Condensed Matter Field Theories

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PROBLEM SET 0, DEADLINE: ABOUT 20 MINUTES FROM THE TIME YOU SEE THIS PAPER.

A request: Please try to work out the problems, as far as possible, by yourself (discuss with friends, if necessary). Please do not give in to your urge to "google out the answer".

A note on the problems: Problems are marked by C, T, C/T or E. C means that the problem is *conceptual*, i. e., it helps you to understand and use the conceptual ideas of the subject. Such problems, usually, will not involve long calculations. On the other hand, problems marked with a T have primarily *technical* content. They should help you familiarise with a calculational technique that has been either discussed in class, or is newly introduced in the problem. A suggestion to approach the T problems is to reason out what the answers *ought to be* by conceptual arguments, i. e., always do the physics before you do the math. C/T problems involve both. E stands for *exploratory*, i. e., that will point you to material elsewhere or ask you a question to ponder and think about.

Revision: Revise critical phenomena from your favourite reference.

0/1. (**Level -1, C)** Consider a three site tight binding model with the Hamiltonian $H = -t|1\rangle\langle 2| - t|2\rangle 3 + \text{h. c.} + \epsilon_0 |2\rangle\langle 2|$, where $\epsilon_0 \gg |t|$. Find a effective low energy model.

0/2. (**Level 0, C)** Consider a quantum Heisenberg model of spin-$\frac{1}{2}$s in a 26 dimensional cubic lattice with nearest neighbour exchange couplings. What is the temperature dependence of the the low temperature specific heat for a) ferromagnetic coupling and b) antiferromagnetic coupling.

0/3. (**Level 0, C)** For the Landau theory

$$f(m) = \frac{r}{2}m^2 + \frac{u}{4}m^4 + \frac{v}{6}m^6$$

prove or disprove the statement. If $v > 0$ is fixed, the theory always has continuous phase transitions.

0/4. (**Level 0, C/T)** Consider the spherical model defined on a d cubic lattice with "soft Ising spins" σ_i an nearest neighbor ferromagnetic interaction J. If there are N site in the lattice, we impose a constraint that $\sum_{i=1,\dots,N} \sigma_i^2 = N$. Find the phase diagram of this model. If there are phase transitions, and if they are continuous, find the critical exponents.

0/5. (**Level 1, C/T)** N atoms sit at the centre of an optical cavity an couple to a single cavity mode of frequency ω. Treating the atoms as a two level system split by energy ω_0 that can couple to cavity mode, write out an effective Hamiltonain using the atom operators and the cavity mode operators. Formulate this problem in a path integral language and discuss what happens when the light-atom coupling g is increased. Specifically comment on the symmetry of the system, and the nature of the ground state. Use the fact that N is large to find an exact solution for the ground state for arbitrary g.

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