These lecture notes include some material from Professors Bertossi, Kolaitis, Guagliardo, Vardi, Libkin, Barland, McMahan

Deductive Databases

Lecture Handout 10

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Datalog rules (1)

$$\underbrace{H(\bar{x})}_{\text{head}} :- \underbrace{S_1(\bar{x}_1), \ldots, S_n(\bar{x}_n)}_{\text{body}}$$

Head predicate over variables and constants

Body conjunction of possibly negated atoms (**subgoals**)

- relational atoms (predicates)
- comparisons between variables/constants

- Head variables are implicitly **universally quantified**
- Body variables not in head are **existentially quantified**
- Rule: Body $\rightarrow$ Head (if the body is true, the head is true)

Datalog rules (2)

Example

$$\text{Lucky}(x) \ {:}{-}\ \text{Customer}(x,y,z), \text{Account}(u,z,x,w), w > 10000$$

In relational calculus we could write:

$$\text{Lucky} = \{\, x \mid \exists y,z,u,w\ \text{Customer}(x,y,z) \wedge \text{Account}(u,z,x,w) \wedge w > 10000 \,\}$$

Safety

Every variable (in head or body)
appears in at least one non-negated relational atom

Not safe: $\text{BigNumber}(x) \ {:}{-}\ x > 1000000000$

Datalog programs

Program = set of Datalog rules

Example

$$\text{Parent}(x,y) \ {:}{-}\ \text{Mother}(x,y)$$
$$\text{Parent}(x,y) \ {:}{-}\ \text{Father}(x,y)$$

edb (extensional database) relations **stored** in the database

- can appear only in the body of rules

idb (intensional database) derived relations

- can appear both in the head or body of rules

From relational algebra to Datalog

Every relational algebra expression can be translated into Datalog

Projection $\pi_{\#2}(R)$

$$E(x) \,:\!-\, R(y,x)$$

Selection $\sigma_{\#1\,\textbf{op}\,c}(R)$

$$E(x,y) \,:\!-\, R(x,y), x\;\textbf{op}\;c$$

Product $R \times S$

$$E(x,y,w,z) \,:\!-\, R(x,y), S(w,z)$$

Difference $R - S$

$$E(x,y) \,:\!-\, R(x,y), \neg S(x,y)$$

Union $R \cup S$

$$E(x,y) \,:\!-\, R(x,y)$$
$$E(x,y) \,:\!-\, S(x,y)$$

From relational algebra to Datalog

Let R and S be relations over A, B

$$E = \underbrace{\pi_{A,D}\Big(\underbrace{\sigma_{B=C}\big(\underbrace{R \times \rho_{A\to C,\,B\to D}(S)}_{E_1}\big)}_{E_2}\Big)}_{E_3} \cup \underbrace{\rho_{B\to D}(R - S)}_{E_4}$$

$$\begin{aligned}
E_1(x,y,w,z) &\,:\!-\, R(x,y), S(w,z)\\
E_2(x,y,w,z) &\,:\!-\, E_1(x,y,w,z), y = w\\
E(x,y) &\,:\!-\, E_2(x,u,v,y)\\
E(x,y) &\,:\!-\, R(x,y), \neg S(x,y)
\end{aligned}$$

Limitations of relational algebra/calculus

Parent = table of pairs x, y where x is the parent of y

$$
\begin{aligned}
\text{Parent} &= \{x, y \mid \text{Parent}(x, y)\} \\
\text{Grandparent} &= \{x, y \mid \exists z\ \text{Parent}(x, z) \wedge \text{Parent}(z, y)\} \\
\text{Great-grandparent} &= \{x, y \mid \exists z\ \text{Grandparent}(x, z) \wedge \text{Parent}(z, y)\}
\end{aligned}
$$

For a **given** k, we can express the query Ancestor^k

But
We **cannot express** the Ancestor relation itself
that is: an Ancestor^k query that works for **every** k

Recursion in Datalog

The head relation of a rule can appear in its body

$$
\begin{aligned}
\text{Ancestor}(x, y) &:- \text{Parent}(x, y) \\
\text{Ancestor}(x, y) &:- \text{Ancestor}(x, z), \text{Parent}(z, y)
\end{aligned}
$$

Intuition
x is an ancestor of y if

x is a parent of y

or

x is an ancestor of a parent of y

Dependency graph

IDB predicate P **depends on** (IDB) predicate Q
if there is a rule with P in the head and Q in a subgoal

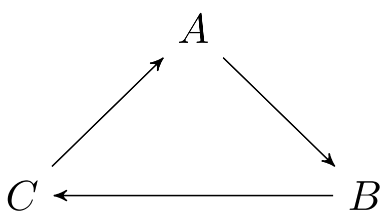
Dependency graph

nodes IDB predicates

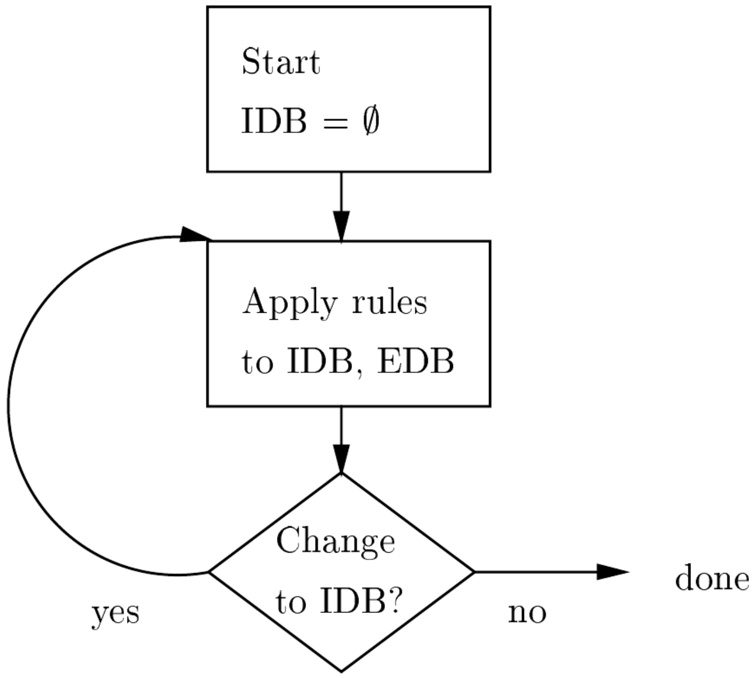
edges $P \to Q$ if P depends on Q

A **cycle** in the dependency graph means the program is recursive

$$
\begin{aligned}
C(x) &:- A(y, x) \\
C(x) &:- S(x, y), y > 1 \\
B(x, y) &:- C(x), P(x, y) \\
A(x, y) &:- B(y, x)
\end{aligned}
$$



Iterative Fixpoint Evaluation



Evaluation of recursive programs

- The **Parent** relation (EDB) never changes
- The **Ancestor** relation (IDB) is initially empty

  $\text{Ancestor}_0 = \varnothing$

- At step $i+1$ compute:

  $\text{Ancestor}_{i+1}(x, y) \text{ :- } \text{Parent}(x, y)$
  $\text{Ancestor}_{i+1}(x, y) \text{ :- } \text{Ancestor}_i(x, z), \text{Parent}(z, y)$

- Stop when a **fixpoint** is reached

  $\text{Ancestor}_{i+1} = \text{Ancestor}_i$

Evaluation of recursive programs

IDB:

Ancestor	
John	Mary
John	Jane
Jane	Louis
Mary	Linda
Louis	Mark
John	Linda
John	Louis
Jane	Mark
John	Mark

EDB:

Parent	
John	Mary
John	Jane
Jane	Louis
Mary	Linda
Louis	Mark

$\text{Ancestor}(x, y) \text{ :- } \text{Parent}(x, y)$

$\text{Ancestor}(x, y) \text{ :- } \text{Ancestor}(x, z), \text{Parent}(z, y)$

Recursion in SQL

Suppose we have a table Parent with attributes name, child

```
WITH RECURSIVE Ancestor(name, descendant) AS (
    SELECT *
    FROM   Parent
    UNION
    SELECT A.name, P.child
    FROM   Ancestor A, Parent P
    WHERE  A.descendant = P.name
)
SELECT * FROM Ancestor ;
```

The definition mimics the structure of the Datalog program

Nonlinear recursion

The head relation can appear **more than once** in its body

$$\text{Ancestor}(x, y) :- \text{Parent}(x, y)$$
$$\text{Ancestor}(x, y) :- \text{Ancestor}(x, z), \text{Ancestor}(z, y)$$

Intuition

x is an ancestor of y if

x is a parent of y

or

x is an **ancestor of an ancestor** of y

Nonlinear recursion in SQL

Not supported

```
WITH RECURSIVE Ancestor(name, descendant) AS (
    SELECT *
    FROM   Parent
    UNION
    SELECT A.name, A2.desc
    FROM   Ancestor A1, Ancestor A2
    WHERE  A1.descendant = A2.name
)
SELECT * FROM Ancestor ;
```

```
ERROR: recursive reference to query "ancestor" must not appear
more than once
```

Recursive programs with negation

Consider the program $P = \{R(x) :- S(x), \neg R(x)\}$

IDB:

R
1
2

EDB:

S
1
2

Step 0 IDB relation R is empty

Step 1 $R = \{1, 2\}$

Step 2 $R = \varnothing$

Step 3 $R = \{1, 2\}$

Step 4 $R = \varnothing$

… **No fixpoint!**

Iteration never ends

Stratification

Partition a program P into a sequence of subprograms $P_1, \ldots, P_n$

- Each subprogram defines one or more IDB relations
- If a relation S is used positively in the definition of R then S must be defined **earlier or simultaneously** with R
- If a relation S is used negatively in the definition of R then S must be defined **strictly before** R

Stratum 0 EDB relations

Stratum i IDB relations that depend
positively on relations in any stratum $j \leq i$
negatively on relations in any stratum $j < i$

Stratification

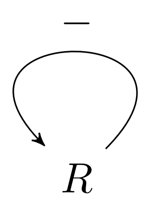
Stratum graph

nodes IDB predicates

edges $P \to Q$ if P depends on Q
label the edge with "–" if Q is a negated subgoal

Stratified program: no cycle involving at least one negated edge

$$R(x) :\!- \; S(x), \neg R(x)$$



More examples

$$\begin{tikzcd} A \arrow[r, bend left] & B \arrow[l, bend left] & C \arrow[l, "-"'] \end{tikzcd}$$

Stratified: $A = B < C$

$$\begin{tikzcd} & A \arrow[dl, "-"'] \arrow[dr, "-"] & \\ B \arrow[rr] & & C \end{tikzcd}$$

Stratified: $C \leq B < A$

$$\begin{tikzcd} A \arrow[rr, bend left, "-"] & B \arrow[l] & C \arrow[l] \end{tikzcd}$$

Not stratified: $A \leq B \leq C < A$

$$\begin{tikzcd} A \arrow[rr, bend left] & & B \arrow[ll, "-"] \arrow[dl, "-"] \\ & C \arrow[ul, "-"] & \end{tikzcd}$$

Not stratified: $A < B \leq A$

Stratified example

Which target nodes cannot be reached from any source node?

$$\begin{aligned} \text{NoReach}(x) &:- \text{Target}(x), \neg\text{Reach}(x) && \text{(rule1)} \\ \text{Reach}(x) &:- \text{Source}(x) && \text{(rule2)} \\ \text{Reach}(x) &:- \text{Reach}(y), \text{Link}(y, x) && \text{(rule3)} \end{aligned}$$

$$\begin{tikzcd} \text{Reach} \arrow[loop above] & \text{NoReach} \arrow[l, "-"'] \end{tikzcd}$$

Stratum 0 Source, Link, Target
Stratum 1 Reach
Stratum 2 NoReach

Evaluation of stratified programs

P partitioned into a **sequence** $P_1, \ldots, P_n$

Gives us an order in which to apply (each group of) rules

At each iteration k, execute each subprogram in sequence

(1) Apply all the rules in P_1

(2) Apply all the rules in P_2

$\vdots$

(n) Apply all the rules in P_n

Evaluation of stratified programs

$$\begin{aligned} \text{Reach}(x) &:- \text{Source}(x) && (P_1) \\ \text{Reach}(x) &:- \text{Reach}(y), \text{Link}(y, x) && (P_1) \\ \text{NoReach}(x) &:- \text{Target}(x), \neg\text{Reach}(x) && (P_2) \end{aligned}$$

EDB

Source
1
4

Link	
1	2
3	4
2	4

Target
2
3
4

IDB

Reach
1
2
4

NoReach
3
4

Further remarks on SQL recursion

- Requires **stratified negation**
- Only **linear recursion**

Problems

Arithmetic operations

introduce new values not present in the database

Multiset semantics

rules must be applied several times

cycles in the data must be detected