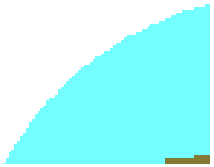


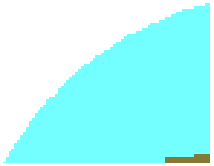
Introduction to Computer Graphics with WebGL

Ed Angel

Professor Emeritus of Computer Science

Founding Director, Arts, Research, Technology and Science Laboratory

University of New Mexico

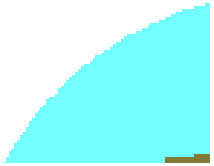


Orthogonal Projection Matrices

Ed Angel

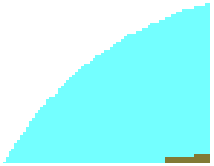
Professor Emeritus of Computer Science

University of New Mexico



Objectives

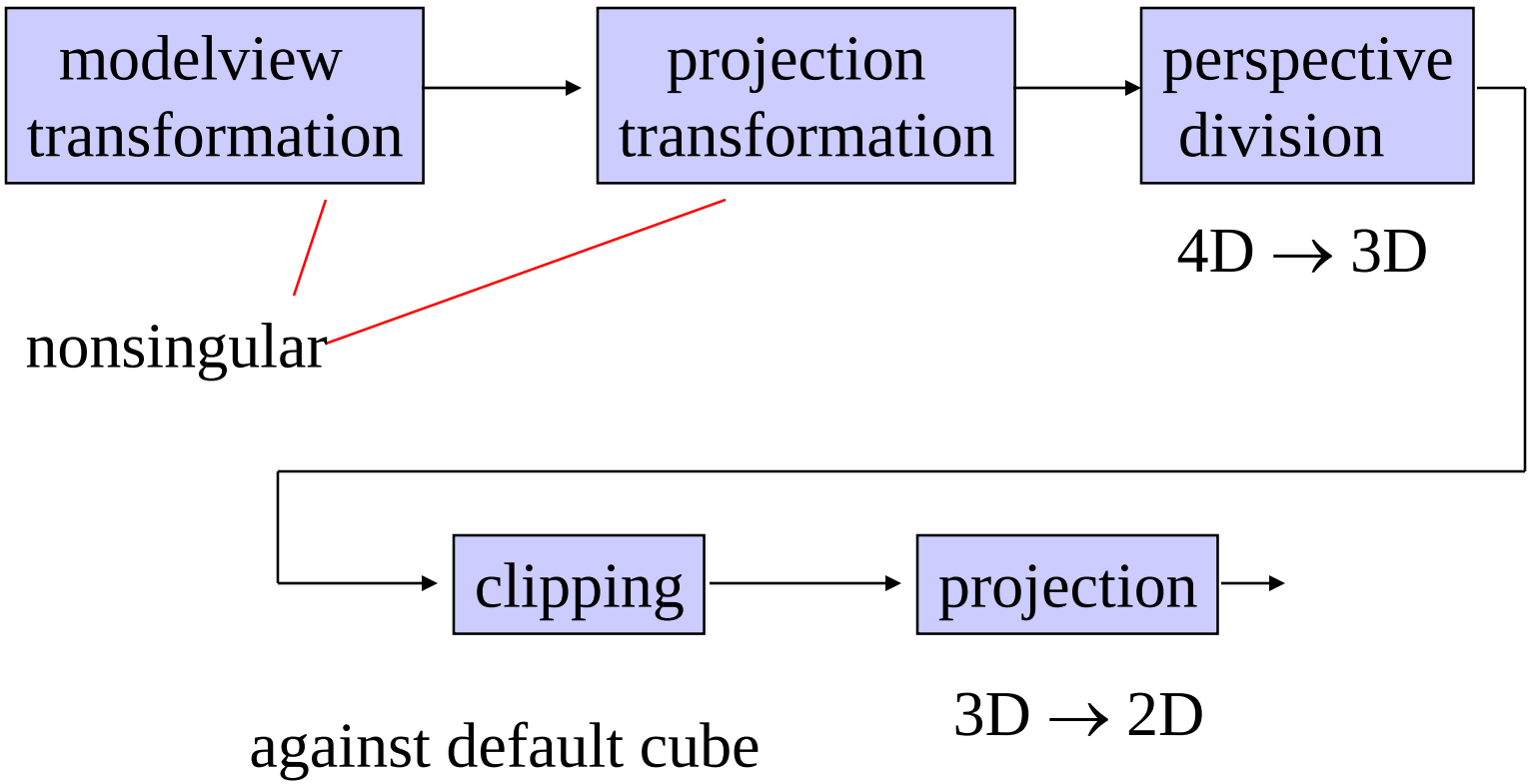
- Derive the projection matrices used for standard orthogonal projections
- Introduce oblique projections
- Introduce projection normalization

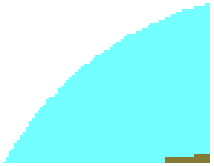


Normalization

- Rather than derive a different projection matrix for each type of projection, we can convert all projections to orthogonal projections with the default view volume
- This strategy allows us to use standard transformations in the pipeline and makes for efficient clipping

Pipeline View





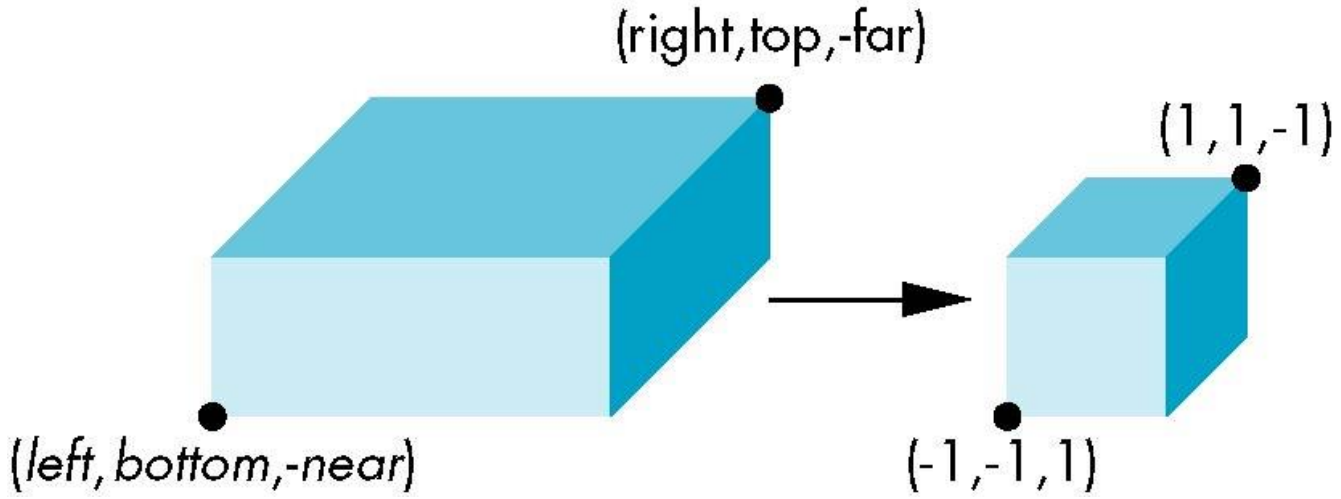
Notes

- We stay in four-dimensional homogeneous coordinates through both the modelview and projection transformations
  - Both these transformations are nonsingular
  - Default to identity matrices (orthogonal view)
- Normalization lets us clip against simple cube regardless of type of projection
- Delay final projection until end
  - Important for hidden-surface removal to retain depth information as long as possible

Orthogonal Normalization

`ortho(left,right,bottom,top,near,far)`

normalization $\Rightarrow$ find transformation to convert specified clipping volume to default



Orthogonal Matrix

- Two steps
  - Move center to origin

    T(-(left+right)/2, -(bottom+top)/2,(near+far)/2))
  - Scale to have sides of length 2

    S(2/(left-right),2/(top-bottom),2/(near-far))

$$\mathbf{P} = \mathbf{ST} = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right-left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{2}{near-far} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Projection

- Set $z = 0$
- Equivalent to the homogeneous coordinate transformation

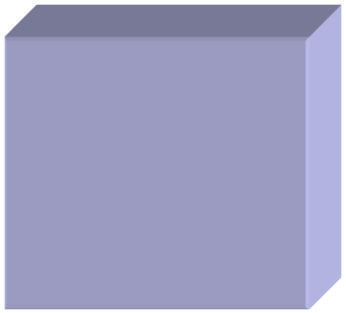
$$\mathbf{M}_{\text{orth}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Hence, general orthogonal projection in 4D is

$$\mathbf{P} = \mathbf{M}_{\text{orth}}\mathbf{ST}$$

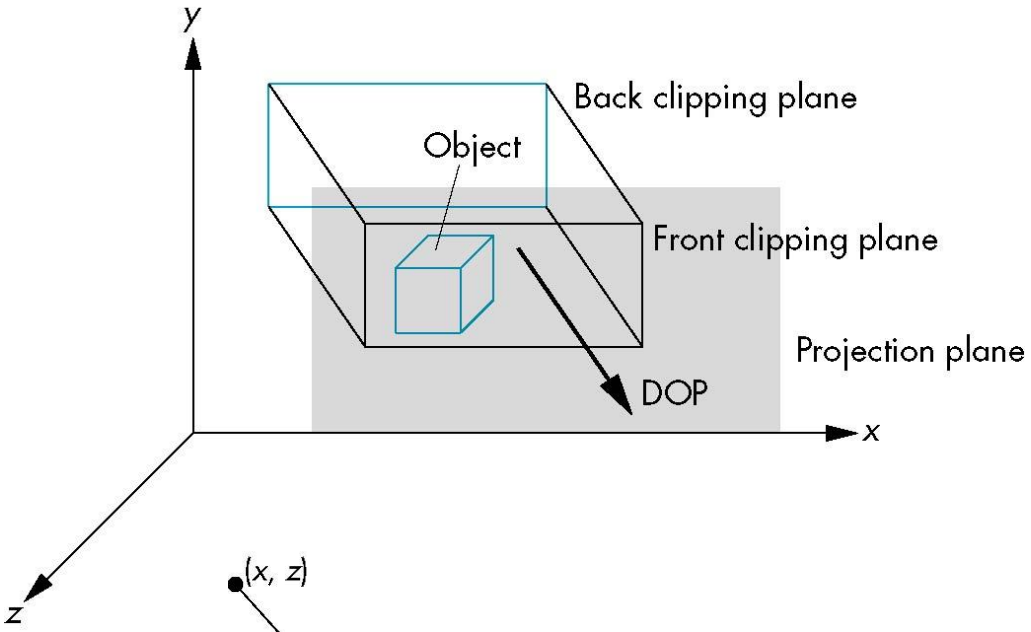
Oblique Projections

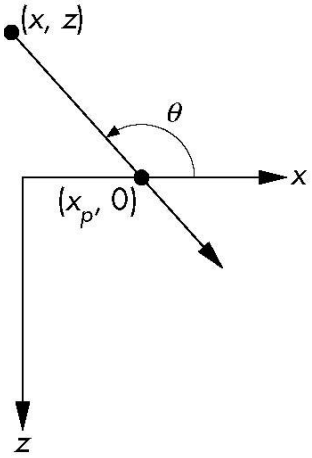
- The OpenGL projection functions cannot produce general parallel projections such as



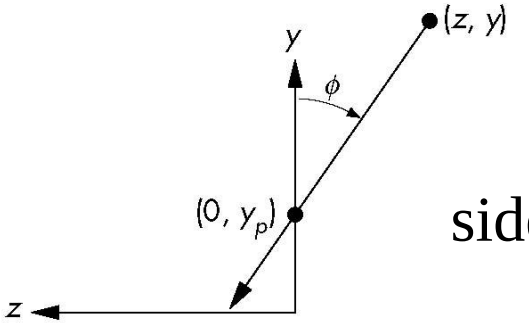
- However if we look at the example of the cube it appears that the cube has been sheared
- Oblique Projection = Shear + Orthogonal Projection

General Shear





top view



side view

Shear Matrix

xy shear (*z* values unchanged)

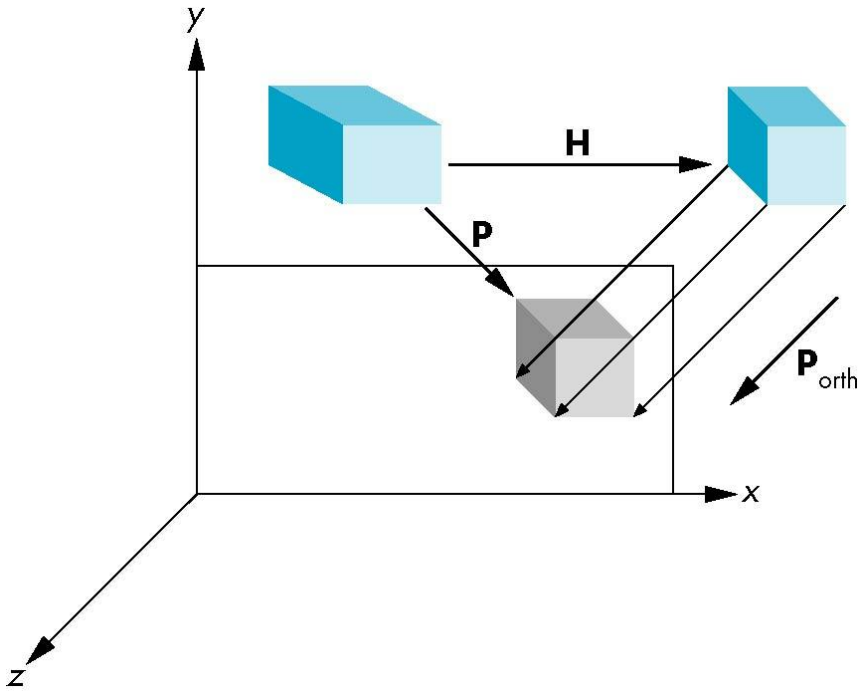
$$\mathbf{H}(\theta,\phi) = \begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\varphi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection matrix

$$\mathbf{P} = \mathbf{M}_{\text{orth}}\,\mathbf{H}(\theta,\phi)$$

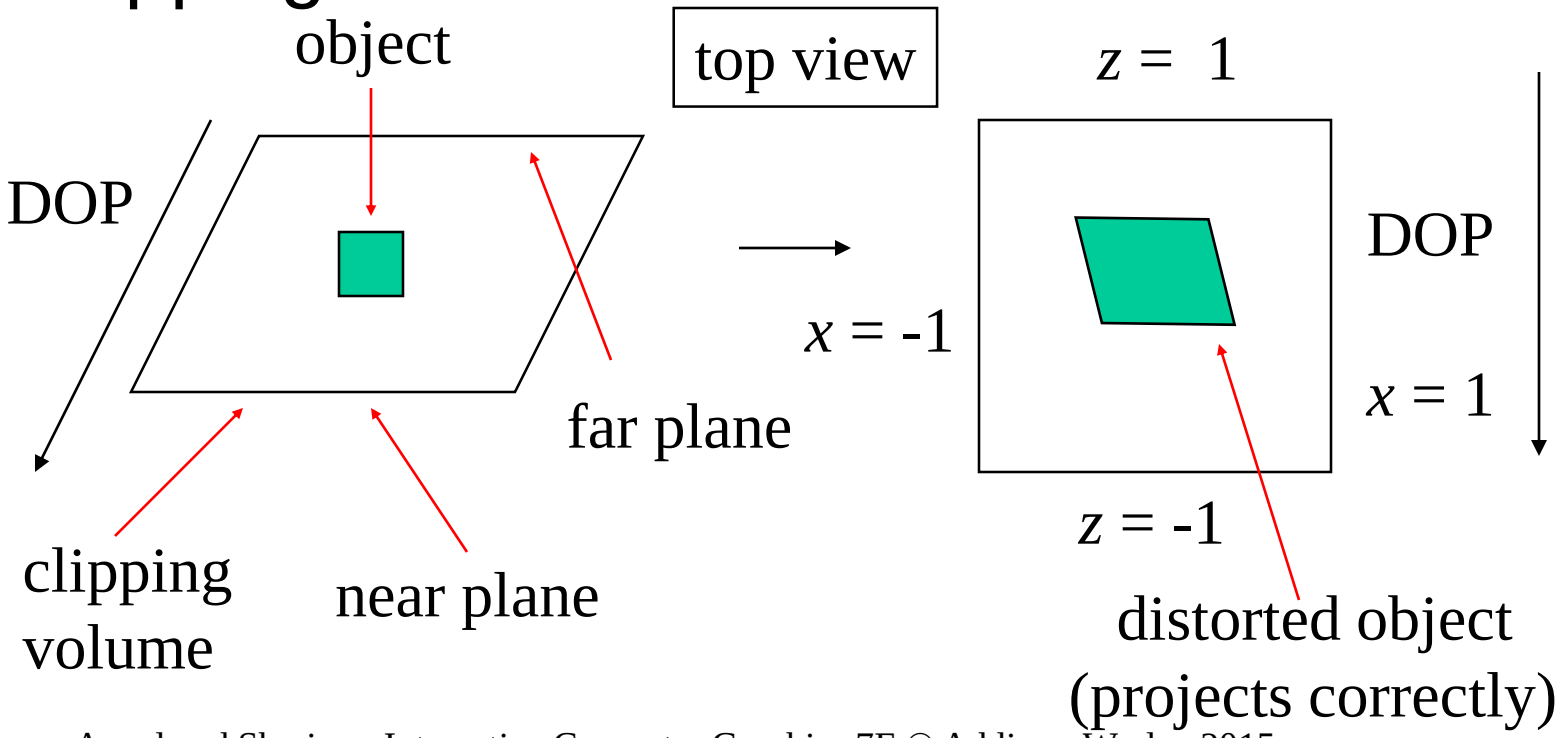
General case: $\mathbf{P} = \mathbf{M}_{\text{orth}}\,\mathbf{STH}(\theta,\phi)$

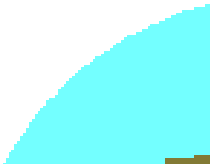
Equivalency



Effect on Clipping

- The projection matrix **P** = **STH** transforms the original clipping volume to the default clipping volume





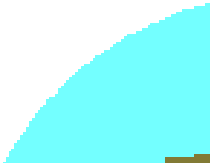
Introduction to Computer Graphics with WebGL

Ed Angel

Professor Emeritus of Computer Science

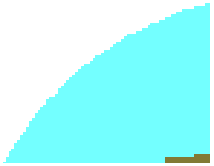
Founding Director, Arts, Research, Technology and Science Laboratory

University of New Mexico



Perspective Projection Matrices

Ed Angel
Professor Emeritus of Computer Science
University of New Mexico



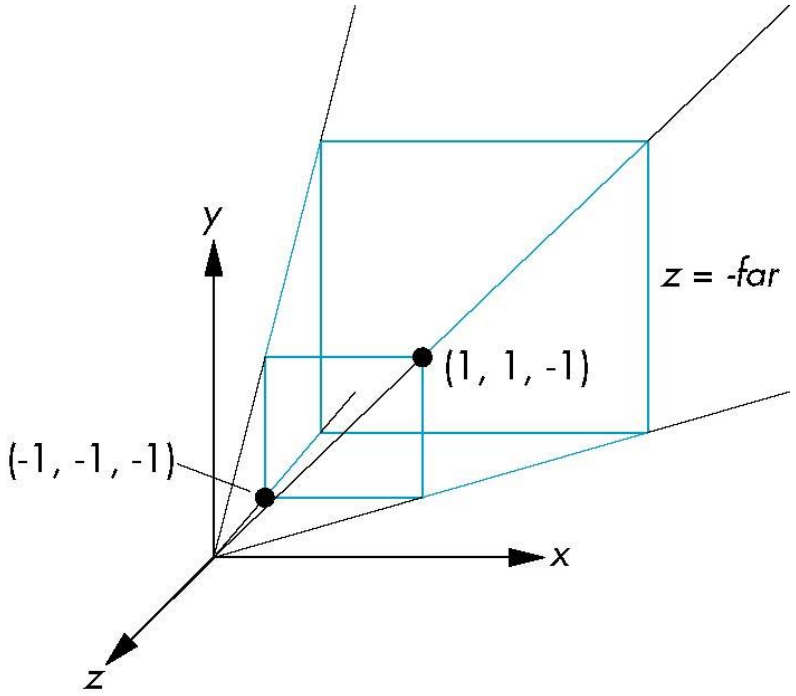
Objectives

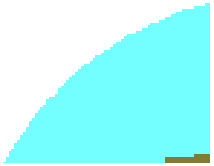
- Derive the perspective projection matrices used for standard WebGL projections

Simple Perspective

Consider a simple perspective with the COP at the origin, the near clipping plane at $z = -1$, and a 90 degree field of view determined by the planes

$x = \pm z, y = \pm z$





Perspective Matrices

Simple projection matrix in homogeneous coordinates

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Note that this matrix is independent of the far clipping plane

Generalization

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

after perspective division, the point $(x, y, z, 1)$ goes to

$$x'' = x/z$$
$$y'' = y/z$$
$$Z'' = -(\alpha+\beta/z)$$

which projects orthogonally to the desired point regardless of α and β

Picking α and β

If we pick

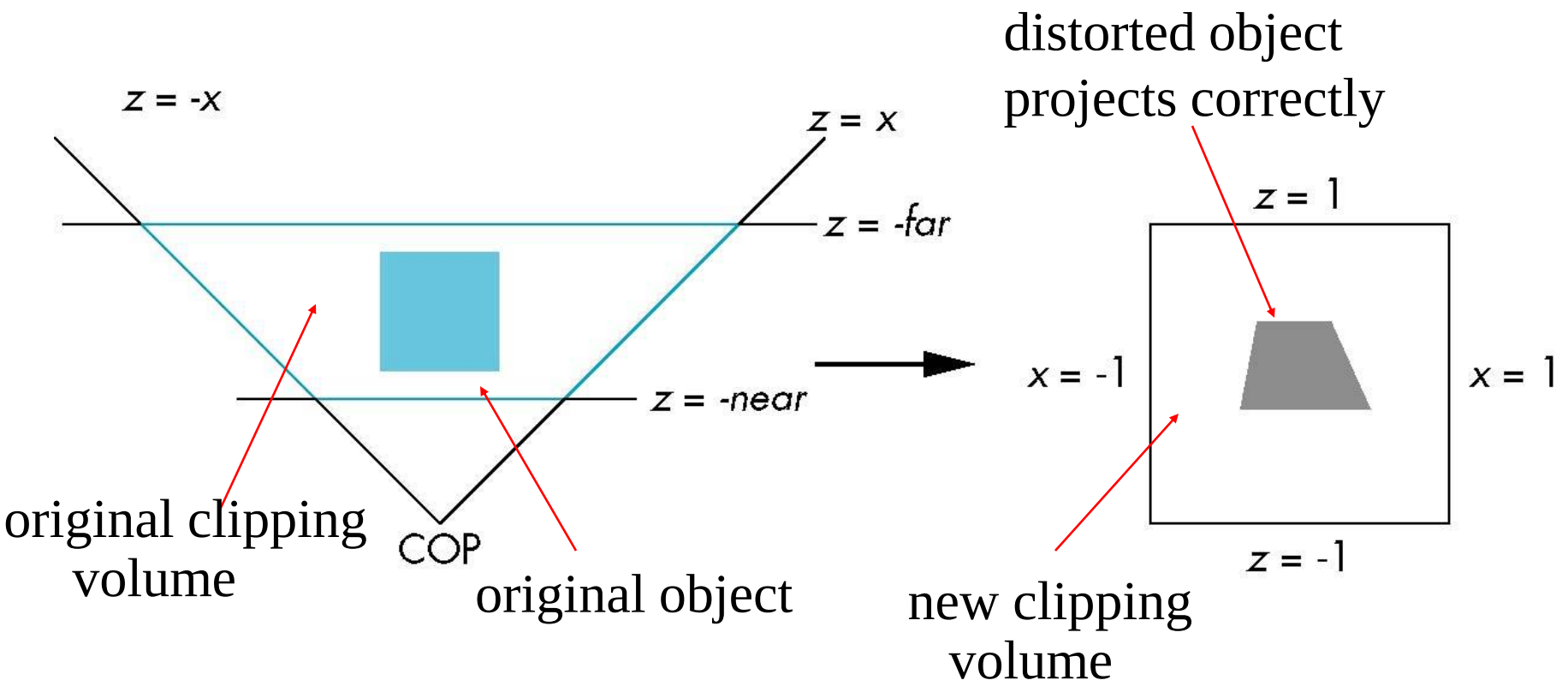
$$\alpha = \frac{\text{near} + \text{far}}{\text{far} - \text{near}}$$

$$\beta = \frac{2\text{near} * \text{far}}{\text{near} - \text{far}}$$

the near plane is mapped to $z = -1$
the far plane is mapped to $z = 1$
and the sides are mapped to $x = \pm 1, y = \pm 1$

Hence the new clipping volume is the default clipping volume

Normalization Transformation

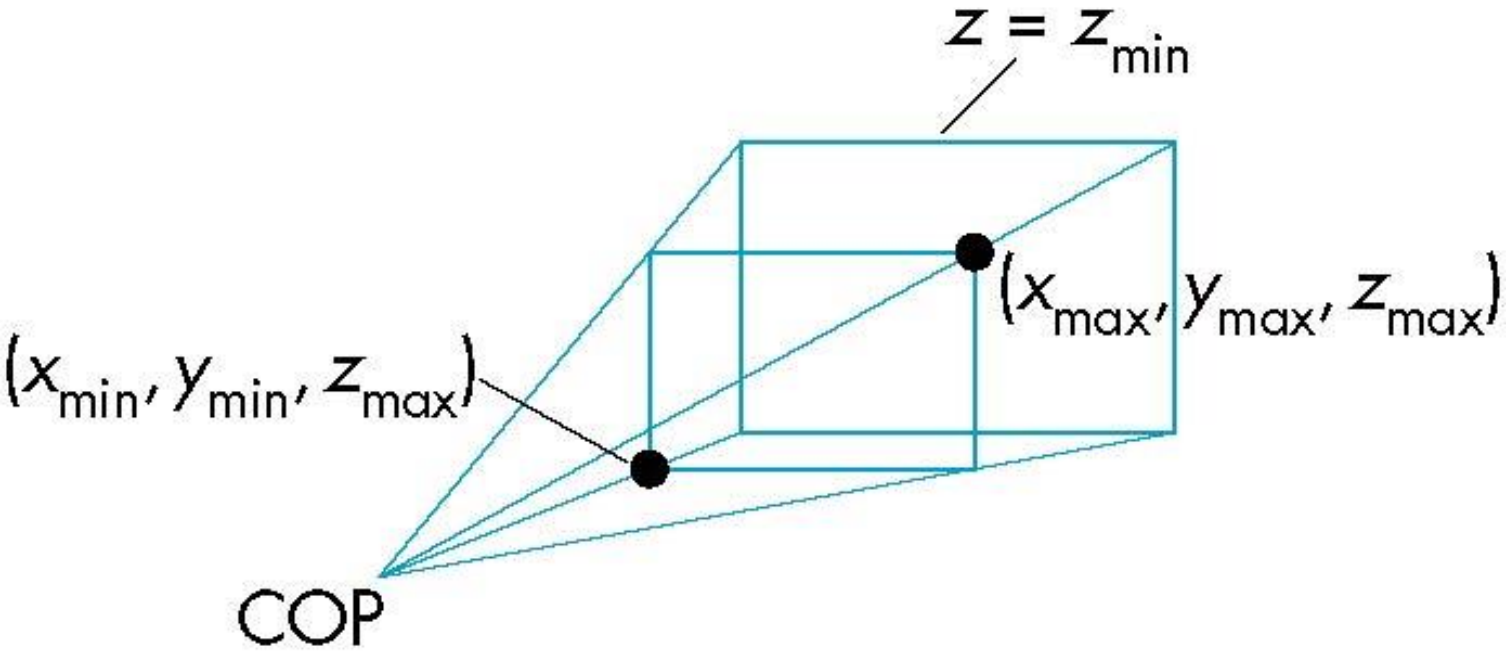


Normalization and Hidden-Surface Removal

- Although our selection of the form of the perspective matrices may appear somewhat arbitrary, it was chosen so that if $z_1 > z_2$ in the original clipping volume then the for the transformed points $z_1' > z_2'$
- Thus hidden surface removal works if we first apply the normalization transformation
- However, the formula $z'' = -(\alpha+\beta/z)$ implies that the distances are distorted by the normalization which can cause numerical problems especially if the near distance is small

WebGL Perspective

- **`gl.frustum`** allows for an unsymmetric viewing frustum (although **`gl.perspective`** does not)



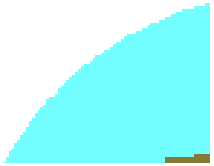
OpenGL Perspective Matrix

- The normalization in **`Frustum`** requires an initial shear to form a right viewing pyramid, followed by a scaling to get the normalized perspective volume. Finally, the perspective matrix results in needing only a final orthogonal transformation

$$\mathbf{P} = \mathbf{NSH}$$

our previously defined perspective matrix

shear and scale



Why do we do it this way?

- Normalization allows for a single pipeline for both perspective and orthogonal viewing
- We stay in four dimensional homogeneous coordinates as long as possible to retain three-dimensional information needed for hidden-surface removal and shading
- We simplify clipping

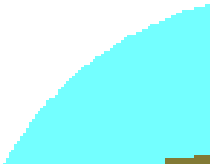
Perspective Matrices

frustum

$$\mathbf{P} = \begin{bmatrix} \frac{2*near}{right-left} & 0 & \frac{right-left}{right-left} & 0 \\ 0 & \frac{2*near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2*far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

perspective

$$\mathbf{P} = \begin{bmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2*far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



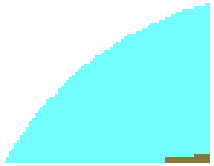
Introduction to Computer Graphics with WebGL

Ed Angel

Professor Emeritus of Computer Science

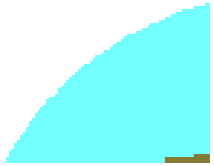
Founding Director, Arts, Research, Technology and Science Laboratory

University of New Mexico



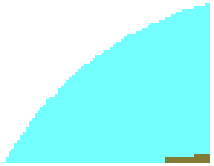
Meshes

Ed Angel
Professor Emeritus of Computer Science
University of New Mexico



Objective

Introduce techniques for displaying polygonal meshes

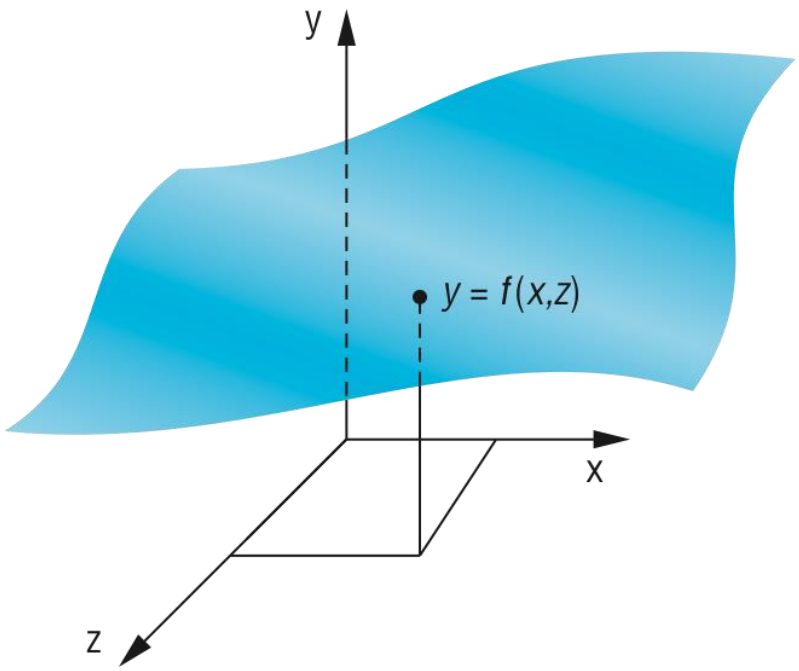


Meshes

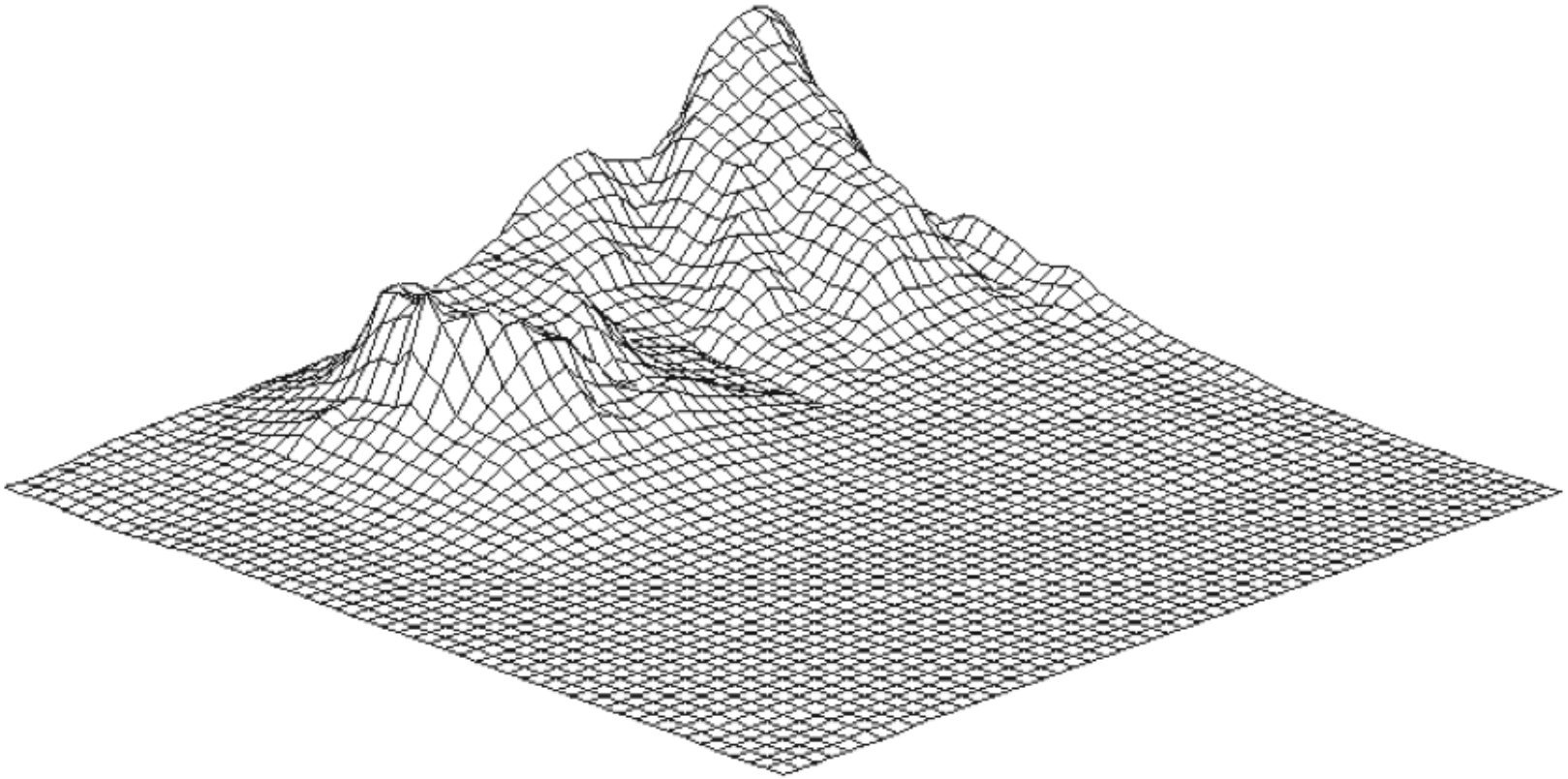
- Polygonal meshes are the standard method for defining and displaying surfaces
  - Approximations to curved surfaces
  - Directly from CAD packages
  - Subdivision
- Most common are quadrilateral and triangular meshes
  - Triangle strips and fans

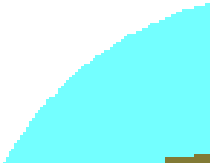
Height Fields

- For each (x, z) there is a unique y
- Sampling leads to an array of y values
- Display as quadrilateral or triangular mesh using strips



Honolulu Plot Using Line Strips

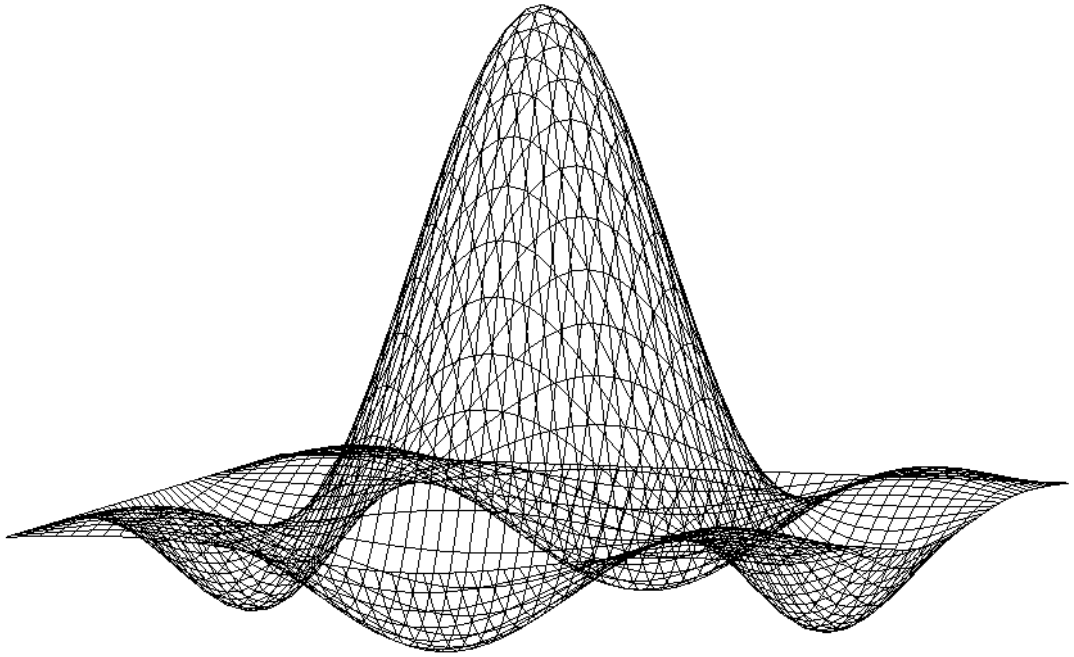




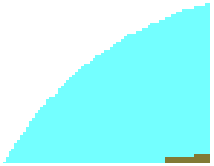
Plot 3D

- Old 2D method uses fact that data are ordered and we can render front to back
- Regard each plane of constant z as a flat surface that can block (parts of) planes behind it
- Can proceed iteratively maintaining a visible top and visible bottom
  - Lots of little line intersections
- Lots of code but avoids all 3D

Lines on Back and Hidden Faces



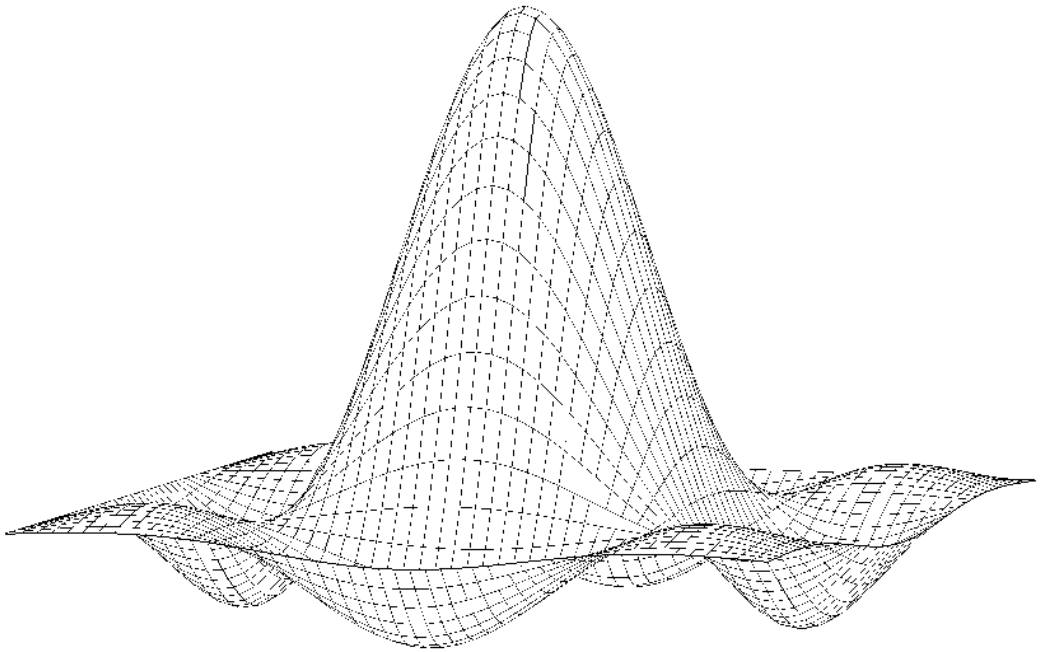
sombrero or Mexican hat function $(\sin \pi r)/(\pi r)$

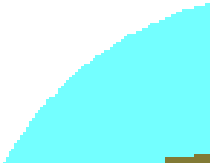


Using Polygons

- We can use four adjacent data points to form a quadrilateral and thus two triangles which can be shaded
- But what if we want to see the grid?
- We can display each quadrilateral twice
  - First as two filled white triangles
  - Second as a black line loop

Hat Using Triangles and Lines

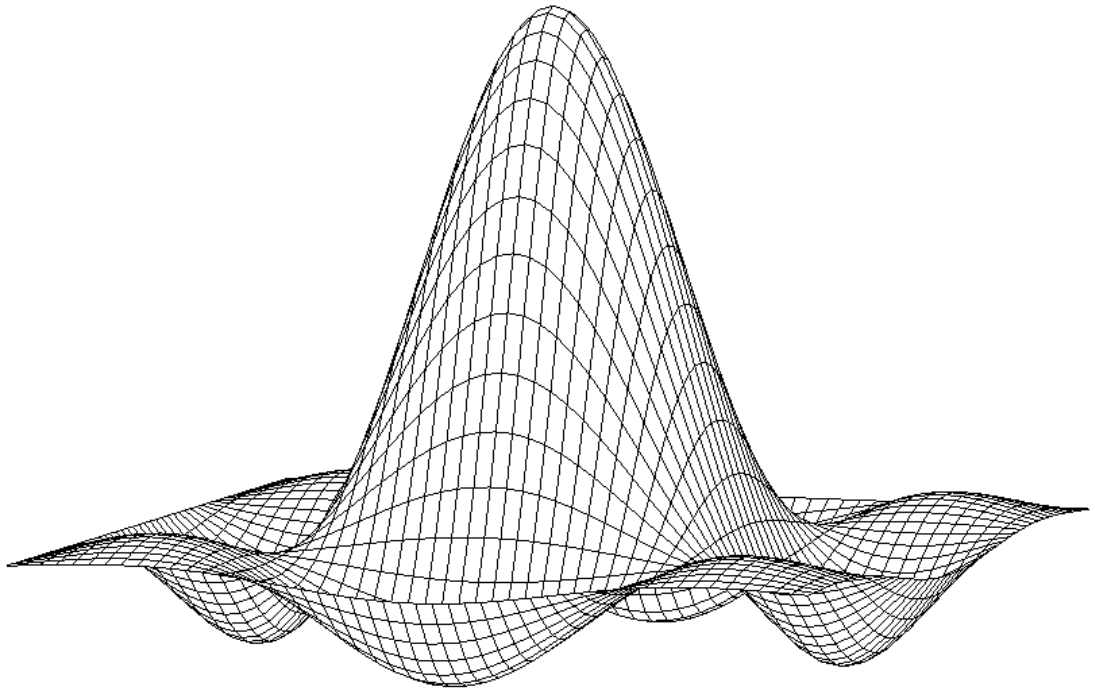


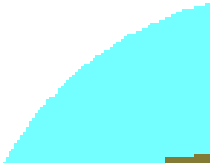


Polygon Offset

- Even though we draw the polygon first followed by the lines, small numerical errors cause some of fragments on the line to be display behind the corresponding fragment on the triangle
- Polygon offset (gl.polygonOffset) moves fragments slight away from camera
- Apply to triangle rendering

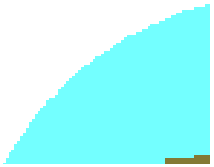
Hat with Polygon Offset





Other Mesh Issues

- How do we construct a mesh from disparate data (unstructured points)
- Technologies such as laser scans can produced tens of millions of such points
- Chapter 12: Delaunay triangulation
- Can we use one triangle strip for an entire 2D mesh?
- Mesh simplification



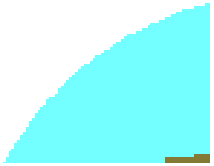
Introduction to Computer Graphics with WebGL

Ed Angel

Professor Emeritus of Computer Science

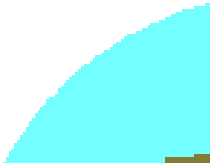
Founding Director, Arts, Research, Technology and Science Laboratory

University of New Mexico



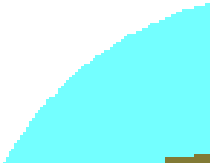
Shadows

Ed Angel
Professor Emeritus of Computer Science
University of New Mexico



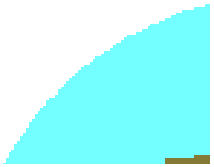
Objectives

- Introduce Shadow Algorithms
- Projective Shadows
- Shadow Maps
- Shadow Volumes



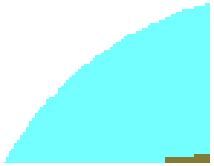
Flashlight in the Eye Graphics

- When do we not see shadows in a real scene?
- When the only light source is a point source at the eye or center of projection
  - Shadows are behind objects and not visible
- Shadows are a global rendering issue
  - Is a surface visible from a source
  - May be obscured by other objects



Shadows in Pipeline Renders

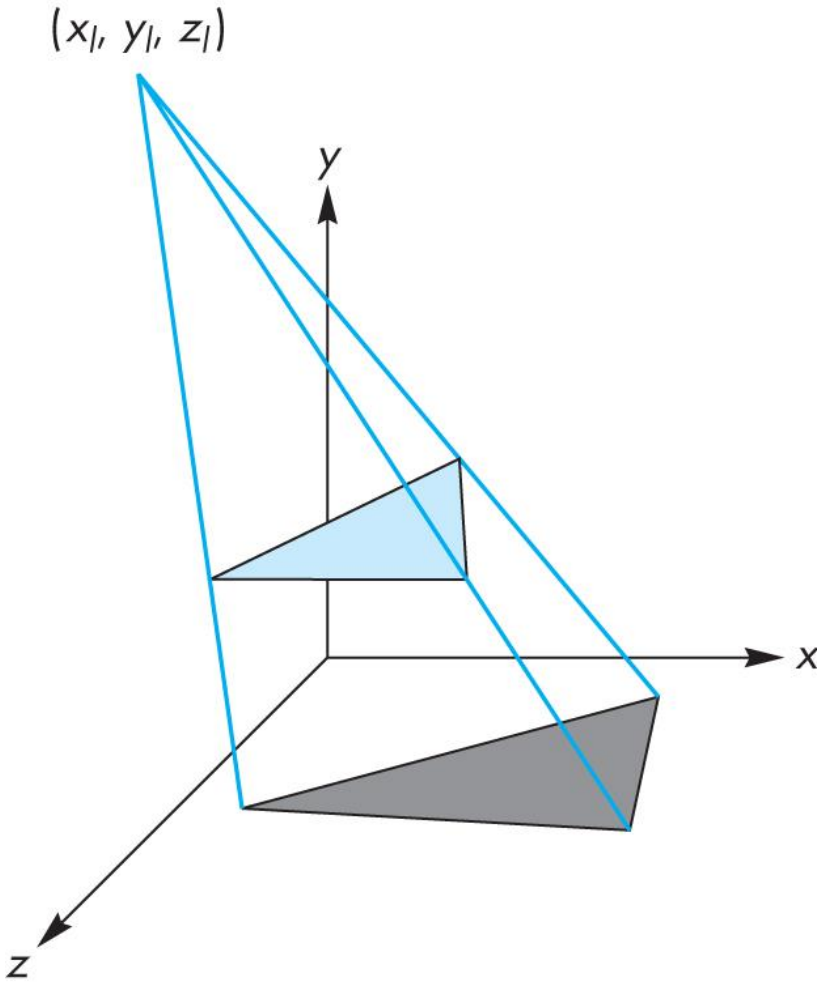
- Note that shadows are generated automatically by a ray tracers
  - feeler rays will detect if no light reaches a point
  - need all objects to be available
- Pipeline renderers work an object at a time so shadows are not automatic
  - can use some tricks: projective shadows
  - multi-rendering: shadow maps and shadow volumes



Projective Shadows

- Oldest methods
  - Used in flight simulators to provide visual clues
- Projection of a polygon is a polygon called a **shadow polygon**
- Given a point light source and a polygon, the vertices of the shadow polygon are the projections of the original polygon's vertices from a point source onto a surface

Shadow Polygon

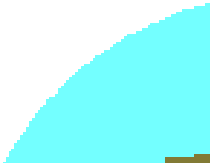


Computing Shadow Vertex

1. Source at (x_l, y_l, z_l)
2. Vertex at (x, y, z)
3. Consider simple case of shadow projected onto ground at $(x_p, 0, z_p)$
4. Translate source to origin with $T(-x_l, -y_l, -z_l)$
5. Perspective projection

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{-y_l} & 0 & 0 \end{bmatrix}$$

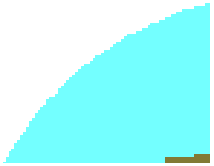
6. Translate back



Shadow Process

1. Put two identical triangles and their colors on GPU (black for shadow triangle)
2. Compute two model view matrices as uniforms
3. Send model view matrix for original triangle
4. Render original triangle
5. Send second model view matrix
6. Render shadow triangle

- Note shadow triangle undergoes two transformations
- Note hidden surface removal takes care of depth issues



Generalized Shadows

- Approach was OK for shadows on a single flat surface
- Note with geometry shader we can have the shader create the second triangle
- Cannot handle shadows on general objects
- Exist a variety of other methods based on same basic idea
- We'll pursue methods based on projective textures

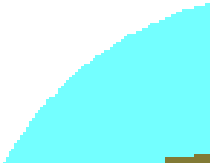
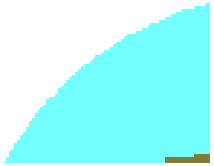


Image Based Lighting

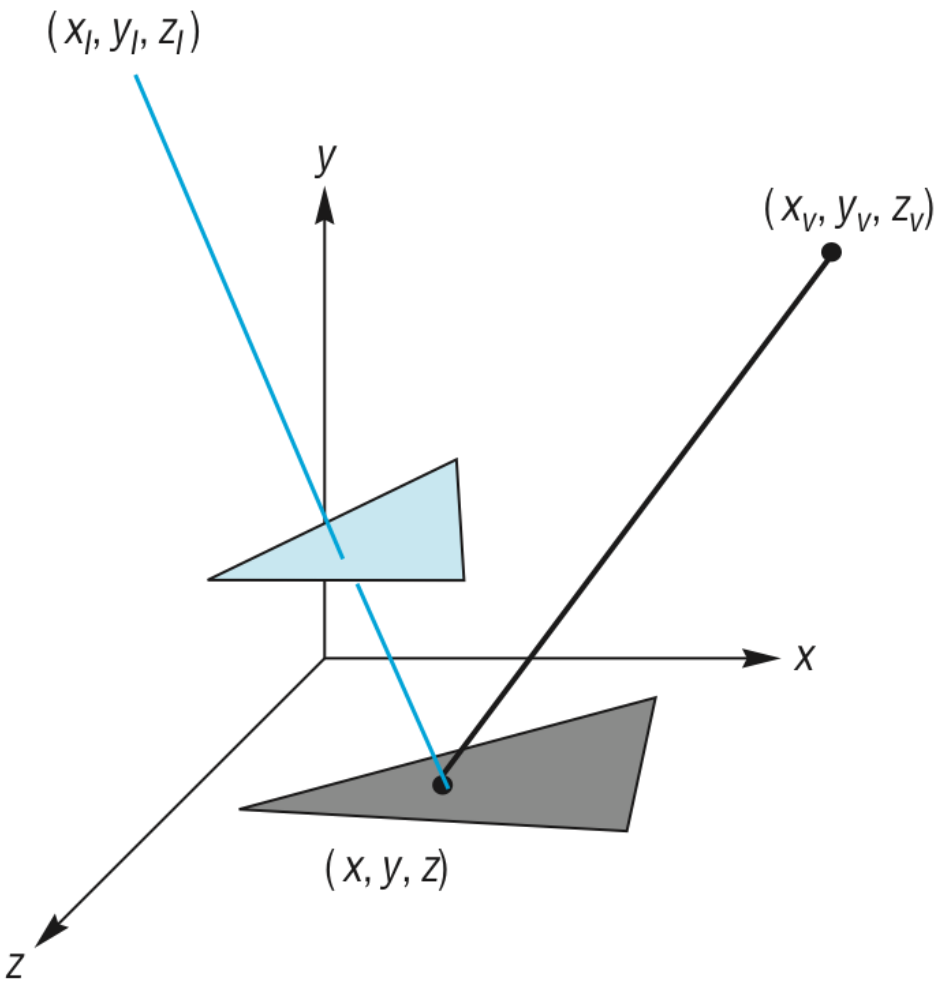
- We can project a texture onto the surface in which case the are treating the texture as a “slide projector”
- This technique the basis of projective textures and image based lighting
- Supported in desktop OpenGL and GLSL through four dimensional texture coordinates
- Not yet in WebGL

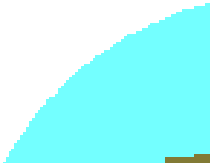


Shadow Maps

- If we render a scene from a light source, the depth buffer will contain the distances from the source to nearest lit fragment.
- We can store these depths in a texture called a **depth map** or **shadow map**
- Note that although we don’t care about the image in the shadow map, if we render with some light, anything lit is not in shadow.
- Form a shadow map for each source

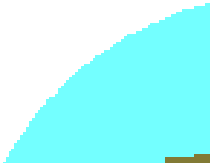
Shadow Mapping





Final Rendering

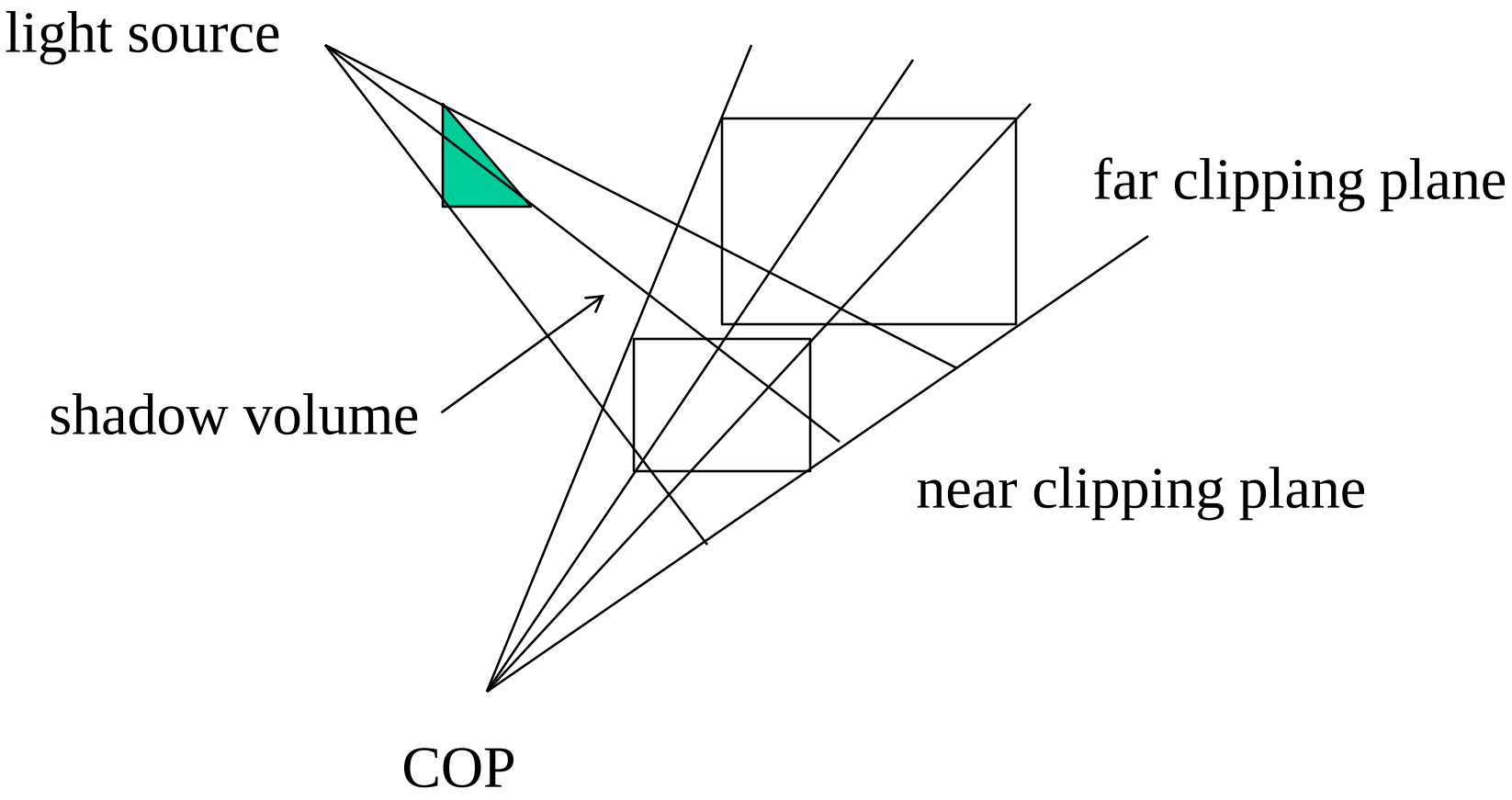
- During the final rendering we compare the distance from the fragment to the light source with the distance in the shadow map
- If the depth in the shadow map is less than the distance from the fragment to the source the fragment is in shadow (from this source)
- Otherwise we use the rendered color

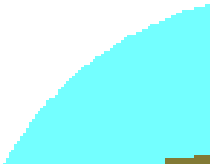


Implementation

- Requires multiple renderings
- We will look at render-to-texture later
  - gives us a method to save the results of a rendering as a texture
  - almost all work done in the shaders

Shadow Volumes





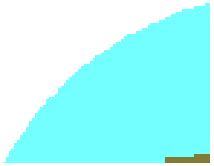
Introduction to Computer Graphics with WebGL

Ed Angel

Professor Emeritus of Computer Science

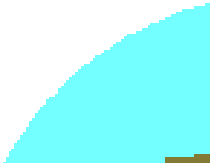
Founding Director, Arts, Research, Technology and Science Laboratory

University of New Mexico



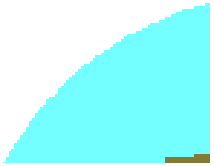
Lighting and Shading I

Ed Angel
Professor Emeritus of Computer Science
University of New Mexico



Objectives

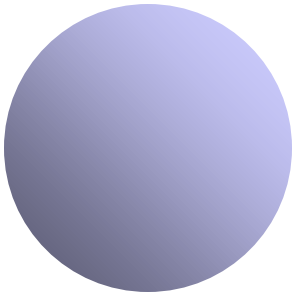
- Learn to shade objects so their images appear three-dimensional
- Introduce the types of light-material interactions
- Build a simple reflection model---the Phong model--- that can be used with real time graphics hardware



Why we need shading

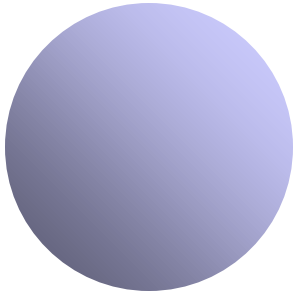
- Suppose we build a model of a sphere using many polygons and color it with `glColor`. We get something like

- But we want



Shading

- Why does the image of a real sphere look like

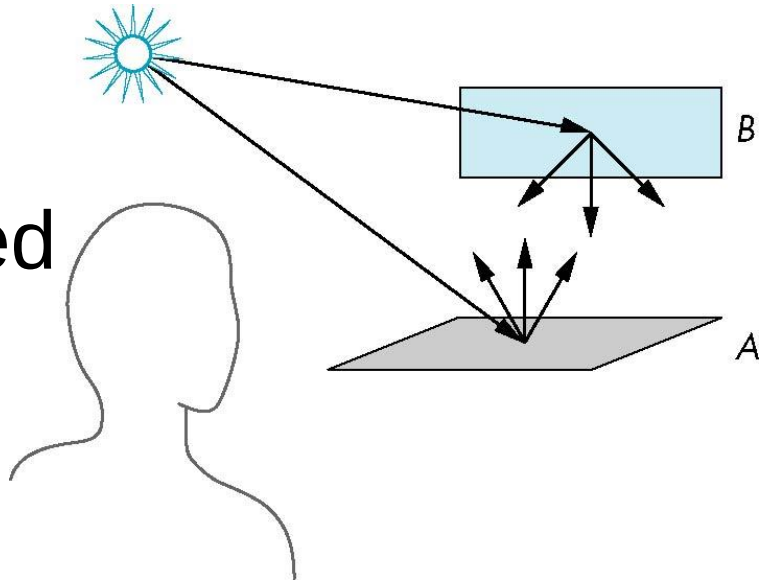


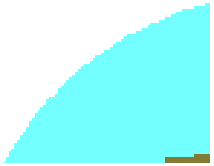
- Light-material interactions cause each point to have a different color or shade
- Need to consider
  - Light sources
  - Material properties
  - Location of viewer
  - Surface orientation

Scattering

- Light strikes A
  - Some scattered
  - Some absorbed
- Some of scattered light strikes B
  - Some scattered
  - Some absorbed
- Some of this scattered light strikes A

  and so on

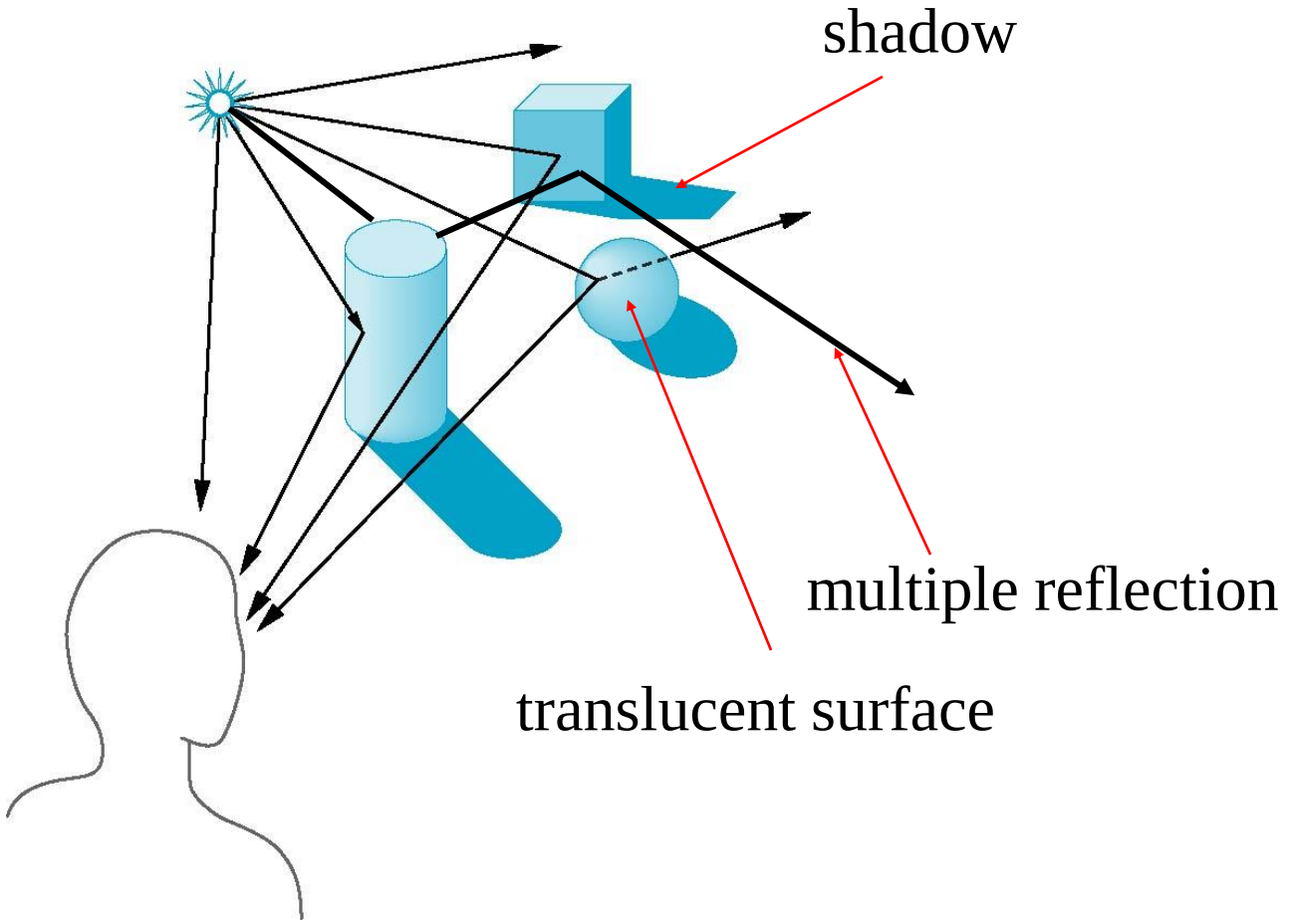


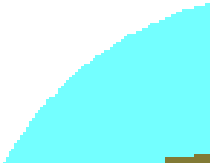


Rendering Equation

- The infinite scattering and absorption of light can be described by the *rendering equation*
  - Cannot be solved in general
  - Ray tracing is a special case for perfectly reflecting surfaces
- Rendering equation is global and includes
  - Shadows
  - Multiple scattering from object to object

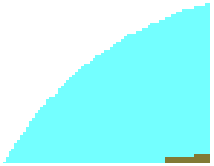
Global Effects





Local vs Global Rendering

- Correct shading requires a global calculation involving all objects and light sources
  - Incompatible with pipeline model which shades each polygon independently (local rendering)
- However, in computer graphics, especially real time graphics, we are happy if things "look right"
  - Exist many techniques for approximating global effects

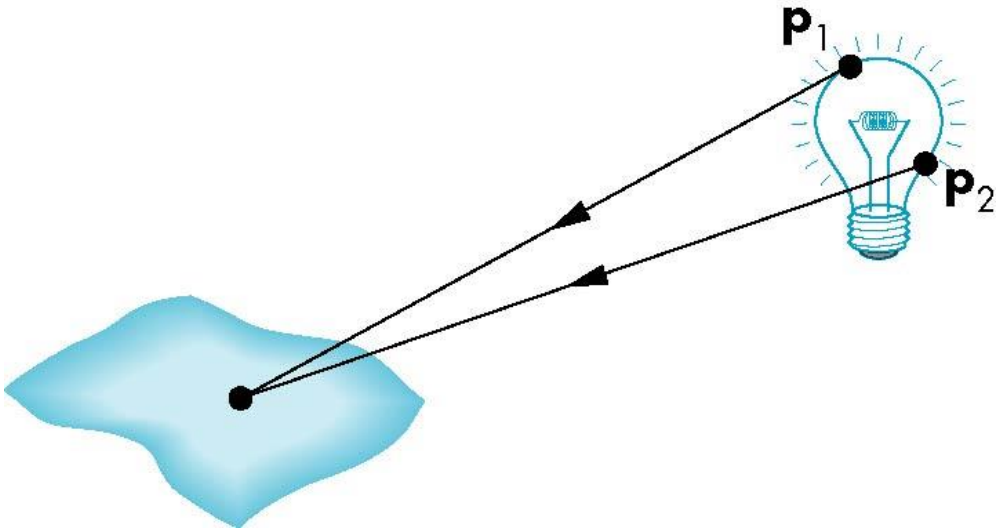


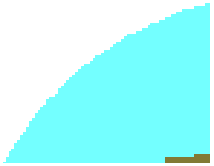
Light-Material Interaction

- Light that strikes an object is partially absorbed and partially scattered (reflected)
- The amount reflected determines the color and brightness of the object
  - A surface appears red under white light because the red component of the light is reflected and the rest is absorbed
- The reflected light is scattered in a manner that depends on the smoothness and orientation of the surface

Light Sources

General light sources are difficult to work with because we must integrate light coming from all points on the source



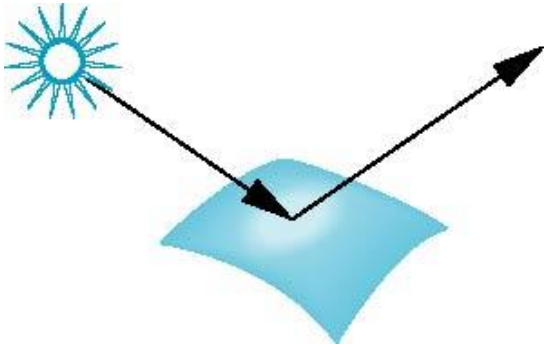


Simple Light Sources

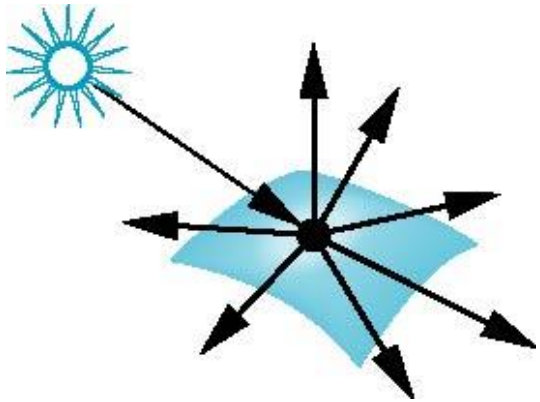
- Point source
  - Model with position and color
  - Distant source = infinite distance away (parallel)
- Spotlight
  - Restrict light from ideal point source
- Ambient light
  - Same amount of light everywhere in scene
  - Can model contribution of many sources and reflecting surfaces

Surface Types

- The smoother a surface, the more reflected light is concentrated in the direction a perfect mirror would reflected the light
- A very rough surface scatters light in all directions



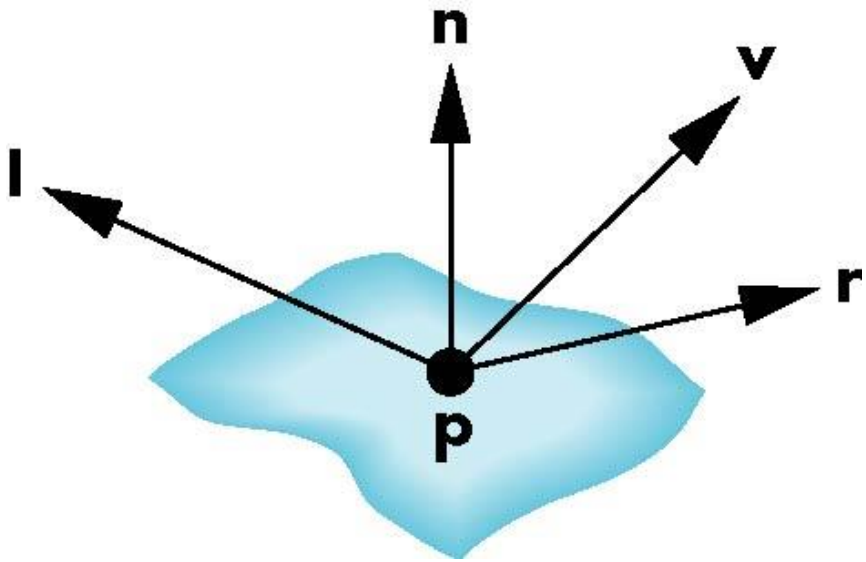
smooth surface



rough surface

Phong Model

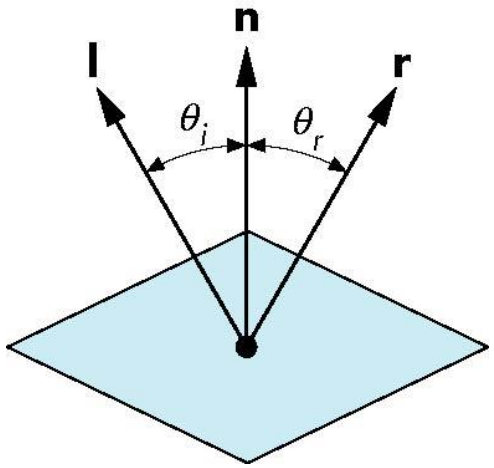
- A simple model that can be computed rapidly
- Has three components
  - Diffuse
  - Specular
  - Ambient
- Uses four vectors
  - To source
  - To viewer
  - Normal
  - Perfect reflector

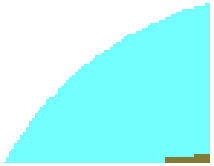


Ideal Reflector

- Normal is determined by local orientation
- Angle of incidence = angle of relection
- The three vectors must be coplanar

$$\mathbf{r} = 2\,(\mathbf{l} \cdot \mathbf{n})\,\mathbf{n} - \mathbf{l}$$



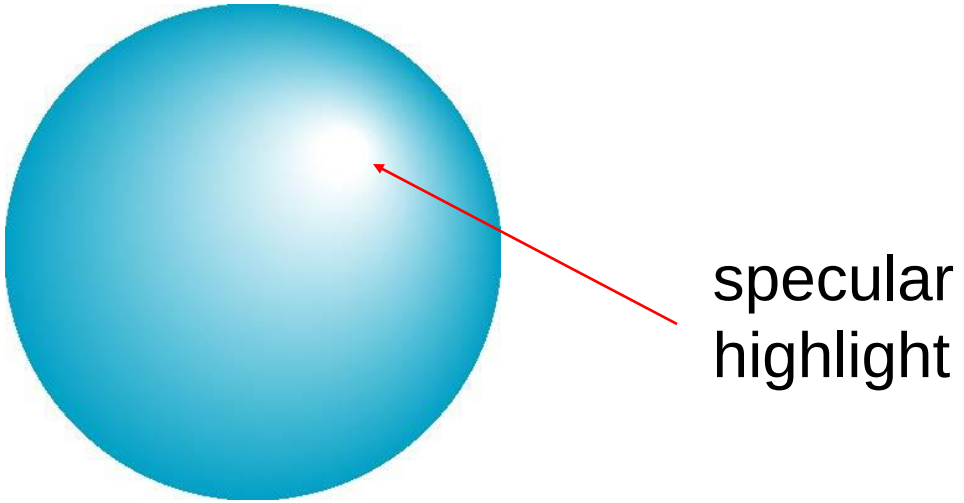


Lambertian Surface

- Perfectly diffuse reflector
- Light scattered equally in all directions
- Amount of light reflected is proportional to the vertical component of incoming light
  - reflected light ~$\cos \theta_i$
  - $\cos \theta_i = \mathbf{l} \cdot \mathbf{n}$ if vectors normalized
  - There are also three coefficients, k_r, k_b, k_g that show how much of each color component is reflected

Specular Surfaces

- Most surfaces are neither ideal diffusers nor perfectly specular (ideal reflectors)
- Smooth surfaces show specular highlights due to incoming light being reflected in directions concentrated close to the direction of a perfect reflection



Modeling Specular Relections

- Phong proposed using a term that dropped off as the angle between the viewer and the ideal reflection increased

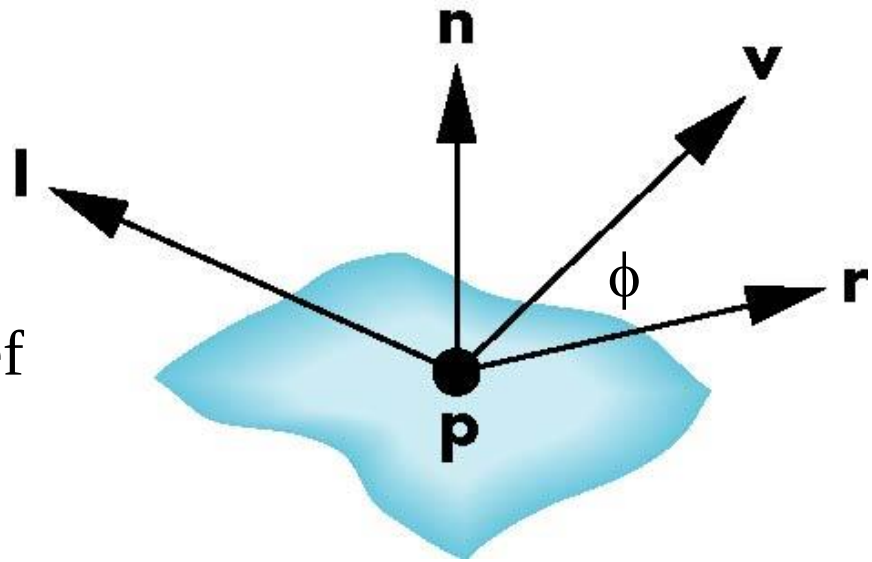
$$\mathbf{I}_r \sim k_s \mathbf{I} \cos^{\alpha}\phi$$

reflected intensity

absorption coef

incoming intensity

shininess coef



The Shininess Coefficient

- Values of α between 100 and 200 correspond to metals
- Values between 5 and 10 give surface that look like plastic