Discrete Structures I

Spring 2018 EXAM # 1

Date: 20^{th} Feb. 2018

Name:.....

Instructions:

- Please take time to read the instructions before beginning to answer the questions.
- Make sure to *print* your name in the designated place.
- This exam consists of **nine** questions spread over **seven** pages, including this page.
- The total number of points for the whole exam is **60**.
- Points for individual questions are also indicated.
- Time allowed is 75 minutes.
- This is a *closed book* exam. However, you may consult a crib sheet and use a non-programmable calculator.
- Show all intermediate steps and justify any auxiliary claims that you make.
- Try to express your ideas with clarity.

1. Write the negation of the following statements in English. Do **not** simply write as "It is not the case that .....". (6 Pts.)

   (a) Some exams are easy.

   (b) Jack and Jill are happy.

   (c) All professors are boring.

2. Write the following statements in the form "if ....., then ....." in English. (6 Pts.)

   (a) You cannot pass this exam unless you studied for it.

   (b) Classes will be canceled only if snowfall exceeds 6 inches.

   (c) It is necessary to wash the professor's car to get an A grade.

3. State the converse and contrapositive of the following statement. *If I flunk this exam, then I will drop out of this course.* (4 Pts.)

   Converse:

   Contrapositive:

4. Prove that $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$ by building truth tables for them. (4 Pts.)

5. Let $P(x)$ be the statement "$x + 3 = 2x$". If the universe of discourse is the set of all integers, what are the truth values of $\exists x P(x)$ and $\forall x P(x)$. Give reasons to support your answer. (4 Pts.)

6. Let $P(m, n)$ be *m is greater than n* where the universe of discourse is the set of all positive integers. What is the truth value of $\exists m \forall n P(m, n)$, and $\forall n \exists m P(m, n)$. Give reasons to support your answer. (6 Pts.)

7. Prove that *disjunctive syllogism* is a valid inference rule. In other words, prove that $\neg p \wedge (p \vee q) \longrightarrow q$ is a tautology using the standard logical equivalences discussed in class. (10 Pts.)

8. Determine whether the following argument is valid or not using only **propositional logic**. If it is valid, provide a derivation of the conclusion justifying each step. If it is not valid, present a scenario in which all premises are true and yet, the conclusion is false. No need to use predicate logic for this question. (10 Pts.)

   If you are not in the tennis tournament, you will not meet Ed. If you are not in the tennis tournament or if you are not in the play, you won't meet Kelly. You meet Kelly or you don't meet Ed. It is false that you are in the tennis tournament and in the play. Therefore, you are in the tennis tournament.

9. Let $I(x)$ be the statement "x has an Internet connection" and $C(x, y)$ be the statement "x and y have chatted over the Internet", where the domain for the variables x and y consist of all students in your class. Translate the following statements into **predicate logic**. (10 Pts.)

   (a) Not everyone in your class has an Internet connection. (1.5 Pts.)

   (b) No one in the class has chatted over the Internet with Zachary. (1.5 Pts.)

   (c) Someone in your class has an Internet connection, but has not chatted over the Internet with anyone in your class. (2 Pts.)

   (d) Everyone in your class with an Internet connection has chatted over the Internet with at least one student in your class. (2 Pts)

   (e) Patricia has chatted over the Internet with everyone except Brandon. (3 Pts.)