Introduction to Computer Graphics with WebGL

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Hierarchical Modeling I

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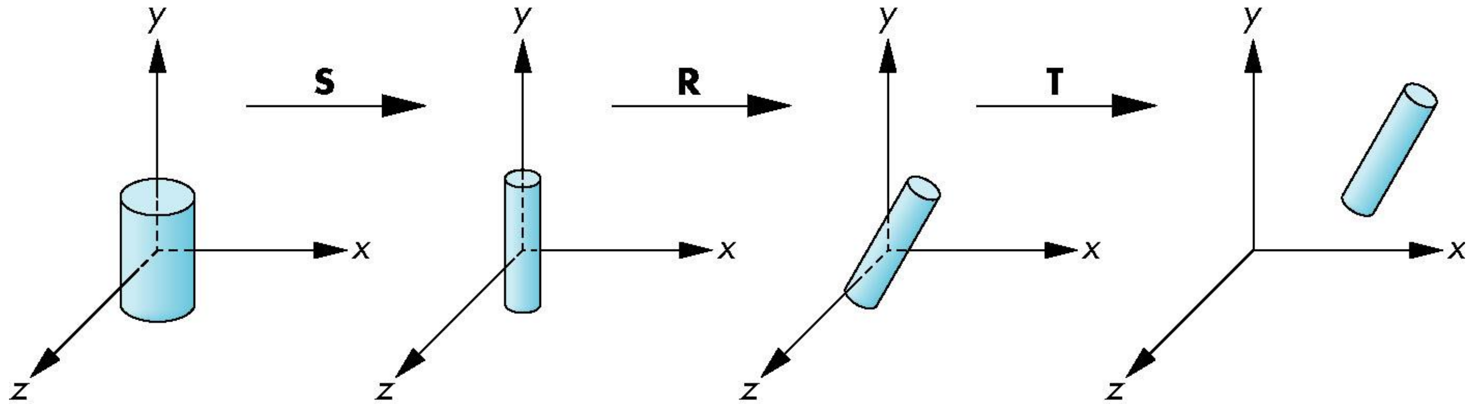
Objectives

- Examine the limitations of linear modeling
  - Symbols and instances
- Introduce hierarchical models
  - Articulated models
  - Robots
- Introduce Tree and DAG models

Instance Transformation

- Start with a prototype object (a *symbol*)
- Each appearance of the object in the model is an *instance*
  - Must scale, orient, position
  - Defines instance transformation



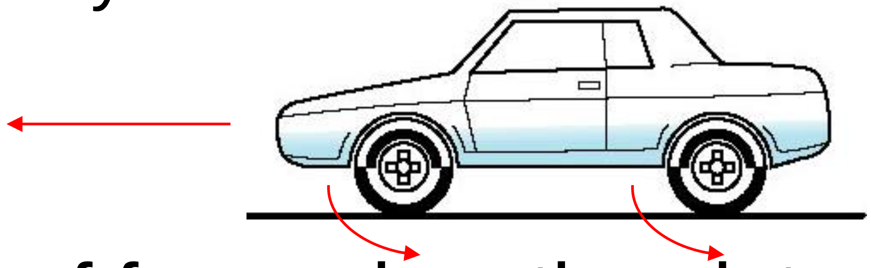
Symbol-Instance Table

Can store a model by assigning a number to each symbol and storing the parameters for the instance transformation

Symbol	Scale	Rotate	Translate
1	s_x, s_y, s_z	$\theta_x, \theta_y, \theta_z$	d_x, d_y, d_z
2			
3			
1			
1			
.			
.			

Relationships in Car Model

- Symbol-instance table does not show relationships between parts of model
- Consider model of car
  - Chassis + 4 identical wheels
  - Two symbols



- Rate of forward motion determined by rotational speed of wheels

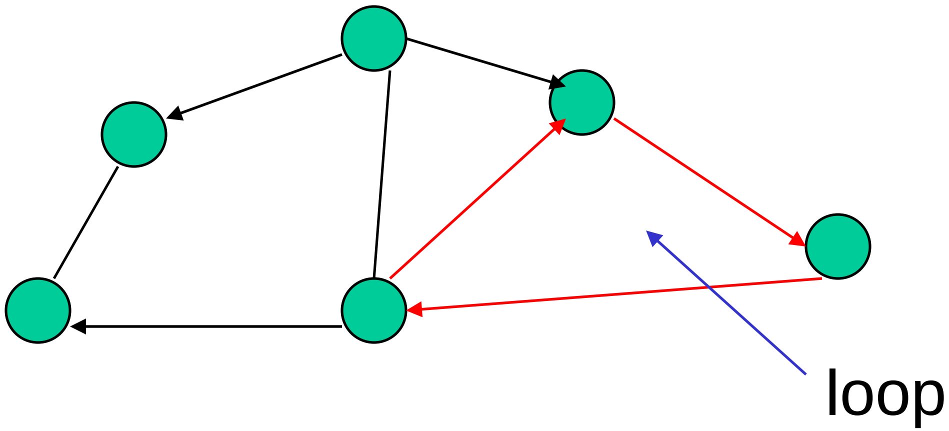
Structure Through Function Calls

```
car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}
```

- Fails to show relationships well
- Look at problem using a graph

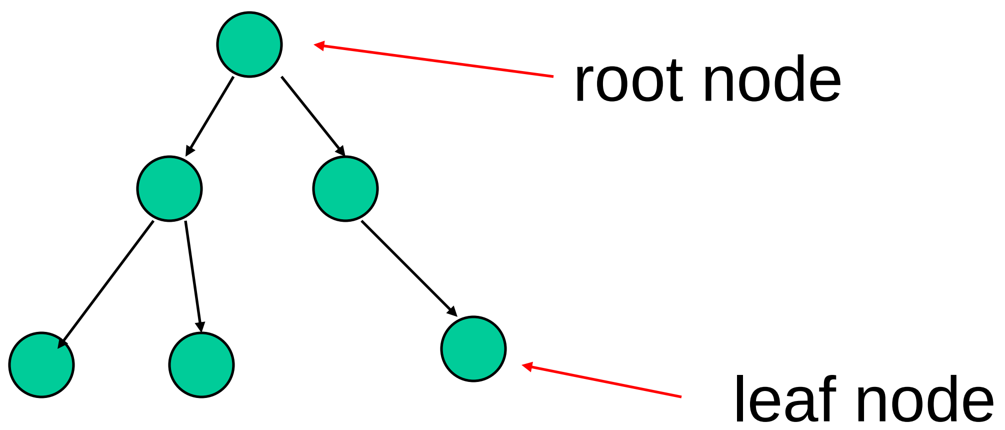
Graphs

- Set of *nodes* and *edges (links)*
- Edge connects a pair of nodes
  - Directed or undirected
- *Cycle*: directed path that is a loop

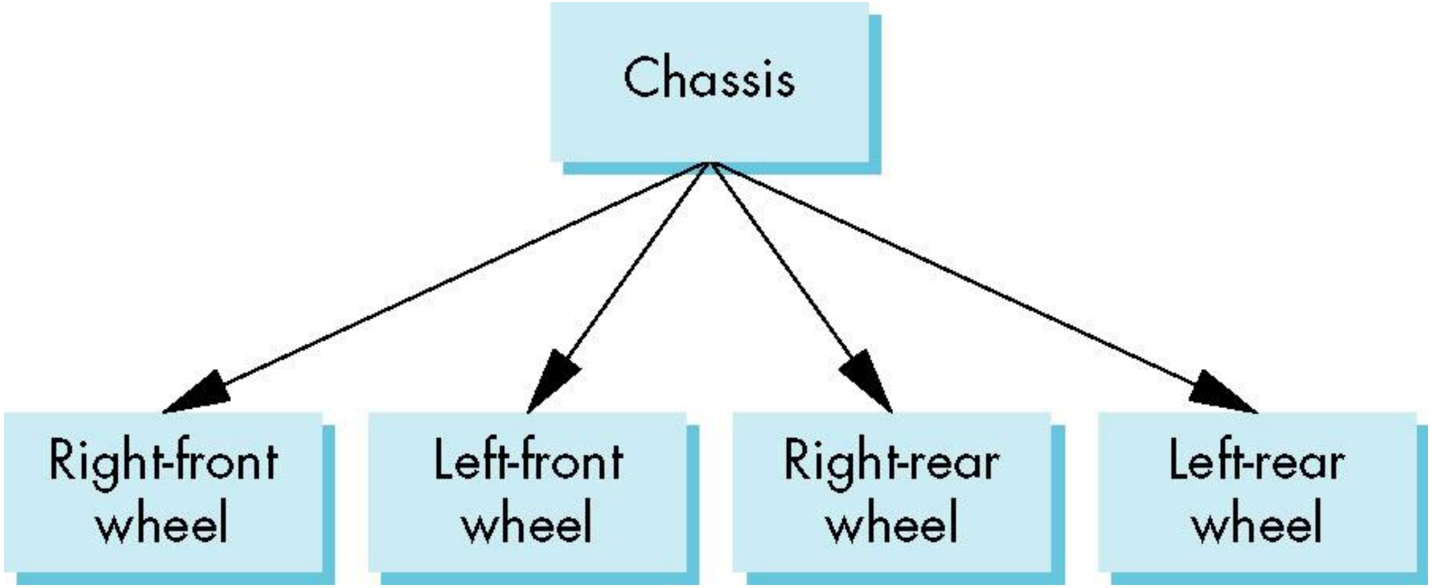


Tree

- Graph in which each node (except the root) has exactly one parent node
  - May have multiple children
  - Leaf or terminal node: no children

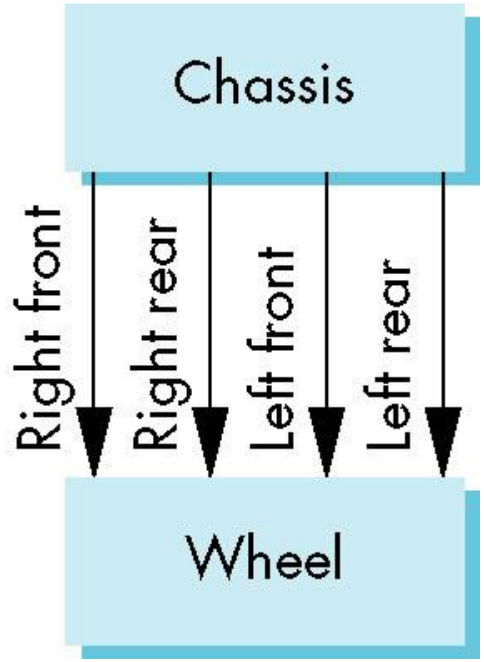


Tree Model of Car



DAG Model

- If we use the fact that all the wheels are identical, we get a *directed acyclic graph*
  - Not much different than dealing with a tree



Modeling with Trees

- Must decide what information to place in nodes and what to put in edges
- Nodes
  - What to draw
  - Pointers to children
- Edges
  - May have information on incremental changes to transformation matrices (can also store in nodes)

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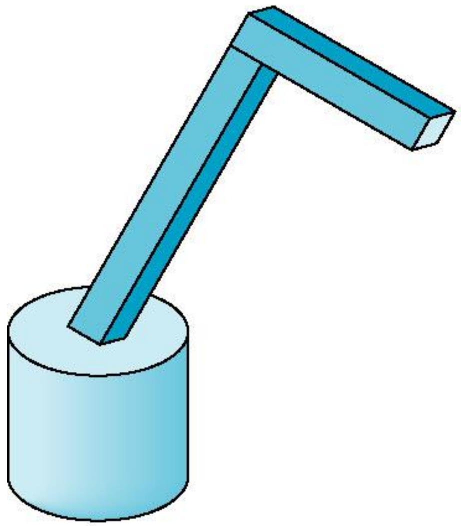
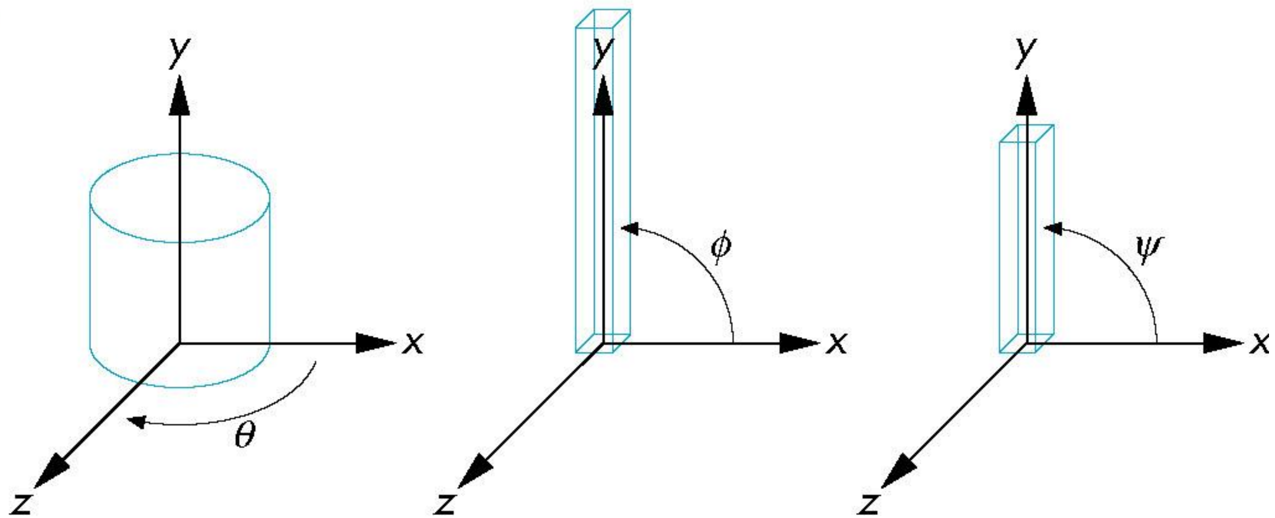
Robot Arm



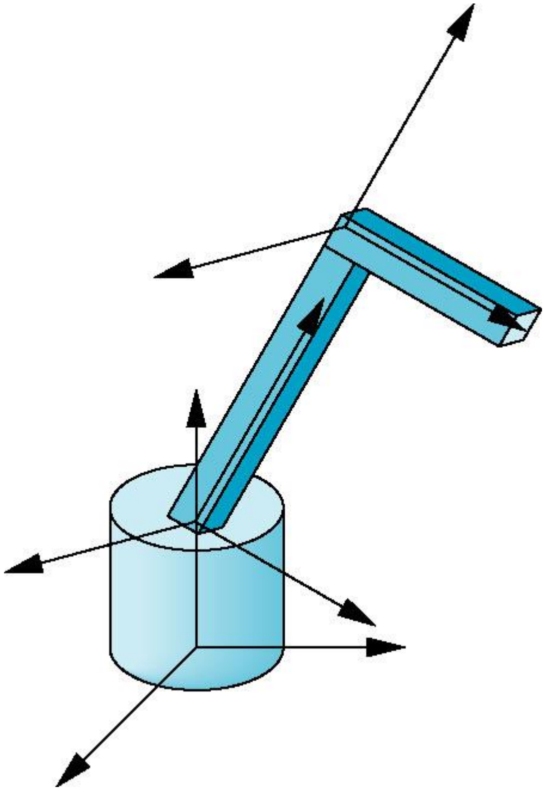
robot arm



parts in their own coodinate systems

Articulated Models

- Robot arm is an example of an *articulated model*
  - Parts connected at joints
  - Can specify state of model by giving all joint angles



Relationships in Robot Arm

- Base rotates independently
  - Single angle determines position
- Lower arm attached to base
  - Its position depends on rotation of base
  - Must also translate relative to base and rotate about connecting joint
- Upper arm attached to lower arm
  - Its position depends on both base and lower arm
  - Must translate relative to lower arm and rotate about joint connecting to lower arm

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Required Matrices

- Rotation of base: $\mathbf{R}_b$
  - Apply $\mathbf{M} = \mathbf{R}_b$ to base
- Translate lower arm relative to base: $\mathbf{T}_{lu}$
- Rotate lower arm around joint: $\mathbf{R}_{lu}$
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu}$ to lower arm
- Translate upper arm relative to upper arm: $\mathbf{T}_{uu}$
- Rotate upper arm around joint: $\mathbf{R}_{uu}$
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu} \mathbf{T}_{uu} \mathbf{R}_{uu}$ to upper arm

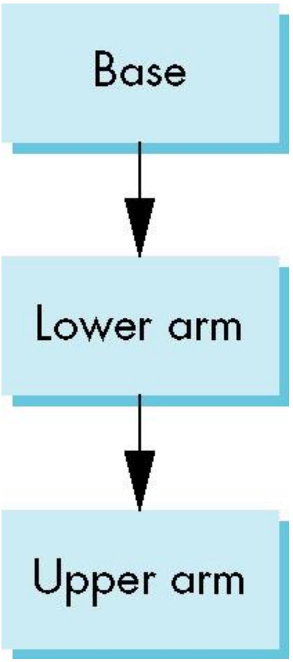
WebGL Code for Robot

```
var render = function() {
   gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT );
   modelViewMatrix = rotate(theta[Base], 0, 1, 0 );
   base();
   modelViewMatrix = mult(modelViewMatrix,
           translate(0.0, BASE_HEIGHT, 0.0));
   modelViewMatrix = mult(modelViewMatrix,
            rotate(theta[LowerArm], 0, 0, 1 ));
   lowerArm();
   modelViewMatrix  = mult(modelViewMatrix,
           translate(0.0, LOWER_ARM_HEIGHT, 0.0));
   modelViewMatrix  = mult(modelViewMatrix,
           rotate(theta[UpperArm], 0, 0, 1) );
   upperArm();
   requestAnimFrame(render);
}
```

Tree Model of Robot

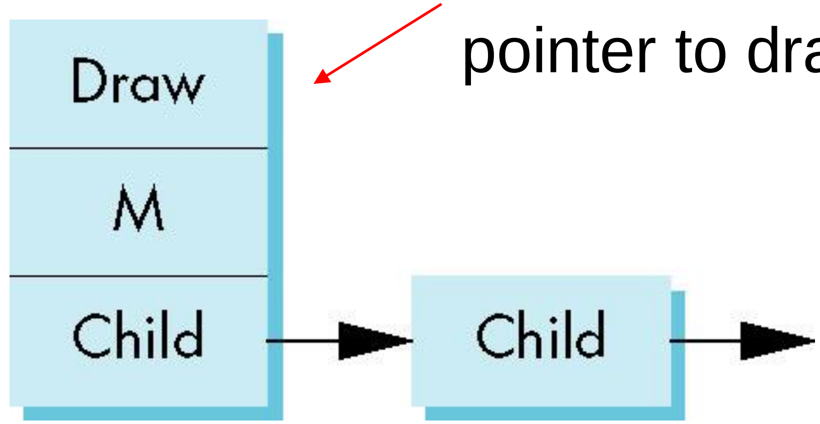
- Note code shows relationships between parts of model
  - Can change "look" of parts easily without altering relationships
- Simple example of tree model
- Want a general node structure

for nodes



Possible Node Structure

Code for drawing part or pointer to drawing function



linked list of pointers to children

matrix relating node to parent

Generalizations

- Need to deal with multiple children
  - How do we represent a more general tree?
  - How do we traverse such a data structure?
- Animation
  - How to use dynamically?
  - Can we create and delete nodes during execution?

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Hierarchical Modeling II

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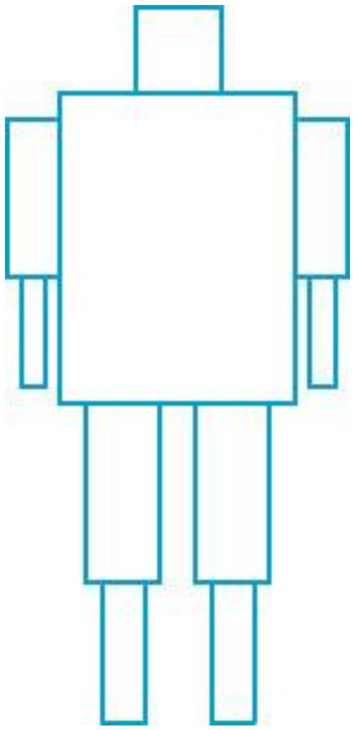
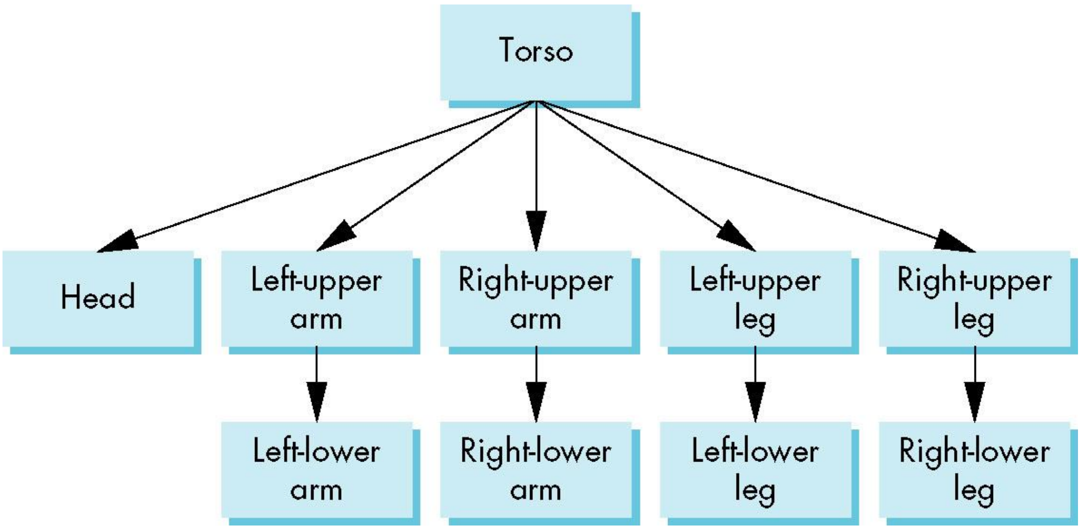
Objectives

- Build a tree-structured model of a humanoid figure
- Examine various traversal strategies
- Build a generalized tree-model structure that is independent of the particular model

Humanoid Figure

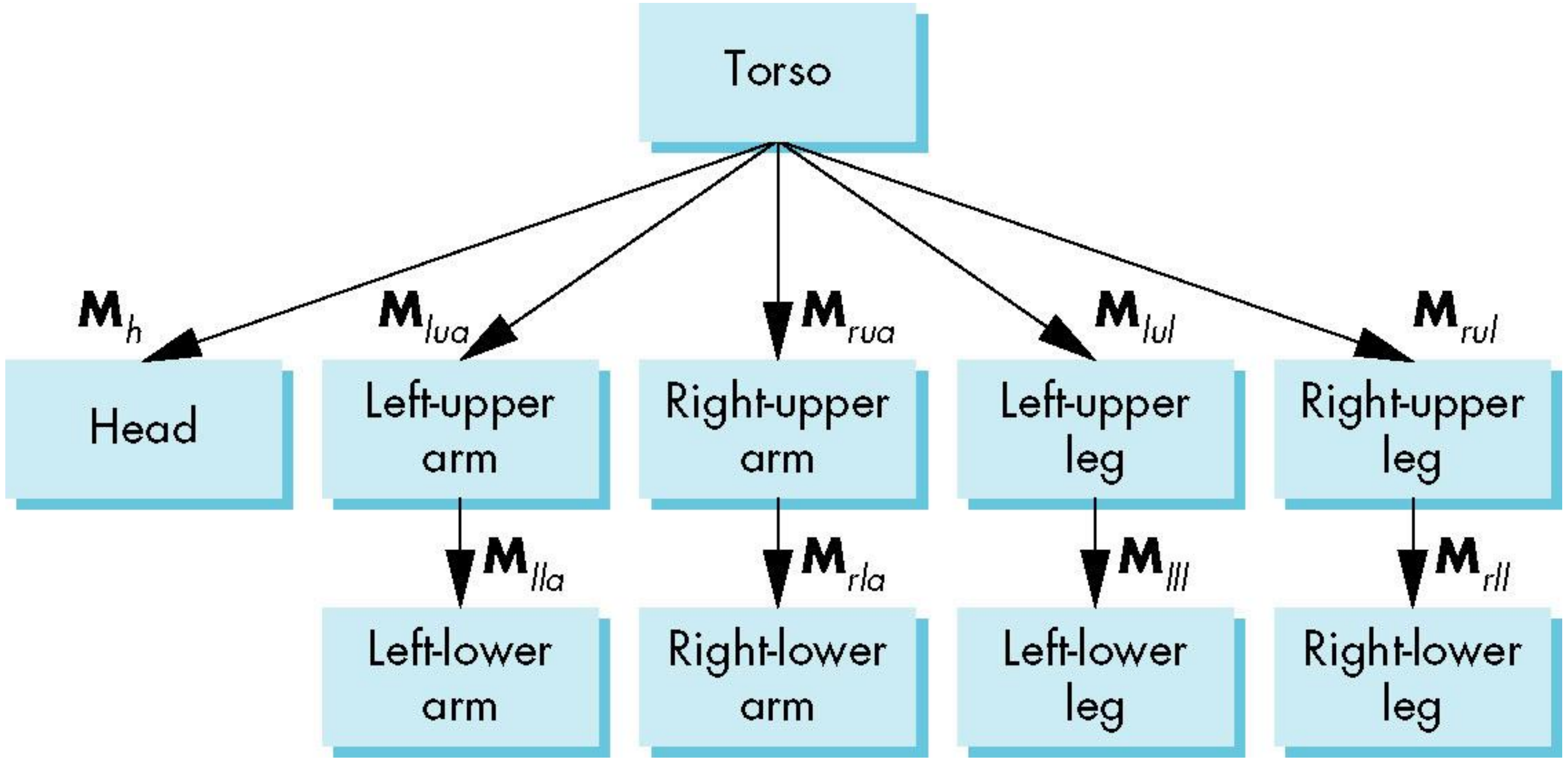




Building the Model

- Can build a simple implementation using quadrics: ellipsoids and cylinders
- Access parts through functions
  - **`torso()`**
  - **`leftUpperArm()`**
- Matrices describe position of node with respect to its parent
  - $\mathbf{M}_{lla}$ positions left lower leg with respect to left upper arm

Tree with Matrices



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Display and Traversal

- The position of the figure is determined by 11 joint angles (two for the head and one for each other part)
- Display of the tree requires a *graph traversal*
  - Visit each node once
  - Display function at each node that describes the part associated with the node, applying the correct transformation matrix for position and orientation

Transformation Matrices

- There are 10 relevant matrices
  - $\mathbf{M}$ positions and orients entire figure through the torso which is the root node
  - $\mathbf{M}_h$ positions head with respect to torso
  - $\mathbf{M}_{lua}$, $\mathbf{M}_{rua}$, $\mathbf{M}_{lul}$, $\mathbf{M}_{rul}$ position arms and legs with respect to torso
  - $\mathbf{M}_{lla}$, $\mathbf{M}_{rla}$, $\mathbf{M}_{lll}$, $\mathbf{M}_{rll}$ position lower parts of limbs with respect to corresponding upper limbs

Stack-based Traversal

- Set model-view matrix to $\mathbf{M}$ and draw torso
- Set model-view matrix to $\mathbf{MM}_h$ and draw head
- For left-upper arm need $\mathbf{MM}_{lua}$ and so on
- Rather than recomputing $\mathbf{MM}_{lua}$ from scratch or using an inverse matrix, we can use the matrix stack to store $\mathbf{M}$ and other matrices as we traverse the tree

Traversal Code

```
figure() {
    PushMatrix()
    torso();
    Rotate (…);
    head();
    PopMatrix();
    PushMatrix();
    Translate(…);
    Rotate(…);
    left_upper_arm();
    PopMatrix();
    PushMatrix();
```

save present model-view matrix

update model-view matrix for head

recover original model-view matrix

save it again

update model-view matrix for left upper arm

recover and save original model-view matrix again

rest of code

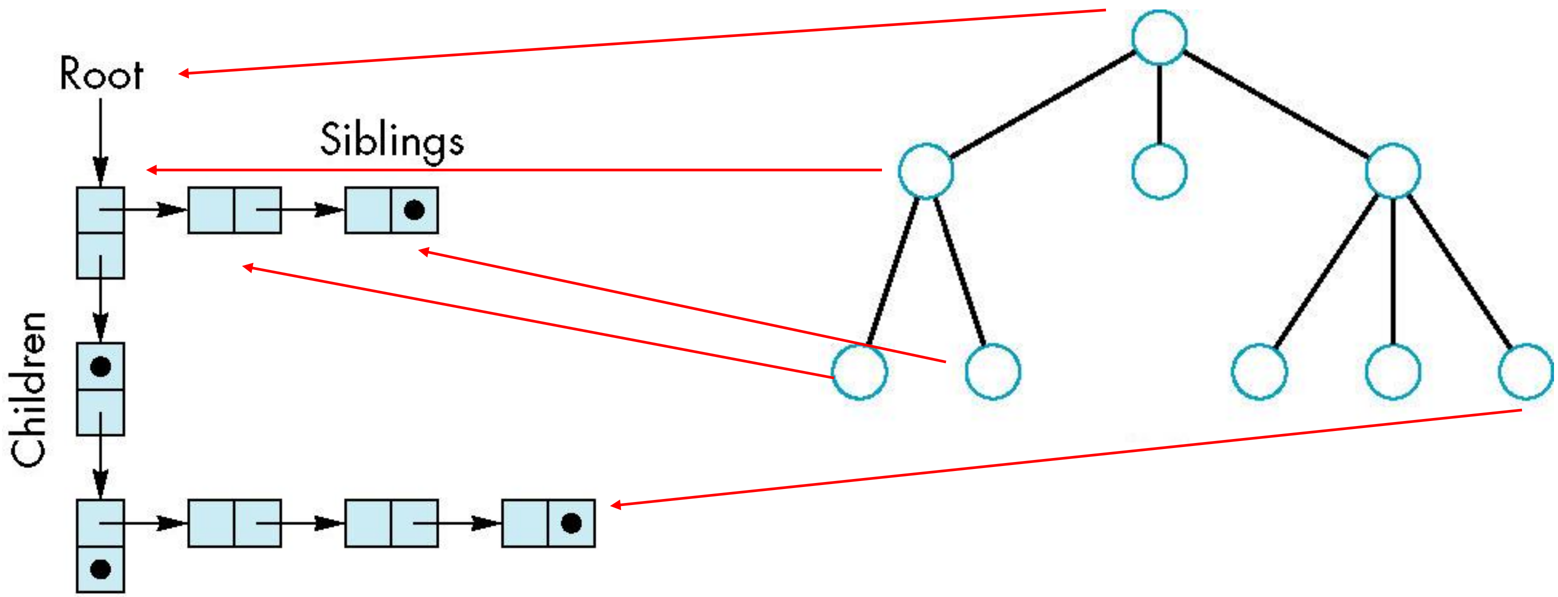
Analysis

- The code describes a particular tree and a particular traversal strategy
  - Can we develop a more general approach?
- Note that the sample code does not include state changes, such as changes to colors
  - May also want to push and pop other attributes to protect against unexpected state changes affecting later parts of the code

General Tree Data Structure

- Need a data structure to represent tree and an algorithm to traverse the tree
- We will use a *left-child right sibling* structure
    - Uses linked lists
    - Each node in data structure is two pointers
    - Left: next node
    - Right: linked list of children

Left-Child Right-Sibling Tree



Tree node Structure

- At each node we need to store
  - Pointer to sibling
  - Pointer to child
  - Pointer to a function that draws the object represented by the node
  - Homogeneous coordinate matrix to multiply on the right of the current model-view matrix
    - Represents changes going from parent to node
    - In WebGL this matrix is a 1D array storing matrix by columns

Creating a treenode

```
function createNode(transform,
            render, sibling, child) {
    var node = {
    transform: transform,
    render: render,
    sibling: sibling,
    child: child,
    }
    return node;
};
```

Initializing Nodes

```
function initNodes(Id) {
    var m = mat4();
        switch(Id) {
        case torsoId:
            m = rotate(theta[torsoId], 0, 1, 0 );
          figure[torsoId] = createNode( m, torso, null, headId );
          break;
      case head1Id:
      case head2Id:
          m = translate(0.0, torsoHeight+0.5*headHeight, 0.0);
          m = mult(m, rotate(theta[head1Id], 1, 0, 0));
          m = mult(m, rotate(theta[head2Id], 0, 1, 0));
          m = mult(m, translate(0.0, -0.5*headHeight, 0.0));
          figure[headId] = createNode( m, head, leftUpperArmId, null);
          break;
```

Notes

- The position of figure is determined by 11 joint angles stored in **`theta[11]`**
- Animate by changing the angles and redisplaying
- We form the required matrices using **`rotate`** and **`translate`**
- Because the matrix is formed using the model-view matrix, we may want to first push original model-view matrix on matrix stack

Preorder Traversal

```
function traverse(Id) {
   if(Id == null) return;
   stack.push(modelViewMatrix);
   modelViewMatrix = mult(modelViewMatrix, figure[Id].transform);
   figure[Id].render();
   if(figure[Id].child != null) traverse(figure[Id].child);
   modelViewMatrix = stack.pop();
   if(figure[Id].sibling != null) traverse(figure[Id].sibling);
 }
var render = function() {
        gl.clear( gl.COLOR_BUFFER_BIT );
        traverse(torsoId);
        requestAnimFrame(render);
}
```

Notes

- We must save model-view matrix before multiplying it by node matrix
  - Updated matrix applies to children of node but not to siblings which contain their own matrices
- The traversal program applies to any left-child right-sibling tree
  - The particular tree is encoded in the definition of the individual nodes
- The order of traversal matters because of possible state changes in the functions

Dynamic Trees

- Because we are using JS, the nodes and the node structure can be changed during execution
- Definition of nodes and traversal are essentially the same as before but we can add and delete nodes during execution
- In desktop OpenGL, if we use pointers, the structure can be dynamic