PH 201 – CLASSICAL MECHANICS (Aug. – Dec. 2017)
Problem sheet – 4

1. Determine the principal moment of inertia for the following homogeneous bodies:
   (a) a thin rod of length l
   (b) a sphere of radius R
   (c) a circular cylinder of radius R and height h
   (d) a rectangular parallelepiped of sides a, b and c
   (e) a circular cone of height h and base radius R
   (f) an ellipsoid of semiaxes a, b and c.

2. Calculate the moment of inertia tensor I for a solid cube of mass M and side length L, with the coordinate axes parallel to the edges of the cube, and the origin at a corner, as shown in Fig.1.

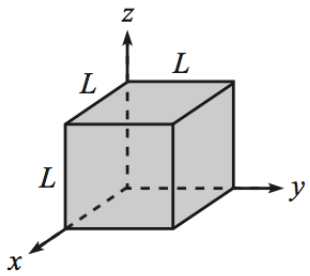

Figure 1: Figure for problem 2

3. Calculate the ratio of the height to the radius of a cylinder such that every axis is a principal axis for the cases:
   (a) the origin is the CM
   (b) the origin is the centre of one of the circular faces

4. Find the kinetic energy of a homogeneous cone of mass m, height h and vertex angle 2α
   (a) rolling on a plane (see Fig. 42 of Mechanics by Landau & Lifshitz)
   (b) whose base rolls on a plane and whose vertex is fixed at a height above the plane equal to the radius of the base so that the axis of the cone is parallel to the plane (see Fig. 43 of Mechanics by Landau & Lifshitz)

5. Given that, the earth is not a spherical top but a flattened rotation ellipsoid with half-axes given by $a = b = 6378$ km (equator) and $c = 6357$ km (pole). Show that the time period of the nutations of earth is
$$T = \frac{2\pi}{\Omega} \simeq 304\,\text{days}$$
where Ω is the angular velocity of the nutations of earth.

   NOTE: For the case of earth, you would often find in textbooks two different kinds of precessions discussed. One is the precession of earth's spin axis with a very long time period often termed as

the precession of equinox which occurs due to the tidal effects induced by the moon and the sun. It has been known since early days of astronomy and its time period is about 26 thousand years. The other is the periodic perturbation of the earth's spin axis, called as nutation, which is much smaller in amplitude than precession and is often regarded as associated motion of precession. In many different contexts, both are often called precessions but the latter is actually nutation of earth. Both precession and nutation are two different types of motions of the earth's spin axis. The measured nutation period of earth, known as *Chandler period*, is about 433 days. Some further details about earth's nutations are available here.

6. A neutron star is a very compact spherical star whose surface vibrates slowly so that the principal moments of inertia are harmonic functions of time, given by

$$I_{zz} = \frac{2}{5}mr^2(1 + \epsilon\cos\omega t),$$

$$I_{xx} = I_{yy} = \frac{2}{5}mr^2\left(1 - \frac{\epsilon}{2}\cos\omega t\right), \quad \epsilon \ll 1.$$

The star also simultaneously rotates with an angular frequency $\Omega(t)$.

(a) show that the z-component of Ω remains nearly constant.

(b) show that $\Omega(t)$ nutates about the z-axis and determine the nutation frequency for $\Omega_z \gg \omega$.

NOTE: In this case, the motion is again nutations which arises due to the very weak harmonic time dependence of the principal moments of inertia of the neutron star. You can check that the frequency of nutations goes to zero in the limit $\epsilon \to 0$, i.e. nutations vanish in this limit.