

# Feature Transformations

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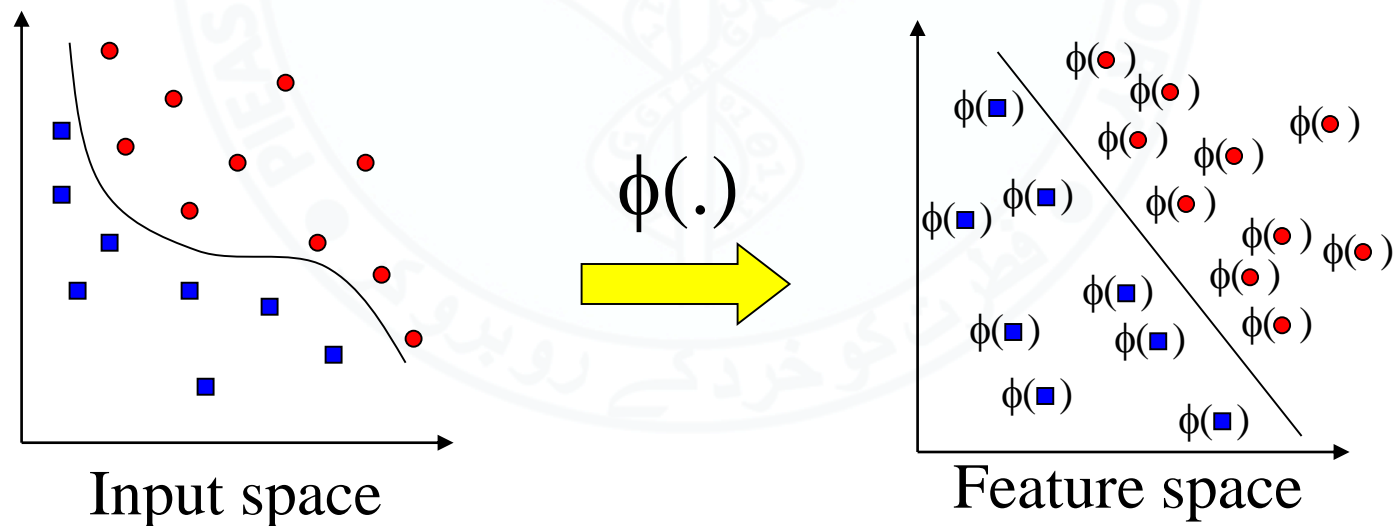
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# Nonlinear Separation through Transformation

- Given a classification problem with a nonlinear boundary, we can, at times, find a mapping or transformation of the feature space which makes the classification problem linear separable in the transformed space



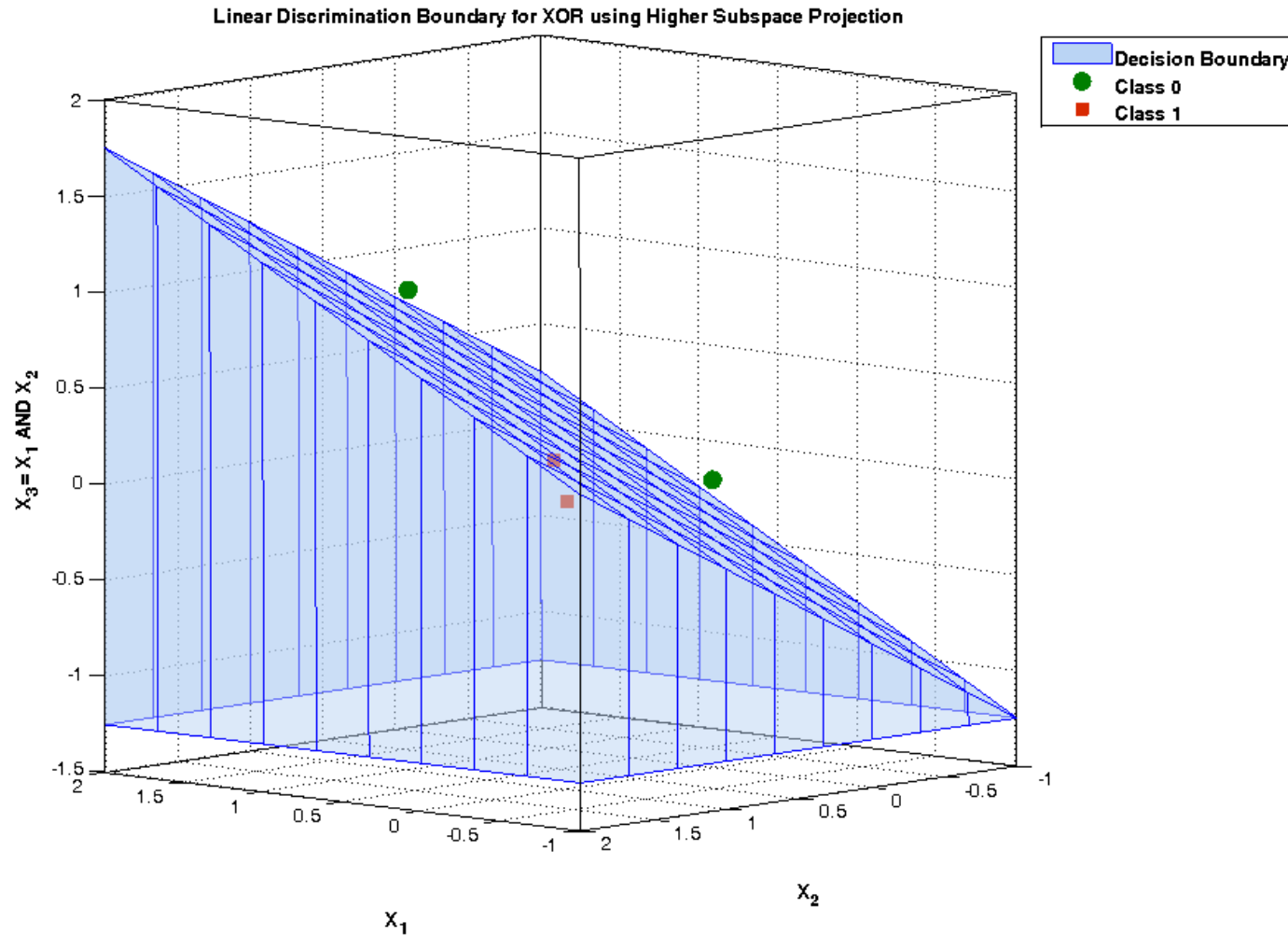
# Examples: Transformation

- Let's define the mapping

$$- \phi \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 x_2 \end{bmatrix}$$

- With a mathematical proof show that the above mapping makes the XOR classification problem linearly separable.

# XOR: Linear Separability



# Another Transformation

- Can the transform make the XOR linearly separable?

$$- \phi \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} (x_1)^2 \\ (x_2)^2 \\ \sqrt{2x_1x_2} \end{bmatrix}$$

- What about this one?

$$- \phi \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} (x_1)^3 \\ (x_2)^3 \\ \sqrt{3}(x_1)^2x_2 \\ \sqrt{3}(x_2)^2x_1 \end{bmatrix}$$

# Examples: Transformation

- Does this mapping do it?

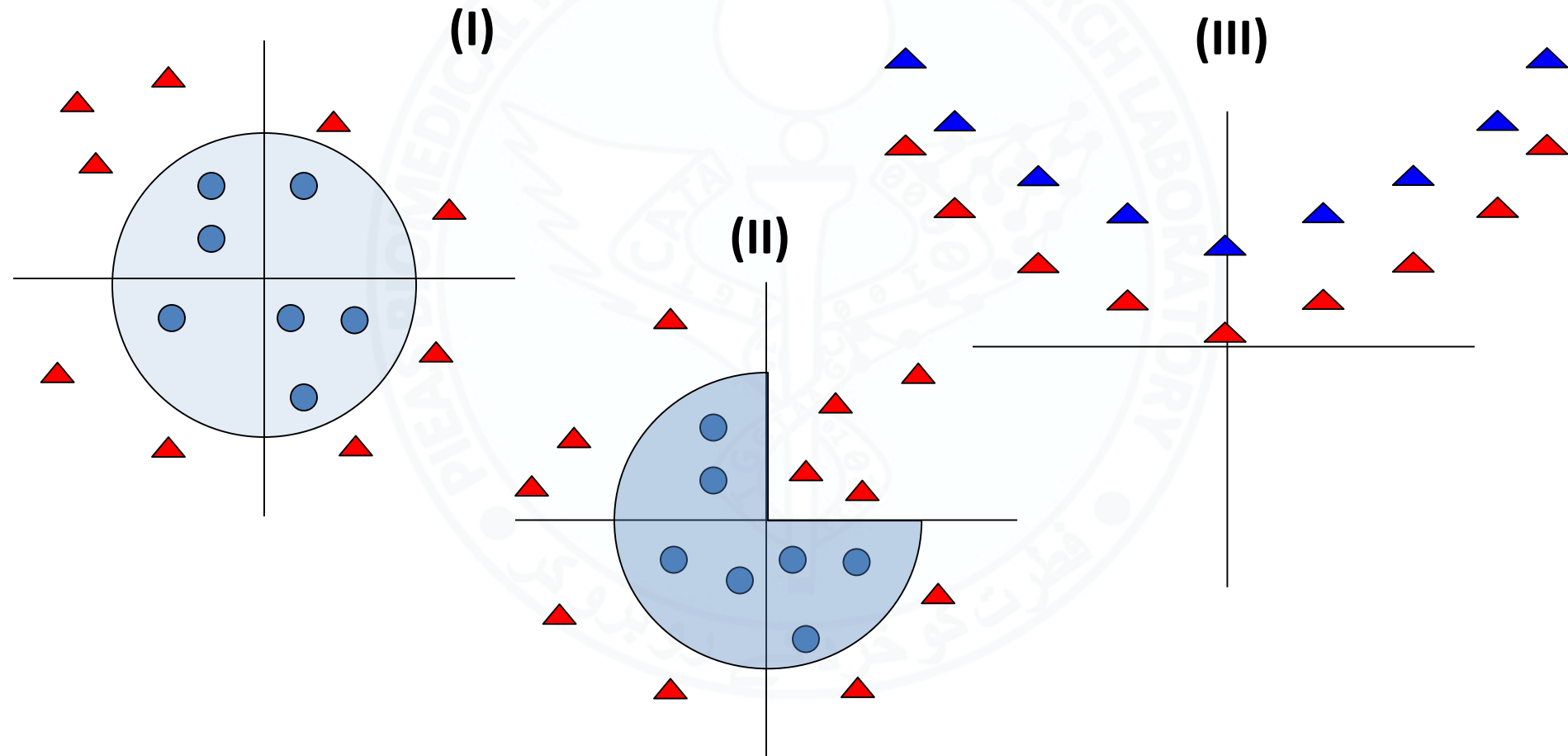
- $\phi \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$

- What about this one?

- $\phi \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = (x_1 + x_2 - 1)^2$

# Transformation Examples

- Can you find a transform that makes the following classification problems linear separable?  
Can you draw the data points in the new transformed feature space?





# End of Lecture

We want to make a machine that will be  
proud of us.

- Danny Hillis