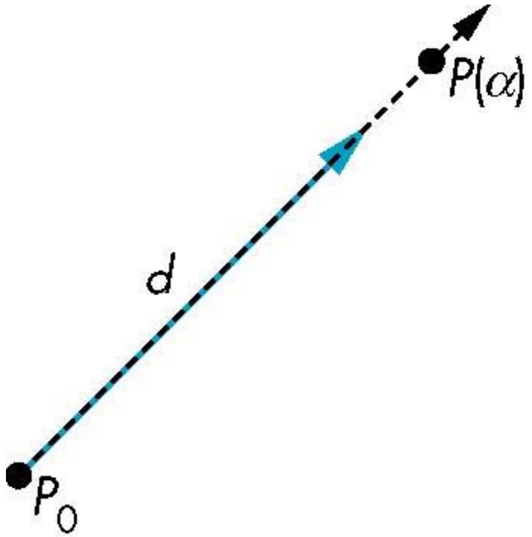
Lines

- Consider all points of the form
  - $P(\alpha)=P_0 + \alpha\, \mathbf{d}$
  - Set of all points that pass through P_0 in the direction of the vector $\mathbf{d}$



Parametric Form

- This form is known as the parametric form of the line
  - More robust and general than other forms
  - Extends to curves and surfaces
- Two-dimensional forms
  - Explicit: $y = mx + h$
  - Implicit: $ax + by + c = 0$
  - Parametric:

$$x(\alpha) = \alpha x_0 + (1-\alpha)x_1$$
$$y(\alpha) = \alpha y_0 + (1-\alpha)y_1$$

Rays and Line Segments

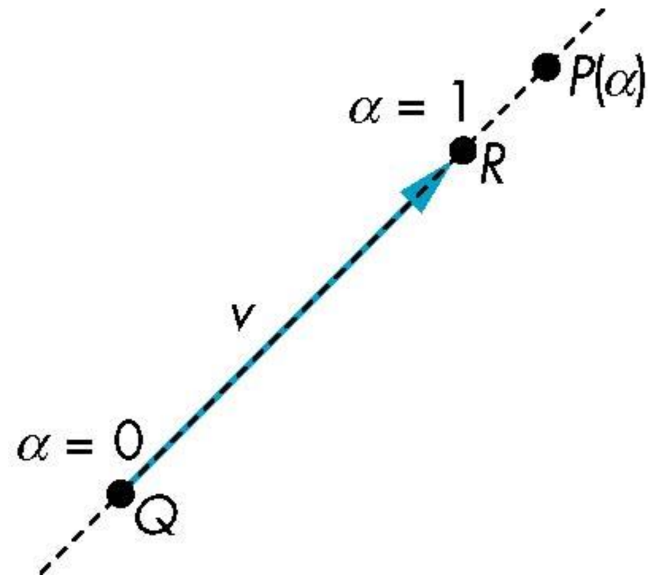
- If $\alpha >= 0$, then $P(\alpha)$ is the *ray* leaving P_0 in the direction $\mathbf{d}$

If we use two points to define v, then

$P(\alpha) = Q + \alpha\,(R-Q) = Q + \alpha v$

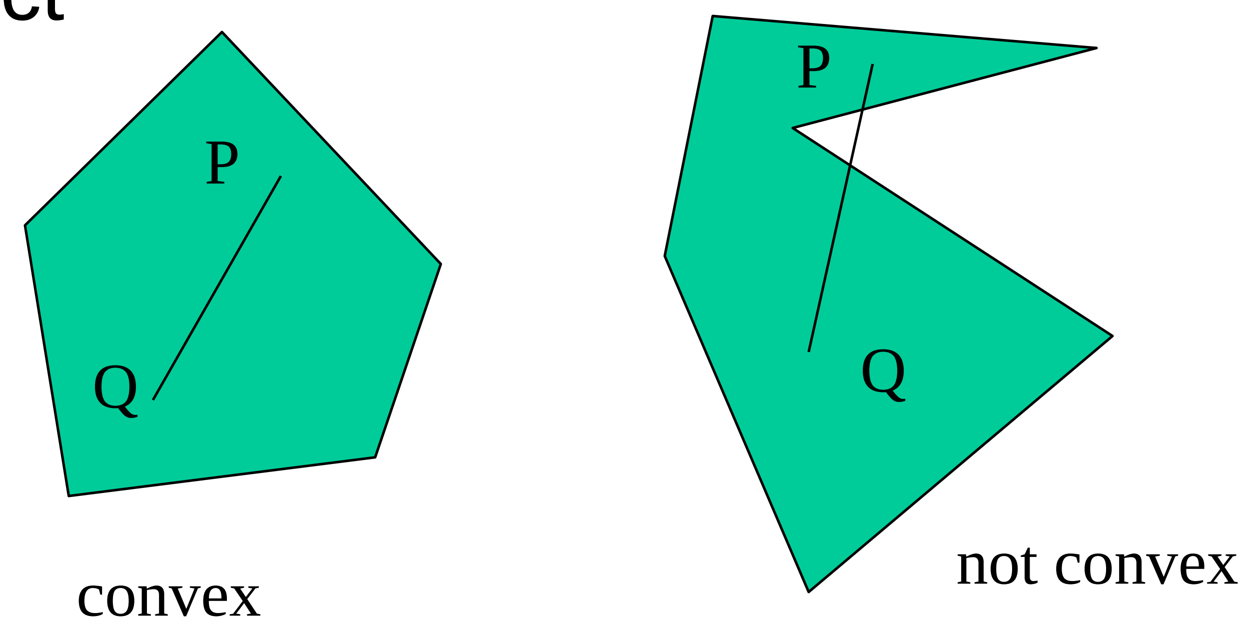
$= \alpha R + (1-\alpha)Q$

For $0 <= \alpha <= 1$ we get all the points on the *line segment* joining R and Q



Convexity

- An object is *convex* iff for any two points in the object all points on the line segment between these points are also in the object



Affine Sums

- Consider the “sum”

$P=\alpha_1 P_1+\alpha_2 P_2+\ldots\ldots+\alpha_n P_n$

Can show by induction that this sum makes sense iff

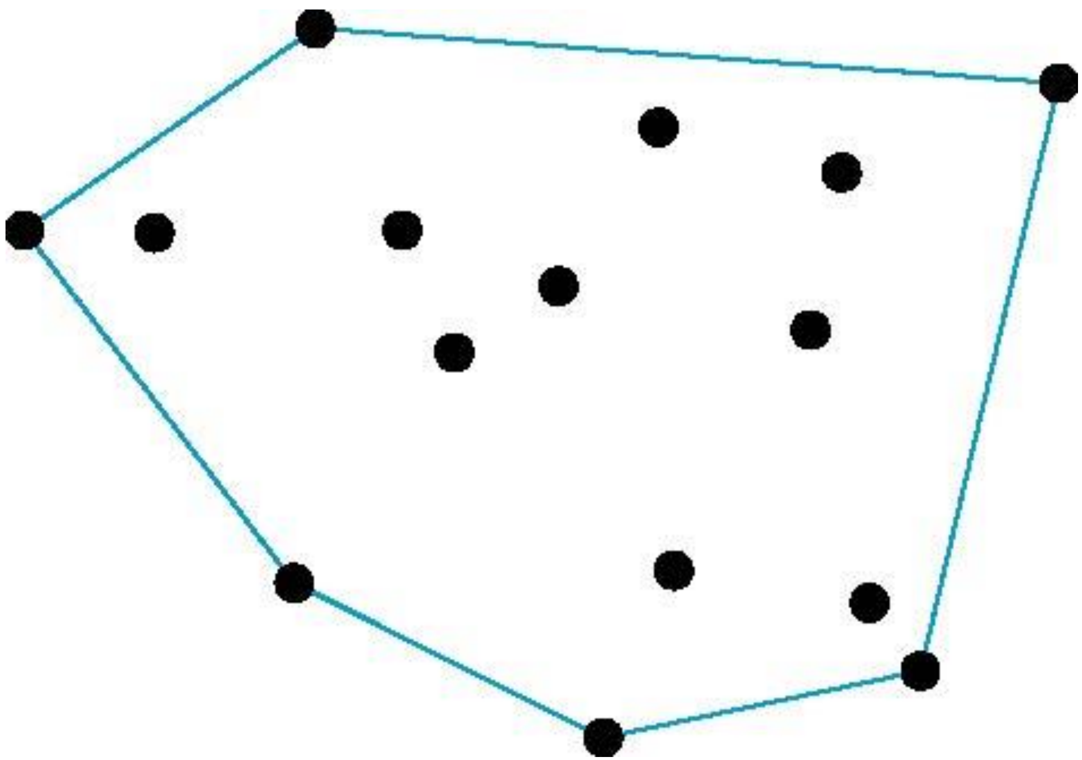
$\alpha_1+\alpha_2+\ldots\ldots\alpha_n=1$

in which case we have the *affine sum* of the points $P_1,P_2,\ldots\ldots P_n$

- If, in addition, $\alpha_i>=0$, we have the *convex hull* of $P_1,P_2,\ldots\ldots P_n$

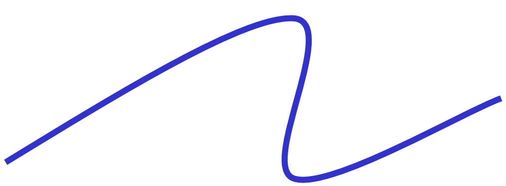
Convex Hull

- Smallest convex object containing $P_1, P_2, \ldots.. P_n$
- Formed by "shrink wrapping" points

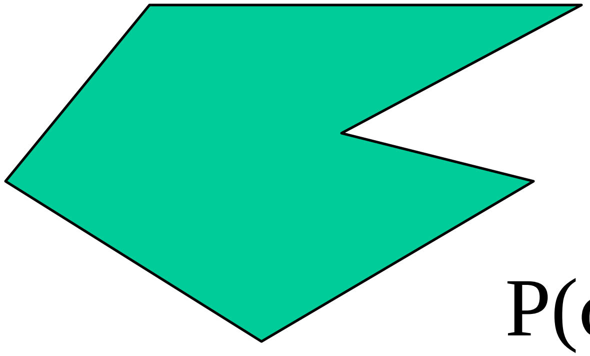


Curves and Surfaces

- Curves are one parameter entities of the form $P(\alpha)$ where the function is nonlinear
- Surfaces are formed from two-parameter functions $P(\alpha, \beta)$
  - Linear functions give planes and polygons



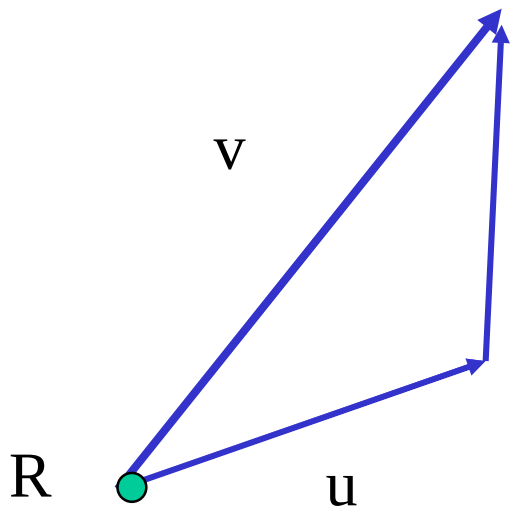
$P(\alpha)$



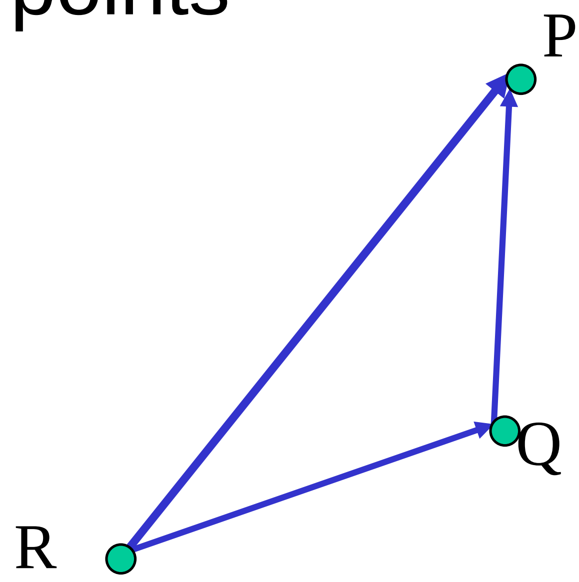
$P(\alpha, \beta)$

Planes

- A plane can be defined by a point and two vectors or by three points



$$P(\alpha,\beta)=R+\alpha u+\beta v$$

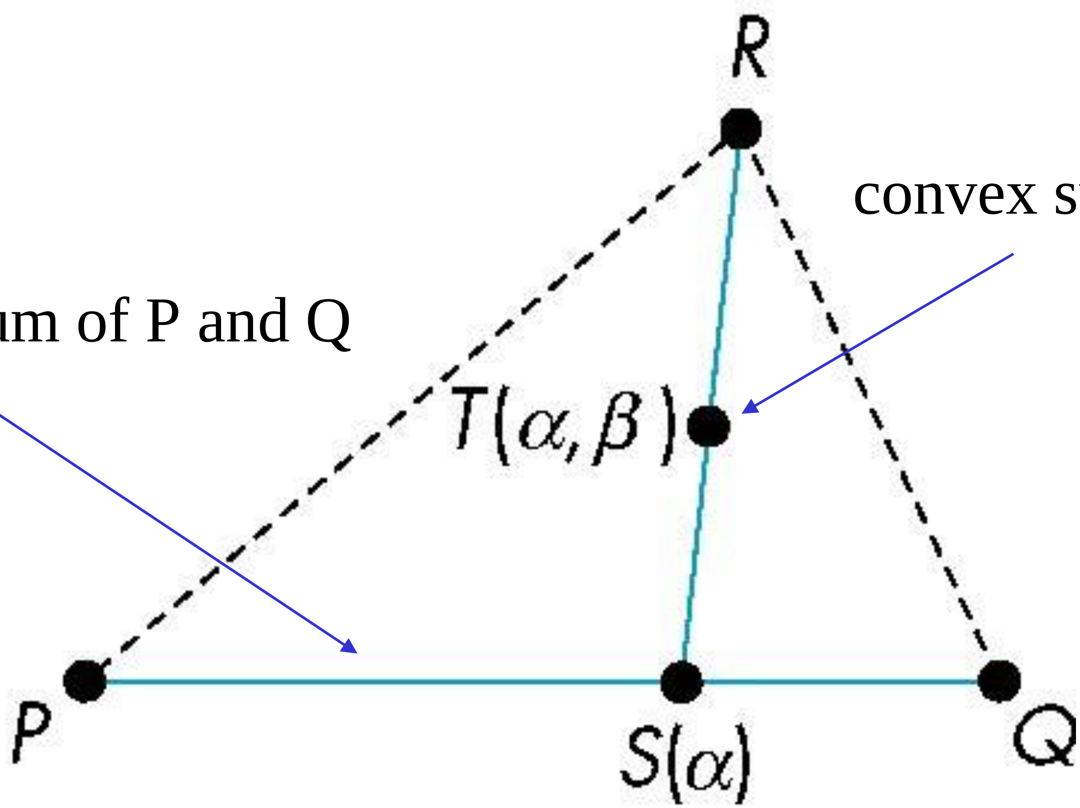


$$P(\alpha,\beta)=R+\alpha(Q-R)+\beta(P-Q)$$

Triangles

convex sum of S(α) and R

convex sum of P and Q



for 0<=α,β<=1, we get all points in triangle

Barycentric Coordinates

Triangle is convex so any point inside can be represented as an affine sum

$$P(\alpha_1, \alpha_2, \alpha_3)=\alpha_1 P+\alpha_2 Q+\alpha_3 R$$

where

$$\alpha_1 +\alpha_2 +\alpha_3 = 1$$

$$\alpha_i >=0$$

The representation is called the **barycentric coordinate** representation of P

Normals

- In three dimensional spaces, every plane has a vector n perpendicular or orthogonal to it called the **normal vector**
- From the two-point vector form $P(\alpha,\beta)=P+\alpha u+\beta v$, we know we can use the cross product to find $n = u \times v$ and the equivalent form

$$(P(\alpha, \beta)-P) \cdot n=0$$