the system is consistent. If so, use **back substitution** to find all the solutions of the system.

A.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

B.

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 1 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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40 pts.

Problem 4. Solve the linear system below. Write down the augmented matrix of the system. Use row operations to reduce the coefficient matrix to RREF. Find all solutions of the system.

Perform the row operations one by one by hand, indicating for each operation what the operation is, what matrix you start with, and what matrix you get.

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= -5 \\ -x_1 - x_2 - x_3 &= 1 \\ x_1 + 2x_2 + 3x_3 &= -4 \end{aligned}$$

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40 pts.

Problem 5. In each part, use row operations to determine if the matrix A is invertible and, if so, to find the inverse. It is not necessary to show the individual row operations (you can just use the `rref` key on the calculator). Show the augmented matrix you start with and the augmented matrix you finish with. Give the matrix entries in fractional form e.g., 1/2 not 0.5.

Be sure to state your conclusion clearly and to give the value of A^{-1} (i.e., $A^{-1} =$ something).

A.

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

B.

$$\begin{bmatrix} 6 & 5 & -16 \\ 1 & 1 & -3 \\ 5 & 4 & -13 \end{bmatrix}$$

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45 pts.

Problem 6. Consider row operations on matrices with 3 rows.

Recall that for each row operation there is a corresponding elementary matrix E so that EA is the same as the matrix obtained by applying the row operation to A.

A. Consider the row operation $R_2 \leftrightarrow R_3$.

- i.) Find the corresponding elementary matrix E.
- ii.) Find the inverse row operation.
- iii.) Find the elementary matrix that corresponds to the inverse row operation.

B. Consider the row operation $R_1 \leftarrow 5R_1$.

- i.) Find the corresponding elementary matrix E.
- ii.) Find the inverse row operation.
- iii.) Find the elementary matrix that corresponds to the inverse row operation.

C. Consider the row operation $R_1 \leftarrow R_1 + 2R_2$

- i.) Find the corresponding elementary matrix E.
- ii.) Find the inverse row operation.
- iii.) Find the elementary matrix that corresponds to the inverse row operation.
