

#### Instructions:

- 1. Please read the problems carefully and answer ALL questions.
- 2. Verify that your exam contains 4 pages + Smith chart.
- 3. You must include the Smith chart in your answer sheet. Do not write your name on it.
- Present your solutions *neatly*. Show the relevant steps, so that partial credit can be awarded.
  BOX your final answers where applicable. Draw figures wherever necessary.

# **QUESTION 1 [15 Marks]**

### TRANSMISSION LINE THEORY

- A 75 Ω lossless T-Line is terminated with an inductive load impedance given by  $Z_L = 45 + j60 \Omega$ .
  - (a) Calculate the load reflection coefficient,  $\Gamma_L$ . (Provide your answer in polar form.) [2]
  - (b) What is the value of the voltage standing wave ratio on the line? [1]
  - (c) Find the percentage time-average incident power that is absorbed by the load. [2]
  - (d) Find the  $V_{\text{max}}$  and  $V_{\text{min}}$  positions nearest to the load. Provide your answers as electrical lengths. [3]

**1B** The voltage and current expressions that were measured on a lossless transmission line are:

$$v(z,t) = 20\cos(2\pi \times 10^8 t - \pi z + \pi/6) + 10\cos(2\pi \times 10^8 t + \pi z)$$

$$i(z,t) = 0.4\cos(2\pi \times 10^8 t - \pi z + \pi/6) - 0.2\cos(2\pi \times 10^8 t + \pi z)$$

- (a) Determine the voltage phasor. [2]
- (b) Determine the current phasor. How could you physically interpret the minus sign? [2]
- (c) Determine the reflection coefficient at the load. [1]
- (d) Determine the impedance of the load. [2]

### **QUESTION 2 [15 Marks]**

#### S-PARAMETERS & SCATTERING MATRIX

**2A** For the "T"-network shown below,  $R_1 = R_2 = 8.56 \ \Omega$  and  $R_3 = 141.8 \ \Omega$ . Assuming a  $50 - \Omega$  system impedance. Find

- (a) The scattering matrix. [4]
- (b) The reflected-to-incident power ratio. [1]
- (c) The transmitted-to-incident power ratio. [1]
- (d) The loss-to-incident power ratio. [1]
- (e) What is the function of this device? [1]



**2B** A four-port network has the scattering matrix shown as follows.

$$[S] = \begin{bmatrix} 0.178 \angle 90^{\circ} & 0.6 \angle 45^{\circ} & 0.4 \angle 45^{\circ} & 0\\ 0.6 \angle 45^{\circ} & 0 & 0 & 0.3 \angle -45^{\circ}\\ 0.4 \angle 45^{\circ} & 0 & 0 & 0.5 \angle -45^{\circ}\\ 0 & 0.3 \angle -45^{\circ} & 0.5 \angle -45^{\circ} & 0 \end{bmatrix}$$

- (a) Is this network lossless? Justify your answer. [2]
- (b) Is this network reciprocal? Why or why not? [1]
- (c) What is the return loss at port 1 when all other ports are terminated with matched loads? [1]
- (d) What is the reflection coefficient seen at port 1 if a short circuit is placed at port 3 and all other ports are terminated with matched loads? [3]

## **QUESTION 3 [20 Marks]**

#### **RECTANGULAR WAVEGUIDES**

**3A** A rectangular air-filled waveguide supports TE propagation mode (shown below). Assume that the following magnetic field in the z direction is obtained:



- (a) What is the mode of operation? [1]
- (b) What are the cutoff wave number  $k_c$  and the free-space wave number k? [2]
- (c) Find the wave impedance for this waveguide? [2]
- (d) Find  $E_x$ ,  $E_y$ ,  $H_x$ , and  $H_y$ . [3]
- **3B** An X-band waveguide has a recommended frequency range of 8.20-12.4 GHz. This WR-90 waveguide has inside dimensions of 2.286 × 1.016 cm.
  - (a) What are the first three waveguide modes and their cutoff frequencies? [3]
  - (b) Why is the recommended frequency range of this waveguide given as 8.20-2.4 GHz? [2]
  - (c) Assume the wave is propagating in the z-direction. Write an equation for  $E_x$  if all of the first three modes are propagating. [2]
  - (d) Assume the wave is propagating in the z-direction. Sketch the magnitude of the  $E_y$  field for the first two modes of propagation. Show only the x y plane. [3]
  - (e) Sketch a method of feeding the  $TE_{03}$  mode in a waveguide. Be sure to show where the feed point(s) are, what their orientation is, and what their relative phases are. [2]

## **QUESTION 4 [20 Marks]** POWER DIVIDERS AND DIRECTIONAL COUPLERS

(a) Design a Wilkinson Power divider for equal power combining. All input and output lines are 50  $\Omega$ . Specify *R* and  $Z_{0,Q}$  below. [2]



- (b) If the input to port 2 is 1 V and the input to port 3 is 6 V, find: (Hint: you will need to use the results of Even and Odd analysis. Note: you have two sources so be smart).
  - $V_{1} = \_ V [2]$   $V_{2} = \_ V [2]$   $V_{3} = \_ V [2]$   $I = \_ A [2]$
- (a) If a quadrature coupler has the following input voltages, what is the output voltage on port 4? [3]
  - $V_{1}^{+} = 1 \angle 30^{\circ} \text{ Volts}$   $V_{2}^{+} = 2 \angle 60^{\circ} \text{ Volts}$   $V_{3}^{+} = 3 \angle 90^{\circ} \text{ Volts}$   $V_{4}^{+} = 4 \angle 120^{\circ} \text{ Volts}$   $[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \quad \textcircled{0} \qquad \qquad \bigcirc 90^{\circ} \text{ Hybrid} \qquad \bigcirc 3$
  - (b) Does the result change when V<sub>1</sub><sup>+</sup> is changed to 1000∠30° volts? Explain why or why not? [2]
- (a) With the neat diagram only, show how one can construct a four port circulator using two magic tees. [2]
  - (b) With the neat diagram, explain the working of a two-hole directional coupler. [3]

# **QUESTION 5 [10 Marks]**

# MICROWAVE FILTERS

**5** Design a stepped-impedance, maximally flat low-pass filter having a cutoff frequency of 2.5 GHz and an attenuation of at least 15 dB at 4 GHz. Use a reference impedance of  $Z_0 = 50 \Omega$ . The highest practical line impedance is 110  $\Omega$ , and the lowest is 10  $\Omega$ . Use a high-impedance line for the first section (closest to the source) and use the minimum

Use a high-impedance line for the first section (closest to the source) and use the minimum number of sections capable of meeting the attenuation requirement.

Sketch your design, and clearly specify the electrical lengths, widths of each section, and impedances of all lines. Assume FR4 fiberglass PCB with  $\varepsilon_r = 4.2$  and d = 1.5 mm.



# **QUESTION 6 [10 Marks]** IMPEDANCE MATCHING

6 Use the provided Smith Chart to design a lumped element, lossless, L-section matching network to match a load impedance of  $Z_L = (100 - j50) \Omega$  to a T-line with a characteristic impedance of  $Z_0 = 50 \Omega$  at a frequency of 1 GHz.

Draw the obtained matching networks with the elements and their values clearly indicated on each possible solution.

### \*END OF EXAMINATION\*

### CONSTANTS

 $\varepsilon_o = 8.854 \times 10^{-12}$  F/m,  $\mu_o = 4\pi \times 10^{-7}$  H/m,  $\eta_o = 377 \Omega$ ,  $c = 3 \times 10^8$  m/s 1 Np = 8.686 dB

## **USEFUL FORMULAS**

$$H_{z} = A_{mn} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$
$$E_{x} = \frac{-j\omega\mu}{k_{c}^{2}} \frac{\partial H_{z}}{\partial y}, \qquad H_{x} = \frac{-j\beta}{k_{c}^{2}} \frac{\partial H_{z}}{\partial x}$$
$$E_{y} = \frac{j\omega\mu}{k_{c}^{2}} \frac{\partial H_{z}}{\partial x}, \qquad H_{y} = \frac{-j\beta}{k_{c}^{2}} \frac{\partial H_{z}}{\partial y}$$

$$V_{1}^{e} = -j \frac{V_{g2}\sqrt{2}}{2}, \quad V_{2}^{e} = \frac{V_{g2}}{2}$$
$$V_{1}^{o} = 0, \quad V_{2}^{o} = \frac{r/2}{r/2 + 1} V_{g2}$$
$$L_{k}' = \frac{Z_{0}L_{k}}{\omega_{c}}, \quad C_{k}' = \frac{C_{k}}{\omega_{c}Z_{0}}$$
$$\varepsilon_{e} = \frac{\varepsilon_{r} + 1}{2} + \frac{\varepsilon_{r} - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}$$

For FR4 PCB:

 $W/d = 16 @ Z_0 = 10 \Omega, W/d = 1.98 @ Z_0 = 50 \Omega, and W/d = 0.36 @ Z_0 = 110 \Omega$ Table 8.3 – Maximally flat low-pass prototype filter.

N	g1	g2	g3	g4	g5	g6	<b>g</b> 7	g8	g9
1	2.0000	1.0000							
2	1.4142	1.4142	1.0000						
3	1.0000	2.0000	1.0000	1.0000					
4	0.7654	1.8478	1.8478	0.7654	1.0000				
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000			
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000		
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000	
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000