

PROPOSITIONAL LOGIC 1

Fabrizio Santini | COMP 131

VERSION 2.0

	 What is Logic? Propositional Logic Questions?
TODAY ON AI	





300 BC* Stoics philosophers - Propositional Logic

~340BC* <u>Aristotle</u> described the <u>syllogism</u>, quantifiers. a method of formal, mechanical thought.

~500 Boethius – Latin translation of Aristotle works on Syllogism

- ~1100 Pierre Abelard Further framework around the Syllogism
- ~1300 Jean Buridan Expands the framework around Syllogism
- 1847* George Boole Propositional Logic
- 1879* Gottlob Frege First-Order Logic
- 1922* Ludwig Wittgenstein Proof by truth tables
- 1930* Gödel FOL Inference algorithm
- 1960* Martin Davis / Hilary Putnam Tractable Propositional

Logic algorithm

1965* Robinson Tractable FOL algorithm

ARTIFICIAL INTELLIGENCE What is Logic?

Logic is a way of formally representing the **state of the world** and the world's **rules of operation** so that we can make **rational decisions** and learn **new knowledge** based on our **existing one**.

It allows to:

- Express knowledge using a formal language
- To carry out reasoning in that language

ARTIFICIAL INTELLIGENCE What is Logic?

There are several types of logic. Each type is increasingly complex as it captures more advanced concepts:

LANGUAGE	ONTOLOGICAL COMMITMENT	EPISTEMOLOGICAL COMMITMENT
Propositional logic	Facts	True / False / Unknown
First-order logic	Facts, Objects, Relations	True / False / Unknown
Temporal logic	Facts, Objects, Relations, Times	True / False / Unknown
Probability theory	Facts	Degree of belief \in [0, 1]
Fuzzy logic	Facts with degree of truth \in [0, 1]	Known interval value

ARTIFICIAL INTELLIGENCE	What is Logic? LANGUAGE EXAMPLES	104
Programming	languages:	
They are 1	formal and not ambiguous	
• Unfortuna informatio	ately they lacks expressivity as they cannot accommodate partial on	
Natural Lange	uage:	
 Very expr Inference 	possible, but hard to automate The teacher gave the boys an apple.	
A good repres	entation language is:	
 Both form 	nal and can express partial information	
 Can accor 	mmodate inference	
		7

ARTIFICIAL INTELLIGENCE What is Logic?

- Syntax: it is the set of symbols and rules used to express knowledge
- Semantics: it specifies the way symbols and sentences relate to the world
- Inference procedures: they describe the rules for deriving new sentences (and therefore, new semantics) from existing sentences





ARTIFICIAL INTELLIGENCE	Propositional Logic BASIC ELEMENTS	201
A sentence ex	xpresses a possible condition of the world	
A sentence ca	n either true (T) or false (F)	
Most basic pro am_wet, is_rain	positions are called simple , literal or atomic sentences : ning, have_umbrella	

ARTIFICIAL IN	ITELLIGENCE	Propositional Logic GRAMMAR			202
■ Lite true	ral: Symbo (T) or fals	ls or constants e (F)			
= Sym	bols : P, Q,	S,	PROPOSITIONAL L	OGIC GRAMMAR	
■ Sent con sent	tences are nectives to tences :	combined by produce other	Sentence → Literal → Proposition →	Literal Proposition true (T) false (F) <i>Symbol</i> (Sentence)	
\wedge	AND	Conjunction		Sentence Connective Sentence ¬ Sentence	
\vee	OR	Disjunction	$\textbf{Connective} \rightarrow$	$\land, ~\lor, \rightarrow, \leftrightarrow$	
\rightarrow	IMPLIES	Implication / conditional			
\leftrightarrow	IS EQUIVALENT	Biconditional			
	NOT	Negation			

L

ARTIFICIAL INTELLIGENCE	Propositional Logic Sentences and well formed formulas	203
 A sentence is A symbol is a 	a well formed formula that can be defined recursively:	
 If s is a sente 	nce, then $\neg s$ is a sentence	
 If s is a sente 	nce, then (s) is a sentence	
 If s and t are 	sentences, then $(s \land t)$, $(s \lor t)$, $(s \to t)$ and $(s \leftrightarrow t)$ are sentences	
 A sentence re 	esults from a finite number of iterations of the above rules	
Operator press	cedence : ¬, ∧, ∨,→,↔	

ARTIFICIAL INTELLIGENCE Propositional Logic 204 ■ 0 means "It is hot" ■ H means "It is humid" ■ H means "It is humid" ■ R means "It is raining" ■ (0 ∧ H) → R means "IF it is hot AND humid, THEN it is raining" ■ Q → P means "IF it is humid, THEN it is hot" ■ Abetter way to write sentences: ■ Hot: "It is humid" ■ ■ Humid: "It is numid" ■ ■ ■





AR	TIFICIAL INTELLIGE	NCE Propo	sitional Logic	11/50				207
		TRUTH TA	BLES FOR EUGICAL CONNECT	IVES				
	p	q	$\neg p$	$\boldsymbol{p}\wedge \boldsymbol{q}$	$p \lor q$	p ightarrow q	$p \leftrightarrow q$	
	F	F	т	F	F	т	т	
	F	т	т	F	т	т	F	
	т	F	F	F	т	F	F	
	т	т	F	т	т	т	т	

• $\neg p$ is a **negation** of $p:\neg am_wet$

• $p \land q$ is a conjunction of p and $q: am_wet \land \neg is_raining$

• $p \lor q$ is a **disjunction** of p or q: $am_wet \lor \neg is_raining$

• $p \rightarrow q$ is an implication of p (premise) implies q (conclusion): $am_wet \rightarrow is_raining$

• $p \leftrightarrow q$ is a **biconditional** of p if-and-only-if (iff) $q: am_wet \leftrightarrow is_raining$

	and the sta	Descritte
20	nal Logic of Logical connectives	ARTIFICIAL INTELLIGENCE Propositio
NAME	EQUIVALENCE	FORM
Identity laws	p	$p \wedge \mathbf{T}, \ p \vee \mathbf{F}$
Domination laws	т	$p \vee \mathbf{T}$
	F	$p \wedge F$
Idempotent laws	p	$p \lor p, p \land p$
Double negation law	p	$\neg(\neg p)$
Commutative laws	$q \lor p$	$p \lor q$
	$q \wedge p$	$p \wedge q$
Associative laws	$p \lor (q \lor r)$	$(p \lor q) \lor r$
	$p \wedge (q \wedge r)$	$(p \land q) \land r$
Distributive laws	$(p \lor q) \land (p \lor r)$ $(p \land a) \lor (p \land r)$	$p \lor (q \land r)$ $p \land (q \lor r)$
	(F ···	
De Morgan's laws	$\neg p \lor \neg q$ $\neg p \land \neg q$	$\neg (p \land q) \\ \neg (p \lor q)$
Contrapositive equivalence	$\neg q \rightarrow \neg p$	p ightarrow q
Excluded middle	т	$p \lor \neg p$
Negation creates opposite	F	$p \land \neg p$

ARTIFICIAL INTELLIGENCE Propositional Logic					
 Let's decide that: is_raining means: It's raining outside have_umbrella means: I have an umbrella am_wet means: I am wet 	EVALUA Model: Senten Evaluat	TION $\{w = T \ ce: (\neg w$	$r = \mathbf{F}, u$ $\wedge r) \wedge ($	$u = \mathbf{T} \}$ $(\neg r \lor u)$	
	SATISFAC Sentenc Model:	SATISFACTION Sentence: $(\neg w \land r) \land (\neg r \lor u)$ Model: { $w = ?, r = ?, u = ?$ }			
We can condense propositions replacing them	w	r	u	S	
with symbols:	F	F	F		
 r means: It's raining outside 	F	F	т		
• <i>u</i> means: I have an umbrella	F	т	F	F	
• w means: I am wet	F	т	Т		
	т	F	F	F	
	т	F	т	F	
	т	т	F	F	
	Т	т	Т		

ARTIFICIAL INTELLIGENCE	Propositional Logic KNOWLEDGE BASE	210
A knowledge	base is a list of sentences assumed to be true	
KB P entails Q other words, a	, written $P \models Q$, means that whenever P is true , so is Q. In II models of P are also models of Q.	
This also mear same models)	ns that <i>Q</i> is entailed by <i>P</i> iff { <i>P</i> , <i>Q</i> } has the same meaning (the as <i>P</i> .	
<i>Q</i> is entailed by possible world	y <i>P</i> (a set of premises or assumptions) iff there is no logically in which <i>Q</i> is false while all the premises in <i>P</i> are true .	



ARTIFICIAL INTELLIGENCE Propositional Logic PROOF BY REFUTATION

Proof by refutation is a complete inference procedure that tries to prove something demonstrating the opposite:

- A single inference rule or **resolution**
- It uses a **conjunctive normal form** for the knowledge base
- It reduces inference to the problem of checking satisfiability

ARTIFICIAL INTELLIGENCE	Propositional Logic PROOF BY REFUTATION						213
 The process is with the negat 	performed conjoining the <i>KB</i> ion of the query						
 It's possible to query iff the tr in every row 	say that <i>KB</i> entails the -uth table returns false						
		$(w \lor \neg \iota$	ı) ∧ (¬ı	$\iota \wedge r \rightarrow \iota$	$w) \land \neg (n)$	$r \land \neg w)$	
		w	r	u	?		
		F	F	F	т		
Example:		F	F	т	Т		
Given the follow	wing <i>KB</i> :	F	т	F	F		
$1, w \vee \neg u$		F	Т	т	Т		
2. $\neg u \land r \rightarrow w$		т	F	F	F		
		т	F	т	Т		
$S: (r \land \neg w)$		т	т	F	F		
Does <i>KB</i> entail	S (or $KB \models S$)?	Т	т	т	Т		

