Introduction to Computer Graphics with WebGL

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Rendering Curves and Surfaces

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Objectives

- Introduce methods to draw curves
  - Approximate with lines
  - Finite Differences
- Derive the recursive method for evaluation of Bezier curves and surfaces
- Learn how to convert all polynomial data to data for Bezier polynomials

Evaluating Polynomials

- Simplest method to render a polynomial curve is to evaluate the polynomial at many points and form an approximating polyline
- For surfaces we can form an approximating mesh of triangles or quadrilaterals
- Use Horner's method to evaluate polynomials

$$p(u)=c_0+u(c_1+u(c_2+uc_3))$$

- 3 multiplications/evaluation for cubic

deCasteljau Recursion

- We can use the convex hull property of Bezier curves to obtain an efficient recursive method that does not require any function evaluations
  - Uses only the values at the control points
- Based on the idea that “any polynomial and any part of a polynomial is a Bezier polynomial for properly chosen control data”

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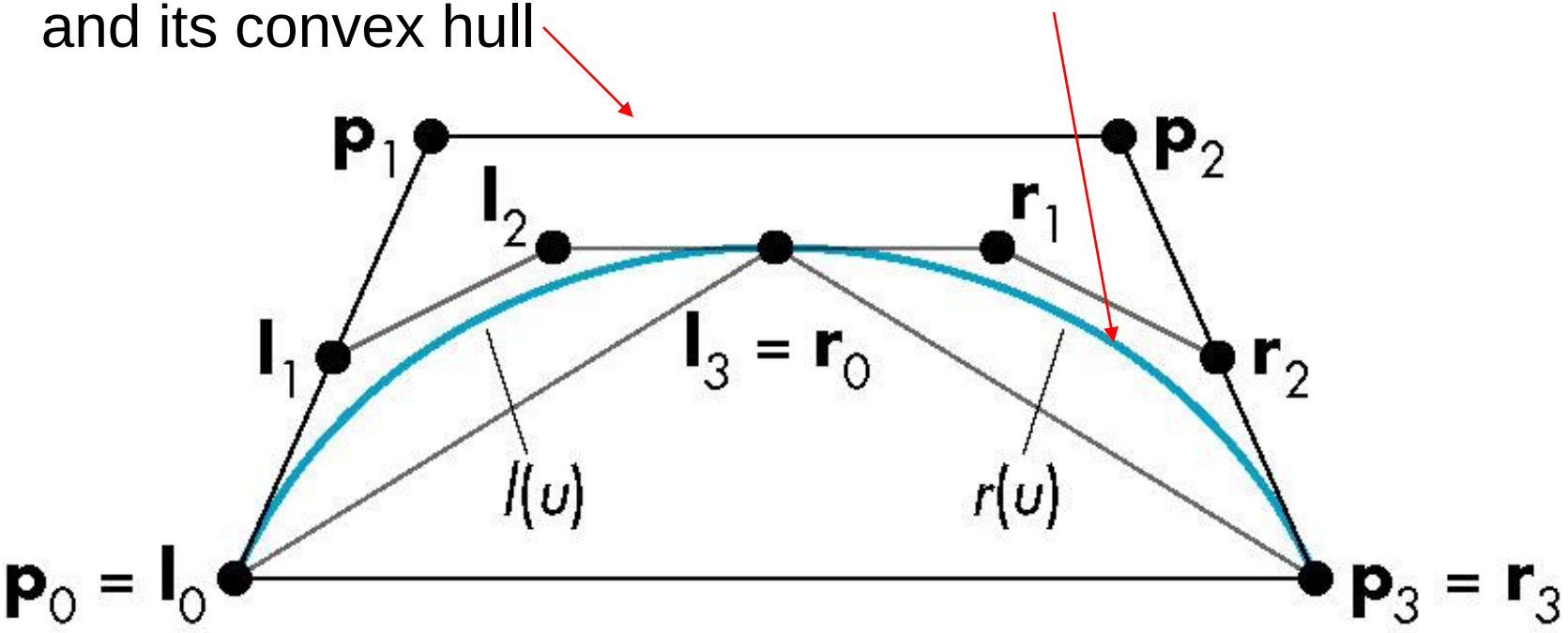
Splitting a Cubic Bezier

p_0, p_1, p_2, p_3 determine a cubic Bezier polynomial and its convex hull



Consider left half $l(u)$ and right half $r(u)$

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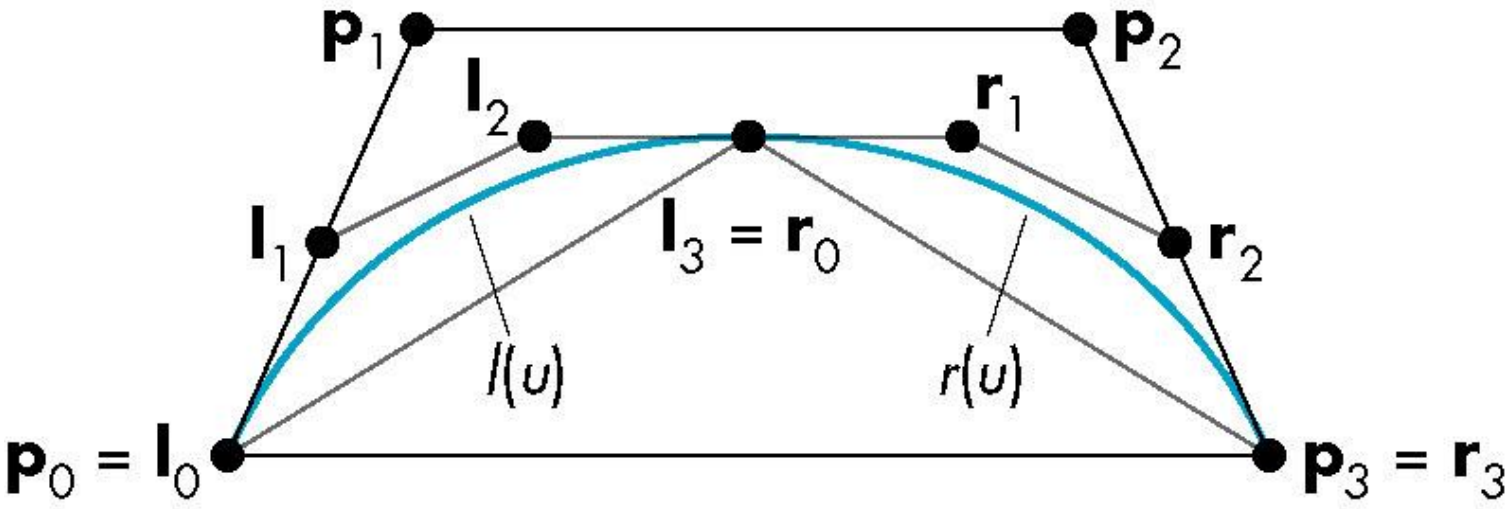
l(u) and r(u)

Since l(u) and r(u) are Bezier curves, we should be able to find two sets of control points $\{l_0, l_1, l_2, l_3\}$ and $\{r_0, r_1, r_2, r_3\}$ that determine them



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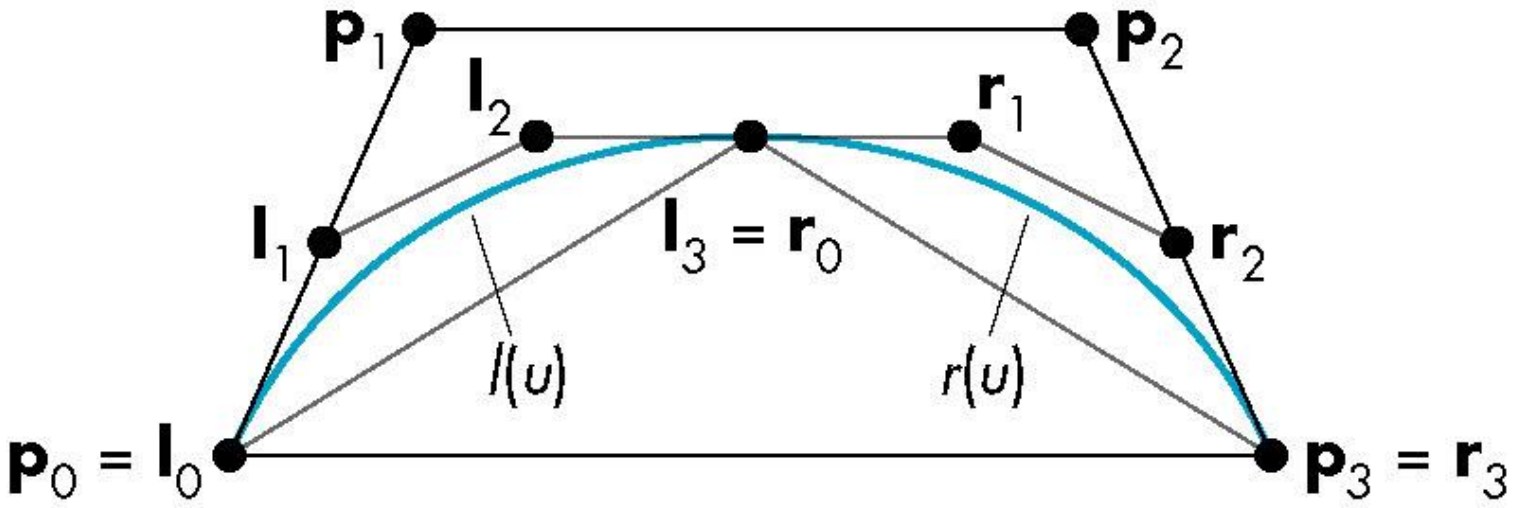
Convex Hulls

$\{l_0, l_1, l_2, l_3\}$ and $\{r_0, r_1, r_2, r_3\}$each have a convex hull that that is closer to p(u) than the convex hull of $\{p_0, p_1, p_2, p_3\}$ This is known as the *variation diminishing property*.

The polyline from l_0 to l_3 $(= r_0)$ to r_3 is an approximation to p(u). Repeating recursively we get better approximations.



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Equations

Start with Bezier equations $p(u)=\mathbf{u}^T M_B \mathbf{p}$

$l(u)$ must interpolate $p(0)$ and $p(1/2)$

$$l(0) = l_0 = p_0$$
$$l(1) = l_3 = p(1/2) = 1/8(p_0 + 3 p_1 + 3 p_2 + p_3)$$

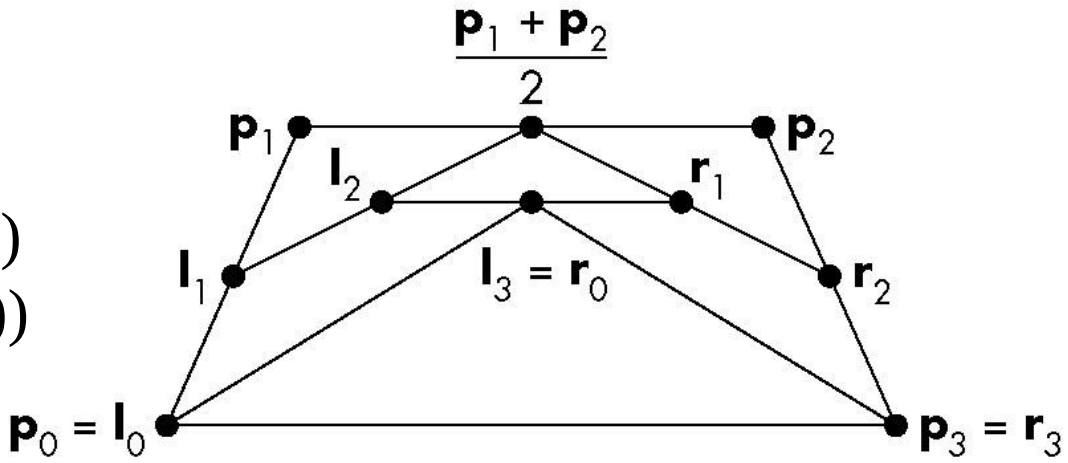
Matching slopes, taking into account that $l(u)$ and $r(u)$ only go over half the distance as $p(u)$

$$l'(0) = 3(l_1 - l_0) = p'(0) = 3/2(p_1 - p_0)$$
$$l'(1) = 3(l_3 - l_2) = p'(1/2) = 3/8(- p_0 - p_1 + p_2 + p_3)$$

Symmetric equations hold for $r(u)$

Efficient Form

$l_0 = p_0$
$r_3 = p_3$
$l_1 = ½(p_0 + p_1)$
$r_1 = ½(p_2 + p_3)$
$l_2 = ½(l_1 + ½(p_1 + p_2))$
$r_1 = ½(r_2 + ½(p_1 + p_2))$
$l_3 = r_0 = ½(l_2 + r_1)$



Requires only shifts and adds!

Every Curve is a Bezier Curve

- We can render a given polynomial using the recursive method if we find control points for its representation as a Bezier curve
- Suppose that $p(u)$ is given as an interpolating curve with control points $\mathbf{q}$

$$p(u)=\mathbf{u}^T\mathbf{M}_I\mathbf{q}$$

- There exist Bezier control points $\mathbf{p}$ such that

$$p(u)=\mathbf{u}^T\mathbf{M}_B\mathbf{p}$$

- Equating and solving, we find $\mathbf{p}=\mathbf{M}_B^{-1}\mathbf{M}_I$

Matrices

Interpolating to Bezier $\mathbf{M}_B^{-1}\mathbf{M}_I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{5}{6} & 3 & -\frac{3}{2} & \frac{1}{3} \\ \frac{1}{3} & -\frac{3}{2} & 3 & -\frac{5}{6} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B-Spline to Bezier $\mathbf{M}_B^{-1}\mathbf{M}_S = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix}$

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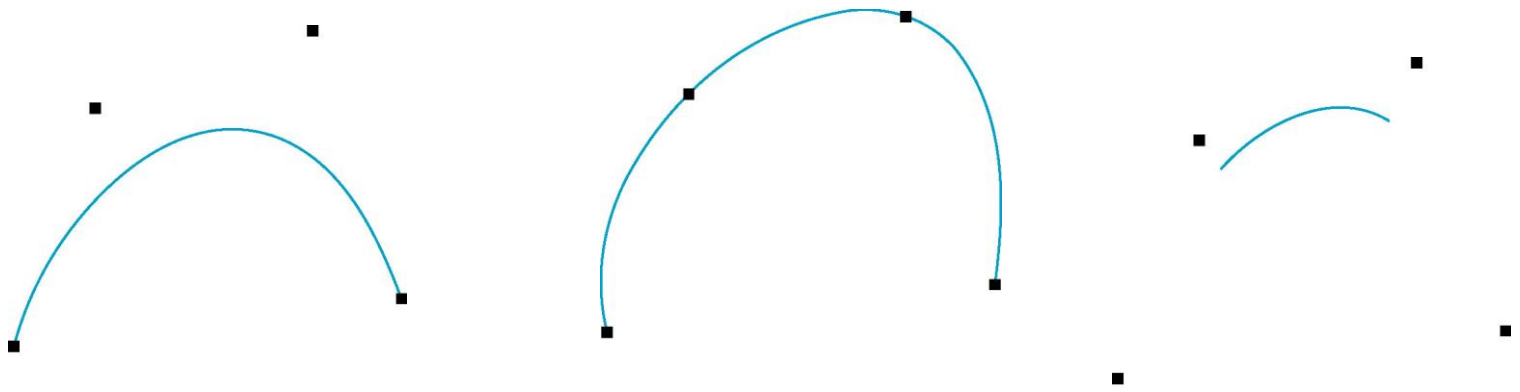
Example

These three curves were all generated from the same original data using Bezier recursion by converting all control point data to Bezier control points



Bezier

Interpolating

B Spline

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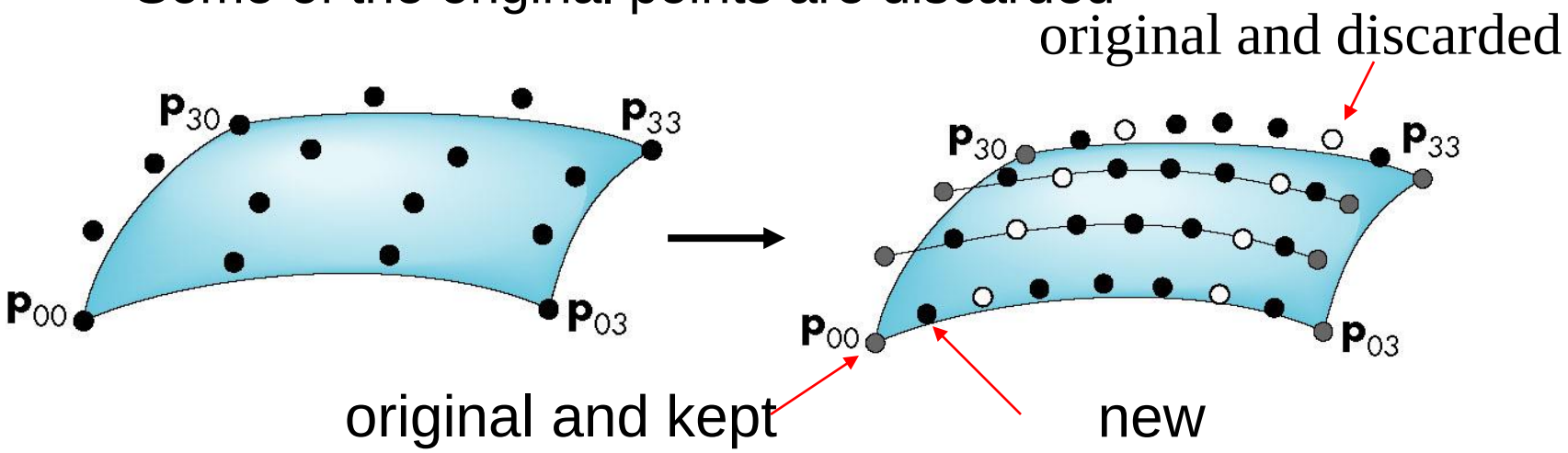
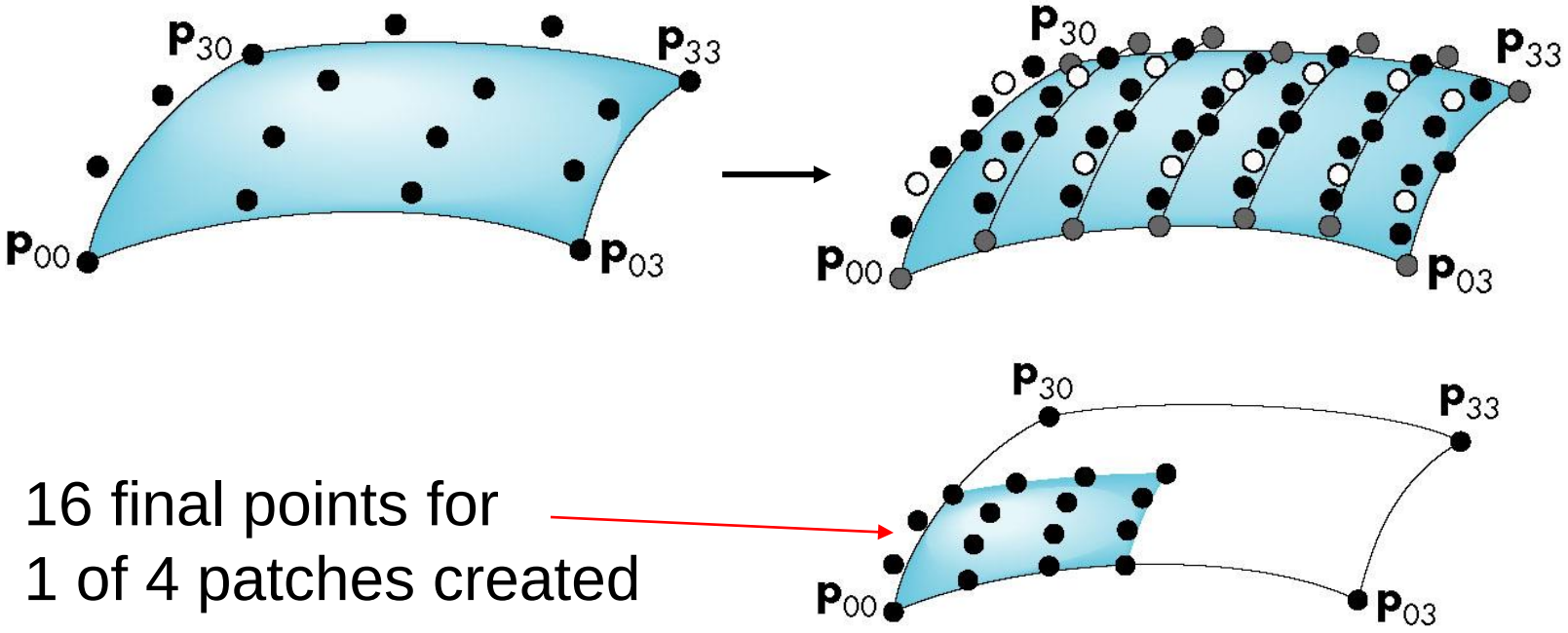
Surfaces

- Can apply the recursive method to surfaces if we recall that for a Bezier patch curves of constant u (or v) are Bezier curves in u (or v)
- First subdivide in u
  - Process creates new points
  - Some of the original points are discarded



Second Subdivision

- New points created by subdivision
- Old points discarded after subdivision
- Old points retained after subdivision



16 final points for
1 of 4 patches created

Normals

- For rendering we need the normals if we want to shade
  - Can compute from parametric equations

$$\mathbf{n} = \frac{\partial \mathbf{p}(u,v)}{\partial u} \times \frac{\partial \mathbf{p}(u,v)}{\partial v}$$

  - Can use vertices of corner points to determine
  - OpenGL can compute automatically

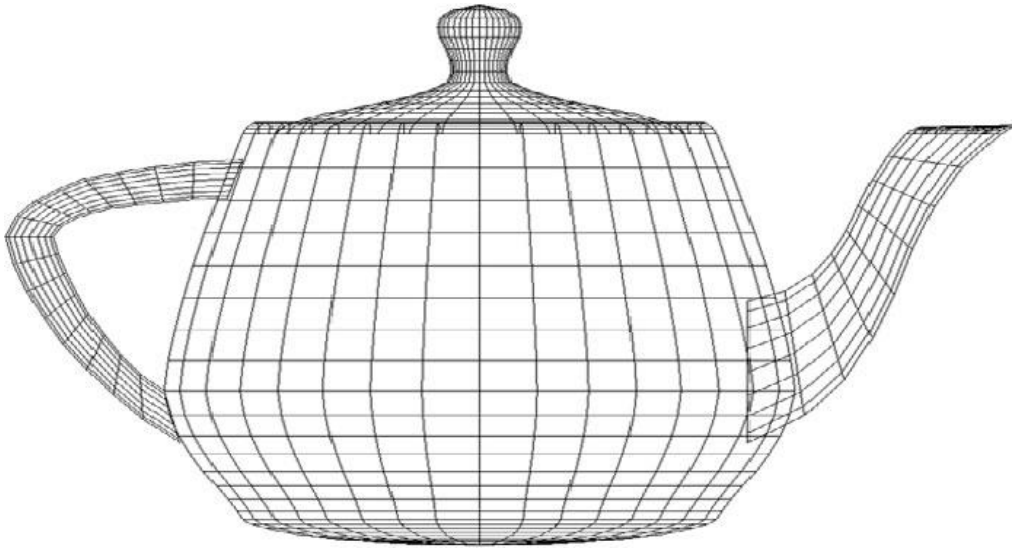
Rendering Other Polynomials

- Every polynomial is a Bezier polynomial for some set of control data
- We can use a Bezier renderer if we first convert the given control data to Bezier control data
  - Equivalent to converting between matrices
- Example: Interpolating to Bezier

$$M_B = M_I M_{BI}$$

Utah Teapot

- Most famous data set in computer graphics
- Widely available as a list of 306 3D vertices and the indices that define 32 Bezier patches



Quadrics

- Any quadric can be written as the quadratic form

  $\mathbf{p}^T\mathbf{A}\mathbf{p}+\mathbf{b}^T\mathbf{p}+c=0$ where $\mathbf{p}=[x, y, z]^T$

  with $\mathbf{A}$, $\mathbf{b}$ and c giving the coefficients

- Render by ray casting
  - Intersect with parametric ray $\mathbf{p}(\alpha)=\mathbf{p}_0+\alpha\mathbf{d}$ that passes through a pixel
  - Yields a scalar quadratic equation
    - No solution: ray misses quadric
    - One solution: ray tangent to quadric
    - Two solutions: entry and exit points

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Rendering the Teapot

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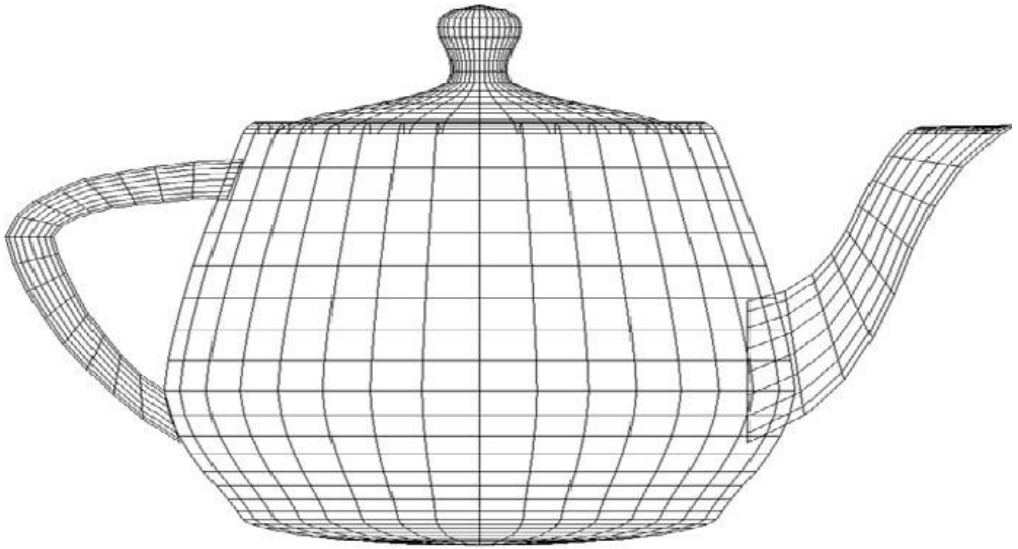
Objectives

- Look at rendering with WebGL
- Use Utah teapot for examples
  - Recursive subdivision
  - Polynomial evaluation
  - Adding lighting

Utah Teapot

- Most famous data set in computer graphics
- Widely available as a list of 306 3D vertices and the indices that define 32 Bezier patches



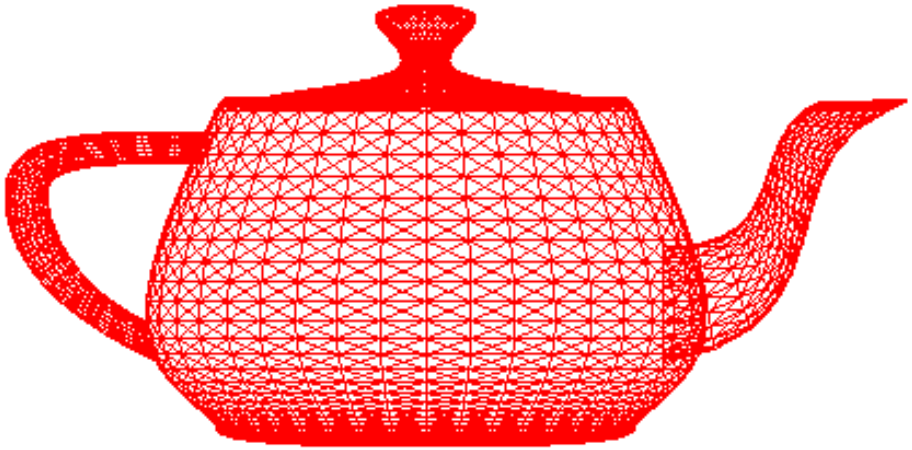
vertices.js

```
var numTeapotVertices = 306;
var vertices = [
vec3(1.4 , 0.0 , 2.4),
vec3(1.4 , -0.784 , 2.4),
vec3(0.784 , -1.4 , 2.4),
vec3(0.0 , -1.4 , 2.4),
vec3(1.3375 , 0.0 , 2.53125),
.
.
.
];
```

patches.js

```
var numTeapotPatches = 32;
var indices = new Array(numTeapotPatches);
    indices[0] = [0, 1, 2, 3,
    4, 5, 6, 7,
    8, 9, 10, 11,
    12, 13, 14, 15
    ];
    indices[1] = [3, 16, 17, 18,
.
.
 ];
```

Evaluation of Polynomials



Bezier Function

```
bezier = function(u) {
    var b = [];
    var a = 1-u;
    b.push(u*u*u);
    b.push(3*a*u*u);
    b.push(3*a*a*u);
    b.push(a*a*a);
   return b;
}
```

Patch Indices to Data

```
var h = 1.0/numDivisions;

   patch = new Array(numTeapotPatches);
   for(var i=0; i<numTeapotPatches; i++)
   patch[i] = new Array(16);
   for(var i=0; i<numTeapotPatches; i++)
       for(j=0; j<16; j++) {
          patch[i][j] = vec4([vertices[indices[i][j]][0],
           vertices[indices[i][j]][2],
              vertices[indices[i][j]][1], 1.0]);
   }
```

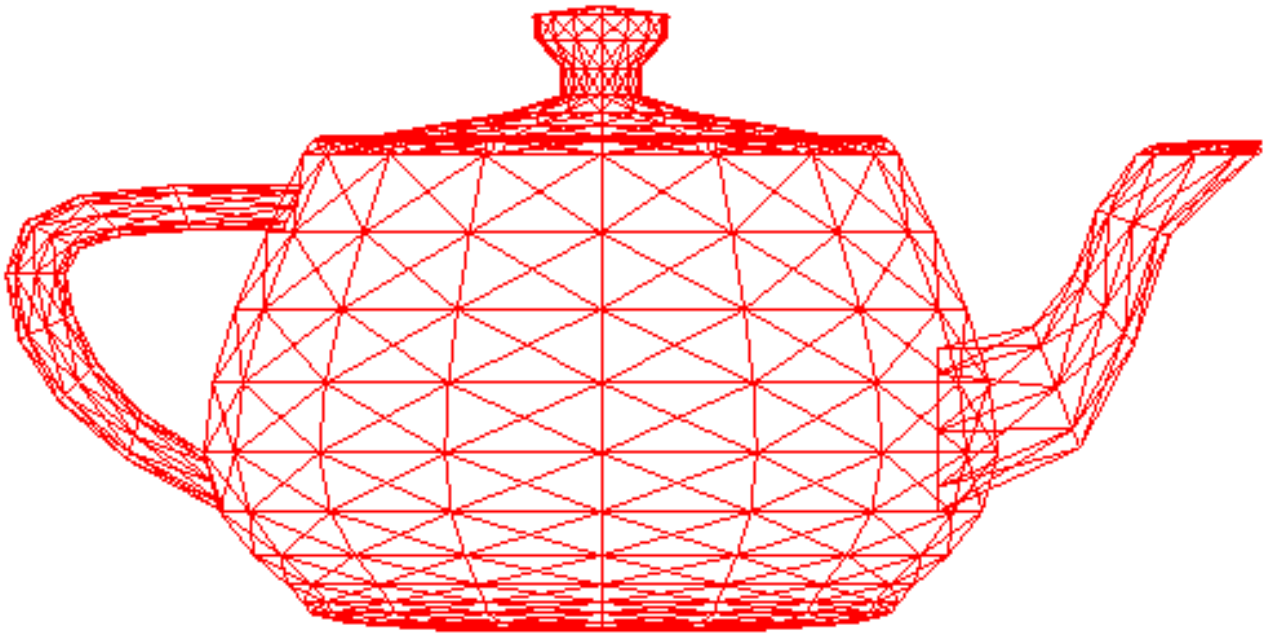
Vertex Data

```
for ( var n = 0; n < numTeapotPatches; n++ ) {
  var data = new Array(numDivisions+1);
  for(var j = 0; j<= numDivisions; j++) data[j] = new Array(numDivisions+1);
  for(var i=0; i<=numDivisions; i++) for(var j=0; j<= numDivisions; j++) {
      data[i][j] = vec4(0,0,0,1);
    var u = i*h;
    var v = j*h;
    var t = new Array(4);
    for(var ii=0; ii<4; ii++) t[ii]=new Array(4);
    for(var ii=0; ii<4; ii++) for(var jj=0; jj<4; jj++)
        t[ii][jj] = bezier(u)[ii]*bezier(v)[jj];
      for(var ii=0; ii<4; ii++) for(var jj=0; jj<4; jj++) {
            temp = vec4(patch[n][4*ii+jj]);
            temp = scale( t[ii][jj], temp);
            data[i][j] = add(data[i][j], temp);
    }
}
```

Quads

```
for(var i=0; i<numDivisions; i++)
   for(var j =0; j<numDivisions; j++) {
       points.push(data[i][j]);
       points.push(data[i+1][j]);
       points.push(data[i+1][j+1]);
       points.push(data[i][j]);
       points.push(data[i+1][j+1]);
       points.push(data[i][j+1]);
       index += 6;
       }
   }
```

Recursive Subdivision



Divide Curve

```
divideCurve = function( c, r , l){
 // divides c into left (l) and right ( r ) curve data
var mid = mix(c[1], c[2], 0.5);
   l[0] = vec4(c[0]);
   l[1] = mix(c[0], c[1], 0.5 );
   l[2] = mix(l[1], mid, 0.5 );
   r[3] = vec4(c[3]);
   r[2] = mix(c[2], c[3], 0.5 );
   r[1] = mix( mid, r[2], 0.5 );
   r[0] = mix(l[2], r[1], 0.5 );
   l[3] = vec4(r[0]);    return;
}
```

Divide Patch

```
dividePatch = function (p, count ) {
   if ( count > 0 ) {
    var a =  mat4();
    var b =  mat4();
    var t = mat4();
    var q = mat4();
    var r = mat4();
    var s = mat4();
// subdivide curves in u direction, transpose results, divide
// in u direction again (equivalent to subdivision in v)
        for ( var k = 0; k < 4; ++k ) {
            var pp = p[k];
            var aa = vec4();
            var bb = vec4();
```

Divide Patch

```
divideCurve( pp, aa, bb );
            a[k] = vec4(aa);
            b[k] = vec4(bb);
          }
      a = transpose( a );
      b = transpose( b );
            for ( var k = 0; k < 4; ++k ) {
            var pp = vec4(a[k]);
            var aa = vec4();
            var bb = vec4();
            divideCurve( pp, aa, bb );
             q[k] = vec4(aa);
             r[k] = vec4(bb);
            }
      for ( var k = 0; k < 4; ++k ) {
            var pp = vec4(b[k]);
            var aa = vec4();
```

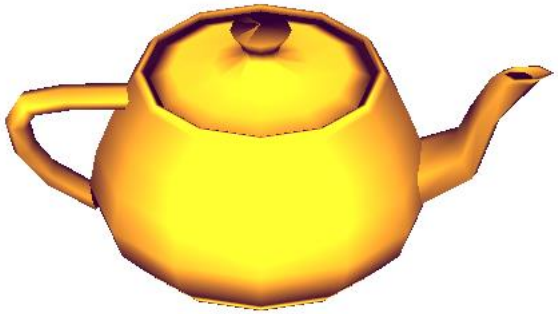
Divide Patch

```
            var bb = vec4();
            divideCurve( pp, aa, bb );
             t[k] = vec4(bb);
            }
// recursive division of 4 resulting patches
        dividePatch( q, count - 1 );
        dividePatch( r, count - 1 );
        dividePatch( s, count - 1 );
        dividePatch( t, count - 1 );
    }
    else {
        drawPatch( p );
    }
    return;
}
```

Draw Patch

```
drawPatch = function(p) {
    // Draw the quad (as two triangles) bounded by
    //   corners of the Bezier patch
    points.push(p[0][0]);
    points.push(p[0][3]);
    points.push(p[3][3]);
    points.push(p[0][0]);
    points.push(p[3][3]);
    points.push(p[3][0]);
    index+=6;
    return;
}
```

Adding Shading



Using Face Normals

```
var t1 = subtract(data[i+1][j], data[i][j]);
var t2  =subtract(data[i+1][j+1], data[i][j]);
var normal = cross(t1, t2);
normal = normalize(normal);
normal[3] =  0;
points.push(data[i][j]);          normals.push(normal);
points.push(data[i+1][j]);        normals.push(normal);
points.push(data[i+1][j+1]);      normals.push(normal);
points.push(data[i][j]);          normals.push(normal);
points.push(data[i+1][j+1]);      normals.push(normal);
points.push(data[i][j+1]);        normals.push(normal);
index+= 6;
```

Exact Normals

```
nbezier = function(u) {
    var b = [];
    b.push(3*u*u);
    b.push(3*u*(2-3*u));
    b.push(3*(1-4*u+3*u*u));
    b.push(-3*(1-u)*(1-u));
    return b;
}
```

Geometry Shader

- Basic limitation on rasterization is that each execution of a vertex shader is triggered by one vertex and can output only one vertex
- Geometry shaders allow a single vertex and other data to produce many vertices
- Example: send four control points to a geometry shader and it can produce as many points as needed for Bezier curve

Tessellation Shaders

- Can take many data points and produce triangles
- More complex since tessellation has to deal with inside/outside issues and topological issues such as holes
- Neither geometry or tessellation shaders supported by ES
- ES 3.1 (just announced) has compute shaders