

FIRST-ORDER LOGIC

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TODAY ON AI

- FOL inference
- Resolution-based inference
- Forward chaining
- Backward chaining
- Questions?

- Resolution-based inference
 - Use **Refutation** to confirm or refute a sentence p (but not to generate all entailed sentences)
 - Requires FOL KB to be reduced to CNF
- Forward chaining
 - Uses **Generalize Modus Ponens** to add new atomic sentences
 - Useful for systems that make inferences as information streams in
 - Requires KB to be in form of first-order definite clauses
- Backward chaining
 - Works backwards from a query to try to construct a proof
 - Can suffer from repeated states and incompleteness
 - Useful for query-driven inference

Resolution-based inference

SECTION 01

- A ground sentence is entailed by a new KB *iff* it is entailed by the original KB. Every FOL KB can be propositionalized so as to preserve entailment.
- An issue arises with function symbols, ground terms are infinitely many, e.g., **Father(Father(Father(John)))**.
- Herbrand (1930) postulated that if a sentence is entailed by an FOL KB, it is entailed by a **finite** subset of the propositional KB.
 - For $n = 0$ to ∞ , create a propositional KB by instantiating with depth- n terms see if is entailed by this KB
 - The problem is that it works if the sentence is entailed; it does not if is not entailed

The **simplest possible** approach is to use what we know about Propositional Logic inference.

We can use both **Forward** and **Backward chaining**.

Two problems in applying PL algorithms to FOL sentences:

1. The **Existential quantifier** $\exists$
2. The **Universal quantifier** $\forall$

Get rid of the existential quantifiers: in a **Existential instantiation** the variable is replaced by a new constant symbol (not in the domain)

CONSTANTS

{Robot, Sq01, Sq12, Sq10}

PREDICATES

{ $\exists x$: Radioactive(x)}

RESULT

Radioactive(C1)

Get rid of the universal quantifiers: in a **Universal instantiation** k copies of the universally quantified sentences are added:

CONSTANTS

{**Robot**, **Sq01**, **Sq12**, **Sq10**}

PREDICATES

{ $\forall x$: **Radioactive**(x) $\rightarrow$ **Unsafe**(x)}

RESULT

Radioactive(**Robot**) $\rightarrow$ **Unsafe**(**Robot**)

Radioactive(**Sq01**) $\rightarrow$ **Unsafe**(**Sq01**)

Radioactive(**Sq12**) $\rightarrow$ **Unsafe**(**Sq12**)

Radioactive(**Sq10**) $\rightarrow$ **Unsafe**(**Sq10**)

Forward chaining

SECTION 02

Like in **Propositional Logic** inference, the algorithm answers queries using the KB to determine new facts until it finds that the query is **true**, or until we've run out of new facts to generate.

Forward chaining works very much like **Breath-first Search**.

```
1 function Forward-Chaining(KB, q) return a substitution, or FAILURE
2   repeat until new is empty
3     new = {}
4     for each  $(p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow c)$  in KB
5       for each  $\theta$  such that  $\text{Subst}(\theta, p_1 \wedge \dots \wedge p_n) = \text{Subst}(\theta, p'_1 \wedge \dots \wedge p'_n)$  for some  $p'_1, \dots, p'_n$  in KB do
6          $c' = \text{Subst}(\theta, c)$ 
7         if  $c'$  does not unify with any sentence in KB + new then
8           new = new +  $c'$ 
9            $\phi = \text{Unify}(c', q)$ 
10          if  $\phi$  is not failure then
11            return  $\phi$ 
12      KB = KB + new
13   return FAILURE
```

Unification is a pattern matching procedure that takes two atomic sentences and returns a failure if they do not match and a substitution list if they do

The substitution list is called the **most general unifier**.

CONSTANTS

{Robot, Sq01, Sq12, Sq10}

PREDICATES

{ $\forall x$: Radioactive(x) $\rightarrow$ Unsafe(x)}

Windy(Sq01)

Empty(Sq10)

RESULT

Radioactive(x) $\cup$ Radioactive(Sq01)

Radioactive(x) $\cup$ Windy(Sq01)

Radioactive(x) $\cup$ Empty(Sq10)

{ x /Sq01}

FAILURE

FAILURE

```

1 function Unify(p, q,  $\theta$ ) return a solution, or FAILURE
2 scan p and q left-to-right to find where p and q are not equal
3 if there is no disagreement
4   return  $\theta$ 
5 let r and s be the different terms
6 if Variable(r) then
7    $\theta$  = Union( $\theta$ , {r/s})
8   return Unify(Subst( $\theta$ , p), Subst( $\theta$ , q),  $\theta$ )
9 else if Variable(s) then
10   $\theta$  = Union( $\theta$ , {s/r})
11  return Unify(Subst( $\theta$ , p), Subst( $\theta$ , q),  $\theta$ )
12 else return FAILURE

```

EXAMPLESAdjacent(Sq01, x) $\cup$ Adjacent(Sq01, Sq02)Adjacent(Sq01, x) $\cup$ Adjacent(y, Sq02)Adjacent(Sq01, x) $\cup$ Adjacent(y, Free(Sq02))Adjacent(Sq01, x) $\cup$ Adjacent(x, Sq02)**RESULT**

{x/Sq02}

{x/Sq02, y/Sq01}

{y/Sq01, x/Free(Sq02)}

FAILURE

Generalized Modus Ponens (GMP)

combines And-Introduction, Universal-Elimination, and Modus Ponens when the Knowledge Base contains only **Implications clauses**:

For $\{p_i, p'_i, q\}$, a substitution θ such that $\text{Subst}(\theta, p'_i) = \text{Subst}(\theta, p_i)$ for all i , then:

$$\frac{p'_1, p'_2, \dots, p'_n, (p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q)}{\text{Subst}(\theta, q)}$$

CONSTANTS

{Robot, Sq01, Sq12, Sq10}

PREDICATES

{ $\forall x$: Radioactive(x) $\wedge$ Empty(x) $\rightarrow$ Unsafe(x)}

Radioactive(Sq01)

Empty(Sq01)

RESULT

Radioactive(Sq01), Empty(Sq01)

(Radioactive(x) $\wedge$ Empty(x) $\rightarrow$ Unsafe(x))

Unsafe(Sq01)

There is an implicit assumption that all variables are universally quantified.

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?



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The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \rightarrow \text{Criminal}(x)$$

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, **has some missiles**, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \rightarrow \text{Criminal}(x)$$
$$\exists x: \text{Owns}(\text{Nono}, x) \wedge \text{Missile}(x)$$

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Is Colonel West a criminal?

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \rightarrow \text{Criminal}(x)$$
$$\text{Owns}(\text{Nono}, \text{M1})$$
$$\text{Missile}(\text{M1})$$

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$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \rightarrow \text{Criminal}(x)$$
$$\forall x: \text{Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \rightarrow \text{Sells}(\text{West}, x, \text{Nono})$$
$$\text{Owns}(\text{Nono}, \text{M1})$$
$$\text{Missile}(\text{M1})$$

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$$\text{Owns}(\text{Nono}, \text{M1})$$
$$\text{Missile}(\text{M1})$$

The law says that it is a crime for an American to sell **weapons** to hostile nations.

The country Nono, an enemy of America, has some **missiles**, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \rightarrow \text{Criminal}(x)$$
$$\text{Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \rightarrow \text{Sells}(\text{West}, x, \text{Nono})$$
$$\text{Missile}(x) \rightarrow \text{Weapon}(x)$$
$$\text{Owns}(\text{Nono}, \text{M1})$$
$$\text{Missile}(\text{M1})$$

The law says that it is a crime for an American to sell weapons to **hostile nations**.

The country Nono, **an enemy of America**, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

American(x) $\wedge$ **Weapon**(y) $\wedge$ **Sells** (x, y, z) $\rightarrow$ **Criminal**(x)

Missile(x) $\wedge$ **Owns**(Nono, x) $\rightarrow$ **Sells**(West, x, Nono)

Missile(x) $\rightarrow$ **Weapon** (x)

Enemy(x, **America**) $\rightarrow$ **Hostile**(x)

Owns(Nono, **M1**)

Missile(**M1**)

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by **Colonel West, who is American**.

Is Colonel West a criminal?

American(x) $\wedge$ **Weapon**(y) $\wedge$ **Sells** (x, y, z) $\rightarrow$ **Criminal**(x)

Missile(x) $\wedge$ **Owns**(Nono, x) $\rightarrow$ **Sells**(West, x, Nono)

Missile(x) $\rightarrow$ **Weapon** (x)

Enemy(x, America) $\rightarrow$ **Hostile**(x)

Owns(Nono, M1)

Missile(M1)

American(West)

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

American(x) $\wedge$ **Weapon**(y) $\wedge$ **Sells** (x, y, z) $\rightarrow$ **Criminal**(x)

Missile(x) $\wedge$ **Owns**(Nono, x) $\rightarrow$ **Sells**(West, x, Nono)

Missile(x) $\rightarrow$ **Weapon** (x)

Enemy(x, America) $\rightarrow$ **Hostile**(x)

Owns(Nono, M1)

Missile(M1)

American(West)

Enemy(Nono, America)

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is **Colonel West** a **criminal**?

American(x) $\wedge$ **Weapon**(y) $\wedge$ **Sells** (x, y, z) $\rightarrow$ **Criminal**(x)

Missile(x) $\wedge$ **Owns**(Nono, x) $\rightarrow$ **Sells**(West, x, Nono)

Missile(x) $\rightarrow$ **Weapon** (x)

Enemy(x, America) $\rightarrow$ **Hostile**(x)

Owns(Nono, M1)

Missile(M1)

American(West)

Enemy(Nono, America)

Criminal(West)?

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

American(x) $\wedge$ **Weapon**(y) $\wedge$ **Sells** (x, y, z) $\rightarrow$ **Criminal**(x)

Missile(x) $\wedge$ **Owns**(Nono, x) $\rightarrow$ **Sells**(West, x, Nono)

Missile(x) $\rightarrow$ **Weapon** (x)

Enemy(x, America) $\rightarrow$ **Hostile**(x)

Owns(Nono, M1)

Missile(M1)

American(West)

Enemy(Nono, America)

Weapon(M1)

Criminal(West)?

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

Weapon(M1)

American(x) $\wedge$ Weapon(y) $\wedge$ Sells(x, y, z) $\rightarrow$ Criminal(x)

Missile(x) $\wedge$ Owns(Nono, x) $\rightarrow$ Sells(West, x, Nono)

Missile(x) $\rightarrow$ Weapon(x)

Enemy(x, America) $\rightarrow$ Hostile(x)

Owns(Nono, M1)

Missile(M1)

American(West)

Enemy(Nono, America)

Hostile(Nono)

Criminal(West)?

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

Weapon(M1)

Hostile(Nono)

American(x) $\wedge$ Weapon(y) $\wedge$ Sells (x, y, z) $\rightarrow$ Criminal(x)

Missile(x) $\wedge$ Owns(Nono, x) $\rightarrow$ Sells(West, x, Nono)

Missile(x) $\rightarrow$ Weapon (x)

Enemy(x, America) $\rightarrow$ Hostile(x)

Owns(Nono, M1)

Missile(M1)

American(West)

Enemy(Nono, America)

Sell(West, M1, Nono)

Criminal(West)?

The law says that it is a crime for an American to sell weapons to hostile nations.

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Is Colonel West a criminal?

Weapon(M1)

Hostile(Nono)

Sell(West, M1, Nono)

American(x) $\wedge$ Weapon(y) $\wedge$ Sells(x, y, z) $\rightarrow$ Criminal(x)

Missile(x) $\wedge$ Owns(Nono, x) $\rightarrow$ Sells(West, x, Nono)

Missile(x) $\rightarrow$ Weapon(x)

Enemy(x, America) $\rightarrow$ Hostile(x)

Owns(Nono, M1)

Missile(M1)

American(West)

Enemy(Nono, America)

Criminal(West): YES

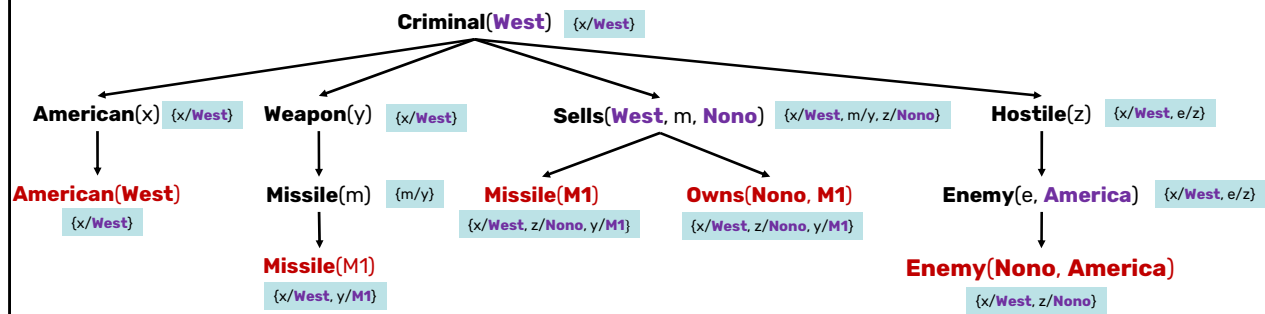
Backward chaining

SECTION 03

Like in Propositional Logic, the basic idea behind **Backward chaining** is to work backward from the goal to the facts that must be asserted for the goal to hold.

Backward chaining proceeds in a **Depth-first Search**.

```
1 function Backward-chaining(KB, goals,  $\theta$ ) return substitution
2   if goals is empty then
3     return  $\theta$ 
4   goal = pop from goals
5    $q' = \text{Subst}(\theta, \text{goal})$ 
6    $\theta'' = \{\}$ 
7   for each r in KB such that  $(p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q)$  do
8      $r' = \text{Standardize}(r)$ 
9      $\theta' = \text{Unify}(q, q')$ 
10    if  $\theta'$  succeeded then
11       $\theta'' = \theta'' + \text{Backward-chaining}(\text{KB}, (p_1 \dots p_n) + \text{goals}, \text{Compose}(\theta, \theta'))$ 
12  return  $\theta''$ 
```



#	RULE
1	$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \rightarrow Criminal(x)$
2	$Missile(x) \wedge Owns(Nono, x) \rightarrow Sells(West, x, Nono)$
3	$Missile(x) \rightarrow Weapon(x)$
4	$Enemy(x, America) \rightarrow Hostile(x)$
5	$Owns(Nono, M_1)$
6	$\exists x Owns(Nono, x) \wedge Missile(x) \equiv Missile(M_1)$
7	$American(West)$
8	$Enemy(Nono, America)$

QUERY

Is Colonel West a criminal?

{x/West, y/M1, z/Nono}

QUESTIONS ?