CS 412 Intro. to Data Mining

Chapter 6. Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

Qi Li, Computer Science, Univ. Illinois at Urbana-Champaign, 2018

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

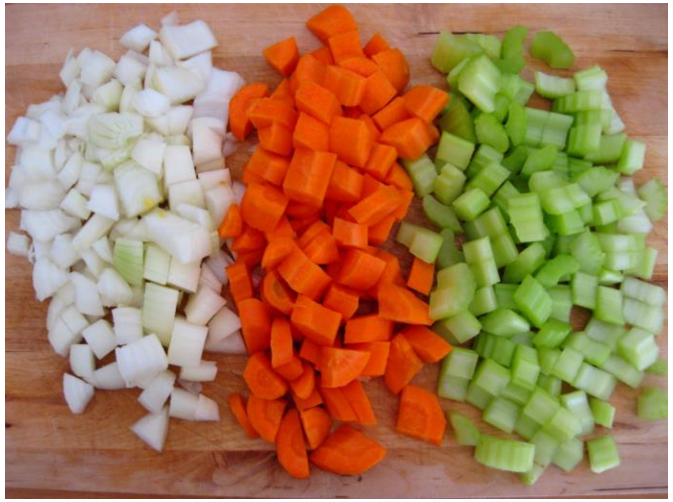
- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Pattern Discovery: Basic Concepts

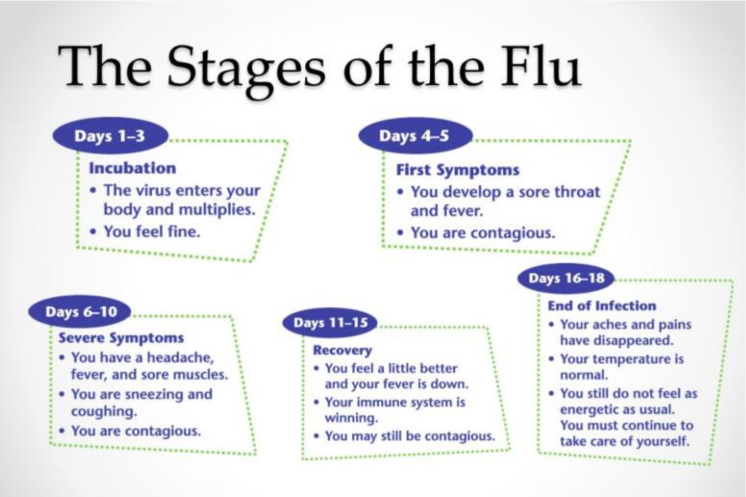
- What Is Pattern Discovery? Why Is It Important?
- Basic Concepts: Frequent Patterns and Association Rules
- Compressed Representation: Closed Patterns and Max-Patterns

What are Patterns?

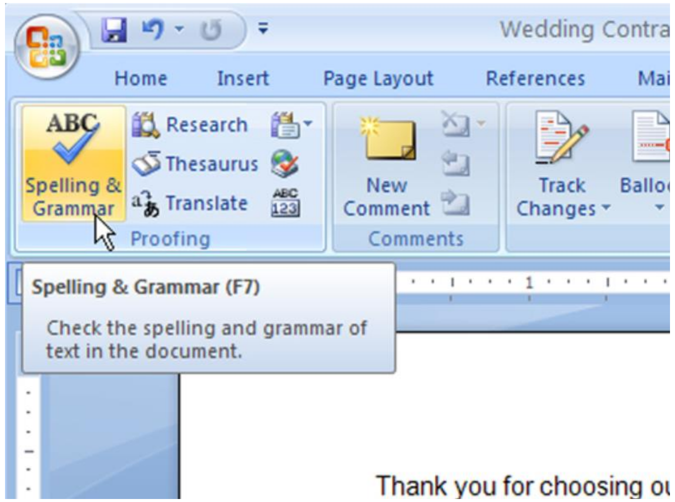
- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets



Frequent item set



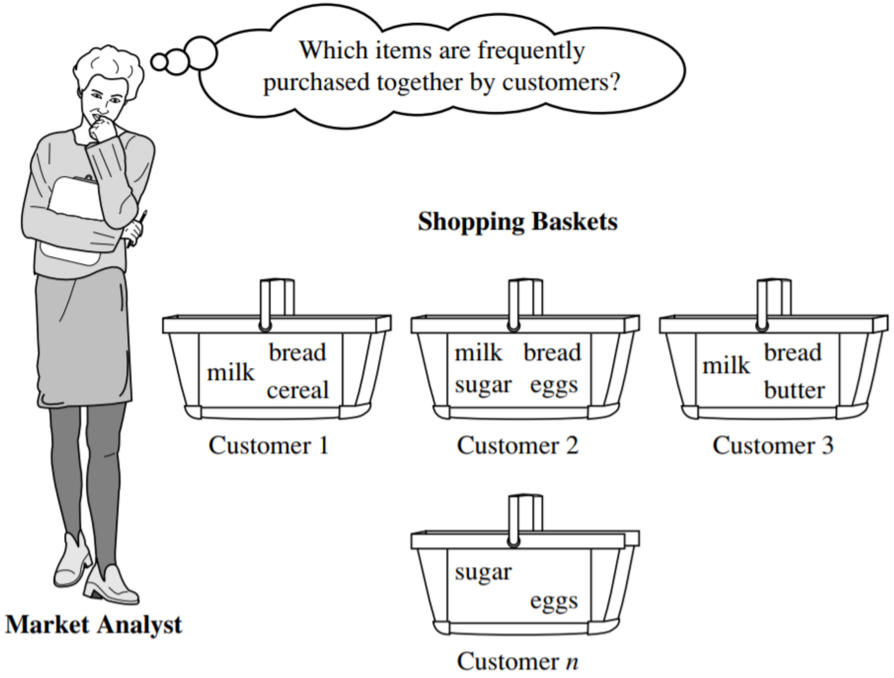
Frequent sequences



Frequent structures

What Is Pattern Discovery?

- Pattern discovery: Uncovering patterns from massive data sets
- It can answer questions such as:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?



Pattern Discovery: Why Is It Important?

- Finding inherent regularities in a data set:
  - spatiotemporal, multimedia, time-series, and stream data
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining **sequential**, structural (e.g., sub-graph) patterns
  - **Classification**: Discriminative pattern-based analysis
  - **Cluster** analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

Basic Concepts: Transactional Database

- Transactional Database (TDB)
  - Each transaction is associated with an identifier, called a TID.

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

Basic Concepts: k-Itemsets and Their Supports

- Itemset: A set of one or more items

$$I = \{ I_1, I_2, \cdots, I_m \}$$

- k-itemset: An itemset containing k items:

  $X = \{x_1, \ldots, x_k\}$

  - Ex. {Beer, Nuts, Diaper} is a 3-itemset
- *(absolute) support (count)*
  - sup{X} = occurrences of an itemset X
    - Ex. sup{Beer} = 3
    - Ex. sup{Diaper} = 4
    - Ex. sup{Beer, Diaper} = 3
    - Ex. sup{Beer, Eggs} = 1

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- *(relative) support*
  - *s*{*X*} = The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
  - Ex. s{Beer} = 3/5 = 60%
  - Ex. s{Diaper} = 4/5 = 80%
  - Ex. s{Beer, Eggs} = 1/5 = 20%

Basic Concepts: Frequent Itemsets (Patterns)

- An itemset (or a pattern) X is *frequent* if the support of X is no less than a *minsup* threshold σ
- Let σ = *50%* (σ: *minsup* threshold)

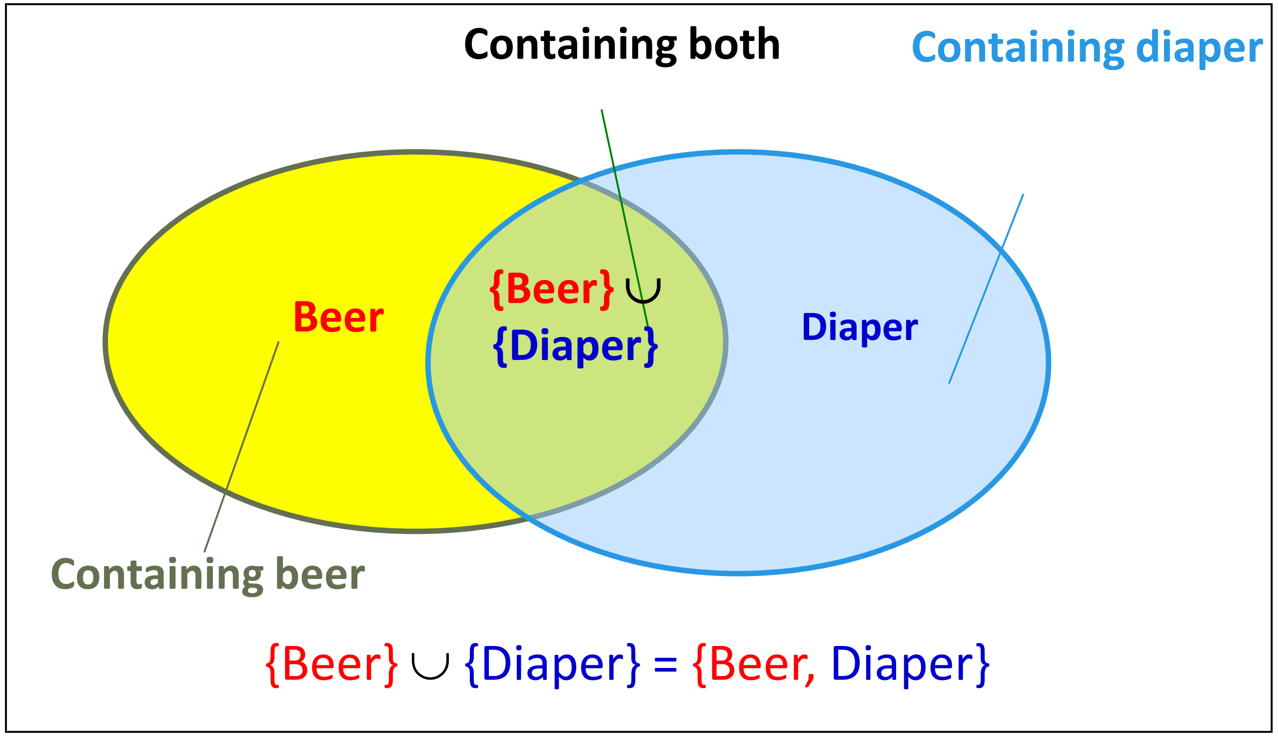
  For the given 5-transaction dataset
  - All the frequent 1-itemsets:
    - Beer: 3/5 (60%); Nuts: 3/5 (60%)
    - Diaper: 4/5 (80%); Eggs: 3/5 (60%)
  - All the frequent 2-itemsets:
    - {Beer, Diaper}: 3/5 (60%)
  - All the frequent 3-itemsets?
    - None

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- Why do these itemsets (shown on the left) form the complete set of frequent *k*-itemsets (patterns) for any *k*?
- **Observation**: We may need an efficient method to mine a complete set of frequent patterns

From Frequent Itemsets to Association Rules

- Comparing with itemsets, association rules can be more telling
  - Ex. *Diaper → Beer*
    - *Buying diapers may likely lead to buying beers*



Note: X ∪ Y: the union of two itemsets

- The set contains both X and Y

Association Rules

- How do we compute the strength of an association rule $X \rightarrow Y$ (Both X and Y are itemsets)?
- We first compute the following two metrics, s and c.
  - Support of $X \cup Y$
    - Ex. s{Diaper, Beer} = 3/5 = 0.6 (i.e., 60%)
  - Confidence of $X \rightarrow Y$
    - *The conditional probability* that a transaction containing X also contains *Y*

    $c = sup(X \rightarrow Y) / sup(X)$
    - Ex. *c* = sup{Diaper, Beer}/sup{Diaper} = ¾ = 0.75

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- In pattern analysis, we are often interested in those rules that dominates the database, and these two metrics ensure the popularity and correlation of X and Y.

Mining Frequent Itemsets and Association Rules

- **Association rule mining**
  - Given two thresholds: *minsup, minconf*
  - Find all of the rules, $X \rightarrow Y$ (s, c)
    - such that, s ≥ *minsup* and *c* ≥ *minconf*

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Let *minconf = 50%*
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

(Q: Are these all rules?)

- **Observations:**
  - Mining association rules and mining frequent patterns are very close problems
  - Scalable methods are needed for mining large datasets

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain (minsup = 1)?
  - $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Let's have a try

1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, …, $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, …, $\{a_{100}\}$: 1,

2-itemsets: $\{a_1, a_2\}$: 2, …, $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 …, …, $\{a_{99}, a_{100}\}$: 1,

…, …, …, …

99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1

- The total number of frequent itemsets:

$$\binom{100}{1} + \binom{100}{2} + \binom{100}{3} + \cdots + \binom{100}{100} = 2^{100} - 1$$

A too huge set for any one to compute or store!

Expressing Patterns in Compressed Form

- How to reduce the redundancy of the list of all the frequent itemsets?
  - If $\{a_1, \ldots, a_{99}\}$ and $\{a_1, \ldots, a_{100}\}$ have the same support in the database, then we don't need to list both of them
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent,* and there exists *no super-pattern* $Y \supset X$, *with the same support* as X
  - Ex. TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "$\{a_1, \ldots, a_{50}\}$: 2"; P_2: "$\{a_1, \ldots, a_{100}\}$: 1"

Expressing Patterns in Compressed Form: Closed Patterns

- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - Given P_1: "{a_1, …, a_{50}}: 2"; P_2: "{a_1, …, a_{100}}: 1"
  - You will still be able to say: "{a_2, …, a_{40}}: 2", "{a_5, a_{51}}: 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ɔ X
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, …, a_{100}\}$: 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Max-pattern is a lossy compression!
  - We only know a subset of the max-pattern P, $\{a_1, \ldots, a_{40}\}$, is frequent
  - But we do not know the real support of $\{a_1, \ldots, a_{40}\}$, …, any more!
- Thus in many applications, mining close-patterns is more desirable than mining max-patterns

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Efficient Pattern Mining Methods

- The Downward Closure Property of Frequent Patterns
- The Apriori Algorithm
- Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- FPGrowth: A Frequent Pattern-Growth Approach
- Mining Closed Patterns

The Downward Closure Property of Frequent Patterns

- Observation: From TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - We get a frequent itemset: $\{a_1, …, a_{50}\}$
  - Also, its subsets are all frequent: $\{a_1\}$, $\{a_2\}$, …, $\{a_{50}\}$, $\{a_1, a_2\}$, …, $\{a_1, …, a_{49}\}$, …
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items};   // frequent 1-itemset

While (F_k != ∅) **do {**   // when F_k is non-empty

  C_{k+1} := candidates generated from F_k; // candidate generation

  Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

  k := k + 1

  }

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm—An Example

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan → C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan → C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

→ F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

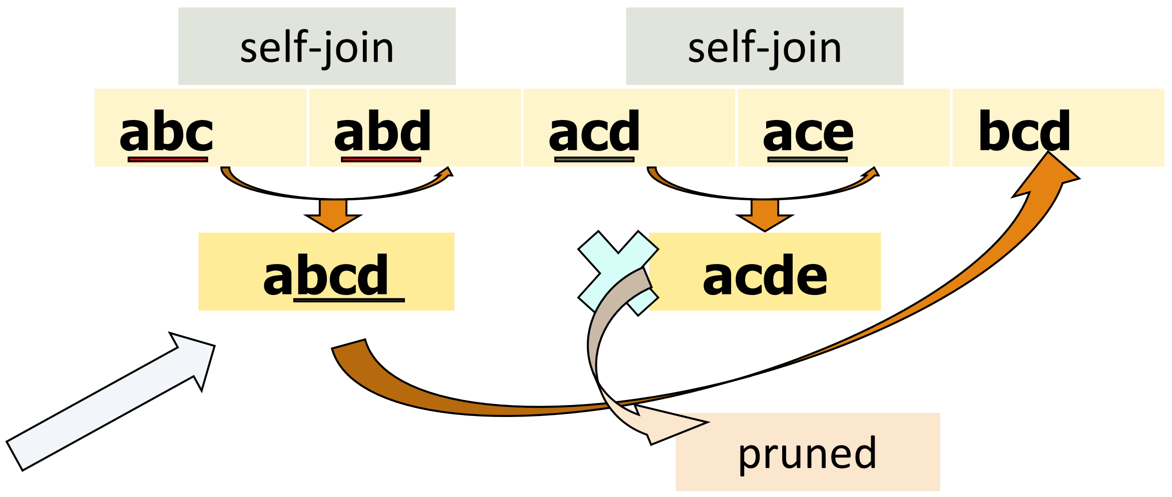
Itemset
{B, C, E}

3rd scan → F_3

Itemset	sup
{B, C, E}	2

Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - F_3 = {*abc, abd, acd, ace, bcd*}
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - C_4 = {*abcd*}



Candidate Generation (Pseudo-Code)

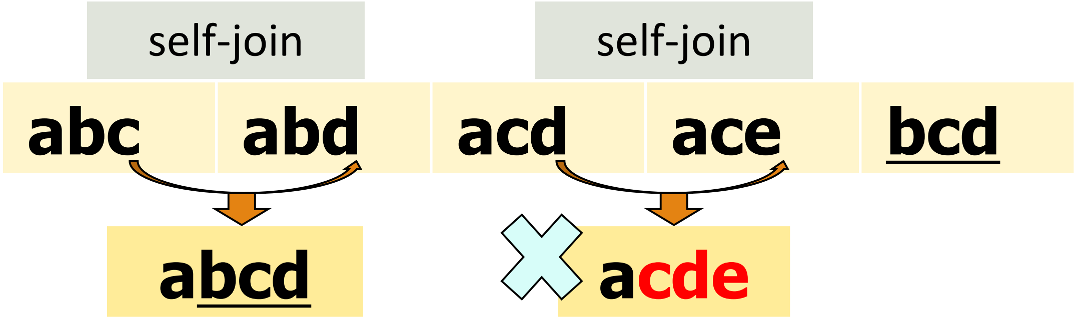
- Suppose the items in F_{k-1} are listed in an order
- // Step 1: Joining

  for each p **in** F_{k-1}

      for each q **in** F_{k-1}

          if $p.item_1 = q.item_1, \ldots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$ **{**

              c = **join**(p, q)



- // Step 2: pruning

              if has_infrequent_subset(c, F_{k-1})

                  continue // prune

              else add c **to** C_k

          }

Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - Partitioning (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)

To be discussed in subsequent slides

- Shrink the number of candidates
  - Hashing (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)

To be discussed in subsequent slides

- Exploring special data structures
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

Partitioning: Scan Database Only Twice

- Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Here is the proof!

TDB_1 + TDB_2 + . . . + TDB_k = TDB

$sup_1(X) < \sigma|TDB_1|$ . . . $sup_2(X) < \sigma|TDB_2|$ . . . $sup_k(X) < \sigma|TDB_k|$ $sup(X) < \sigma|TDB|$

Partitioning: Scan Database Only Twice

- Method: Scan DB twice (A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*)
  - Scan 1: Partition database so that each partition can fit in main memory (why?)
    - Mine local frequent patterns in this partition
  - Scan 2: Consolidate global frequent patterns
    - Find global frequent itemset candidates (those frequent in at least one partition)
    - Find the true frequency of those candidates, by scanning TDB_i one more time

Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Hashing: Different itemsets may have the same hash value: v = *hash*(itemset)
- 1st scan: When counting the 1-itemset, hash 2-itemset to calculate the bucket count
- Observation: A *k*-itemset cannot be frequent if its corresponding hashing bucket count is below the *minsup* threshold
- Example: At the 1st scan of TDB, count 1-itemset, and
  - Hash 2-itemsets in the transaction to its bucket
    - {ab, ad, ce}
    - {bd, be, de}
    - …
  - At the end of the first scan,
    - *if minsup = 80, remove ab, ad, ce, since count{ab, ad, ce}* < 80

Itemsets	Count
{*ab, ad, ce*}	35
{*bd, be, de*}	298
……	…
{*yz, qs, wt*}	58

Hash Table

Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A depth-first search algorithm using set intersection [Zaki et al. @KDD'97]
- Tid-List: List of transaction-ids containing an itemset
- Vertical format: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ae) = \{T_{10}, T_{20}\}$
- Properties of Tid-Lists
  - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac\} = t(d\})$)
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\}$ → Diffset $(ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

Why Mining Frequent Patterns by Pattern Growth?

- Apriori: A *breadth-first search* mining algorithm
  - First find the complete set of frequent k-itemsets
  - Then derive frequent (k+1)-itemset candidates
  - Scan DB again to find true frequent (k+1)-itemsets

Why Mining Frequent Patterns by Pattern Growth?

- Motivation for a different mining methodology
  - Can we develop a *depth-first search* mining algorithm?
  - For a frequent itemset ρ, can subsequent search be confined to only those transactions that containing ρ?
- Such thinking leads to a frequent pattern growth approach:
  - FPGrowth (J. Han, J. Pei, Y. Yin, "Mining Frequent Patterns without Candidate Generation," SIGMOD 2000)

Prerequisite: Find frequent 1-itemset

TID	Items in the Transaction
100	*{f, a, c, d, g, i, m, p}*
200	*{a, b, c, f, l, m, o}*
300	*{b, f, h, j, o, w}*
400	*{b, c, k, s, p}*
500	*{a, f, c, e, l, p, m, n}*

1. Scan DB once, find single item frequent pattern:

   Let min_support = 3

   f:4, a:3, c:4, b:3, m:3, p:3

2. Sort frequent items in frequency descending order, f-list

   F-list = f-c-a-b-m-p

Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	{*f, a, c, d, g, i, m, p*}	*f, c, a, m, p*
200	{*a, b, c, f, l, m, o*}	*f, c, a, b, m*
300	{*b, f, h, j, o, w*}	*f, b*
400	{*b, c, k, s, p*}	*c, b, p*
500	{*a, f, c, e, l, p, m, n*}	*f, c, a, m, p*

3. Scan DB again, find the ordered frequent itemlist for each transaction

Example: Construct FP-tree from a Transaction DB

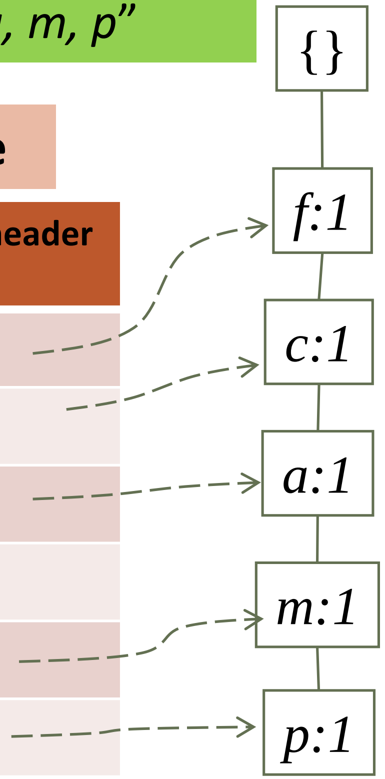
TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting the 1st frequent Itemlist: "*f, c, a, m, p*"

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Example: Construct FP-tree from a Transaction DB

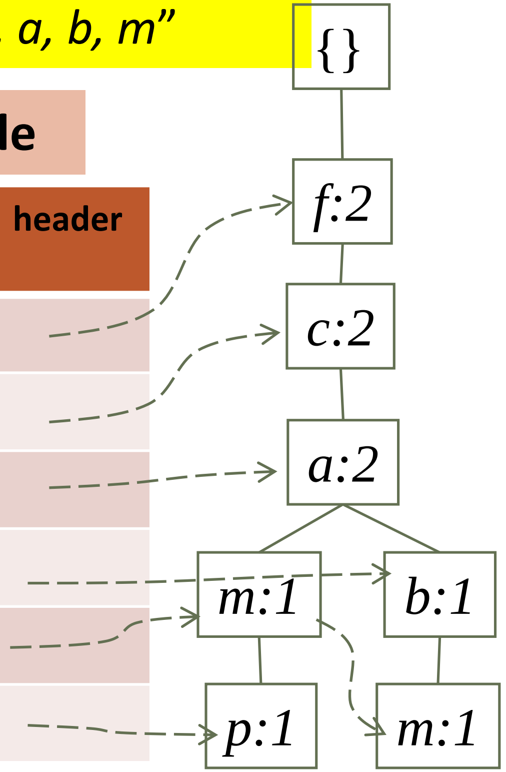
TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting the 2nd frequent itemlist "*f, c, a, b, m*"

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Example: Construct FP-tree from a Transaction DB

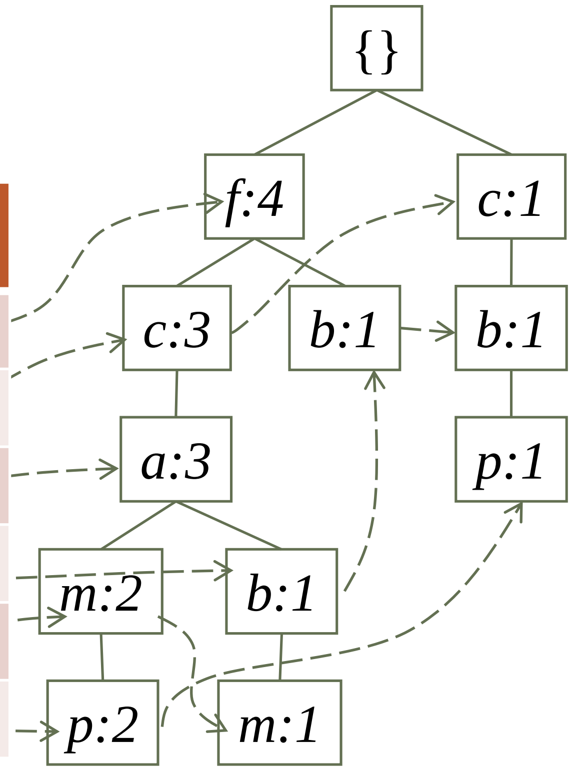
TID	Items in the Transaction	Ordered, frequent itemlist
100	{*f, a, c, d, g, i, m, p*}	*f, c, a, m, p*
200	{*a, b, c, f, l, m, o*}	*f, c, a, b, m*
300	{*b, f, h, j, o, w*}	*f, b*
400	{*b, c, k, s, p*}	*c, b, p*
500	{*a, f, c, e, l, p, m, n*}	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting all the frequent itemlists

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

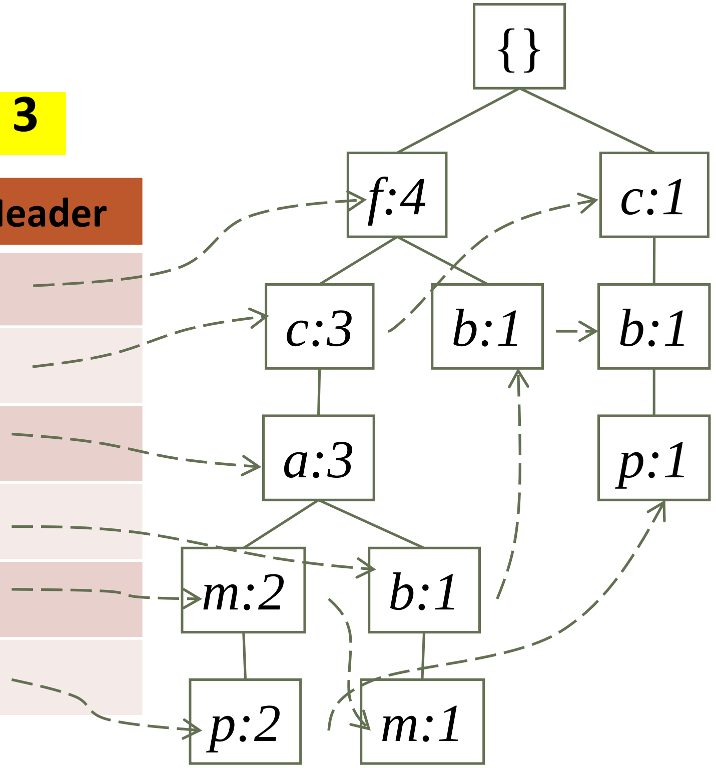


Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - We start to calculate the conditional database from bottom to top (from the least frequent item)
  - Conditional database: the database under the condition that *p* exists
    - *p*'s conditional database (Patterns containing *p*): *fcam:2, cb:1*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

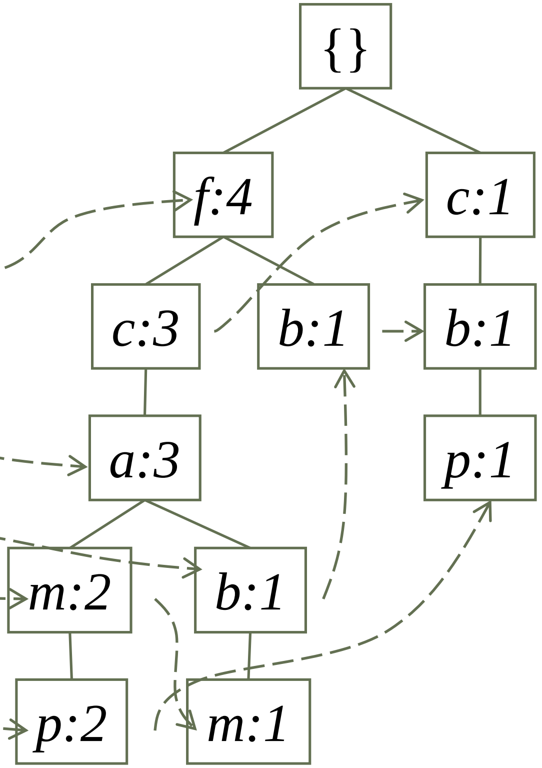
Item	*Conditional database*
p	***fcam:2, cb:1***

Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- *p*’s conditional database (Patterns containing *p*): *fcam:2, cb:1*
- *After calculating p’s conditional database, we calculate m’s conditional database*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

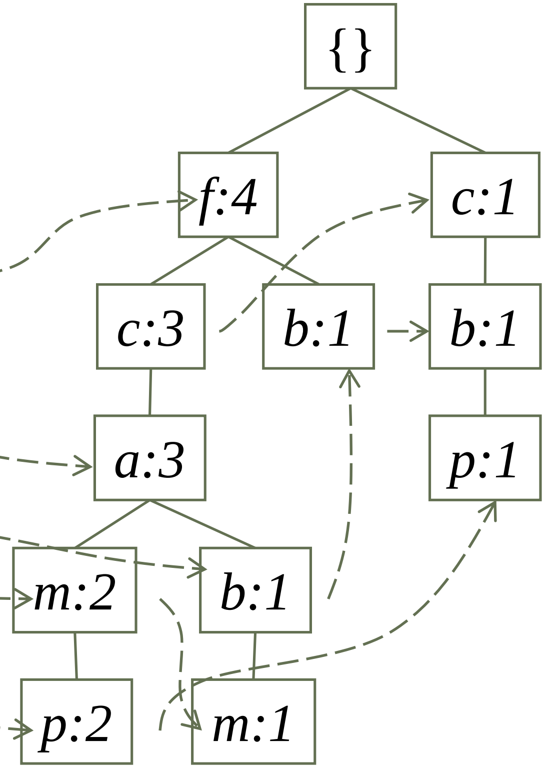
Item	***Conditional database***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- *Repeat and calculate the conditional database of b, a, and c*
- *Since f is the most frequent item, we don't need to compute its conditional dataset*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

Item	*Conditional database*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Mine Each Conditional Database Recursively

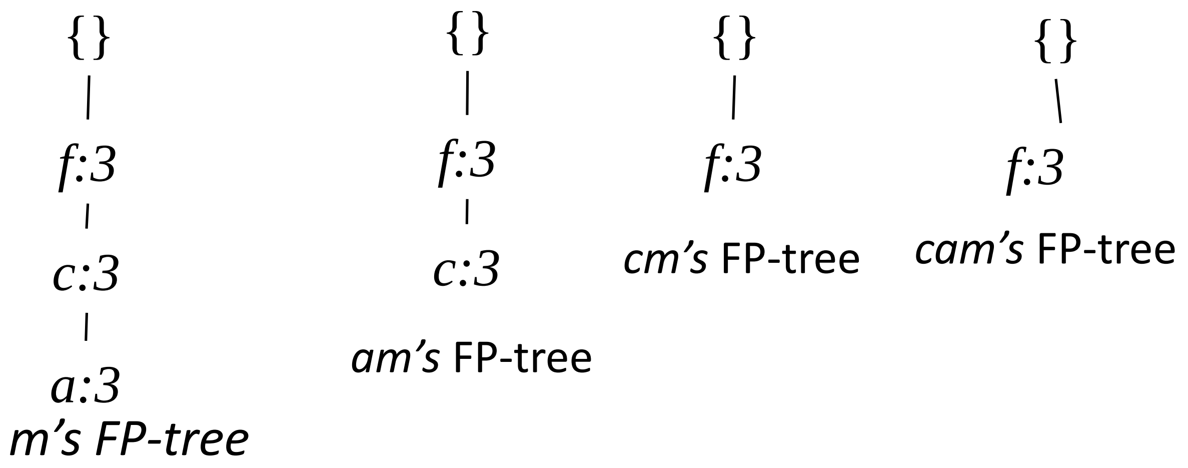
min_support = 3

Conditional Data Bases

item	*cond. data base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

- For each conditional database
  - Mine single-item patterns
  - Construct its FP-tree & mine it

e.g., mining m's FP-tree

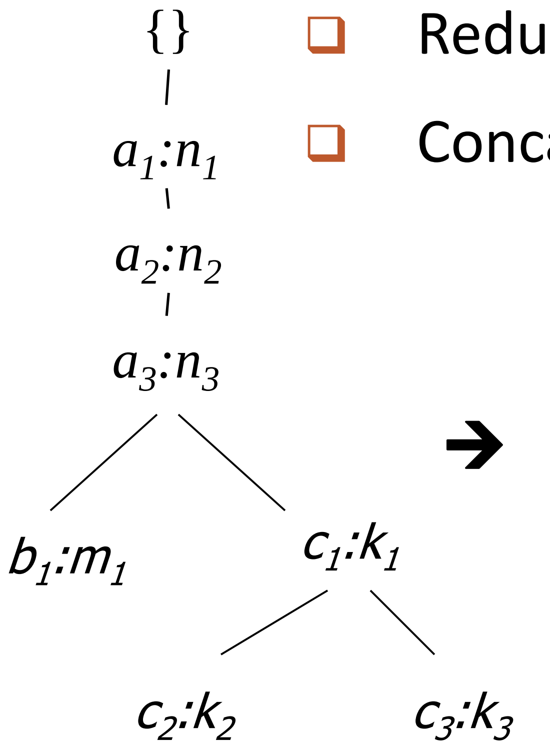


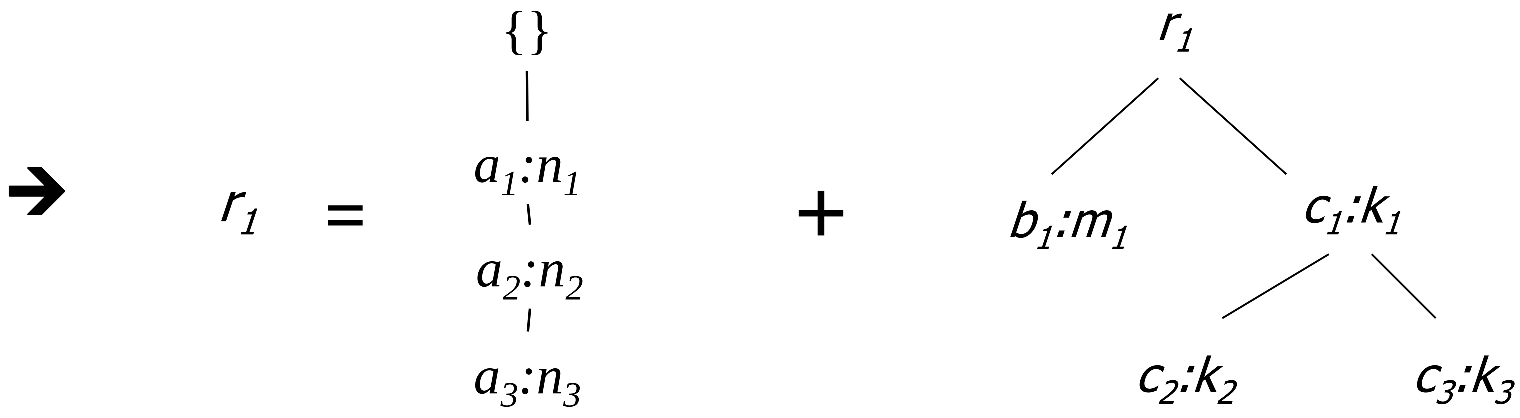
Actually, for single branch FP-tree, all the frequent patterns can be generated in one shot

m: 3
fm: 3, cm: 3, am: 3
fcm: 3, fam:3, cam: 3
fcam: 3

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts





FPGrowth: Mining Frequent Patterns by Pattern Growth

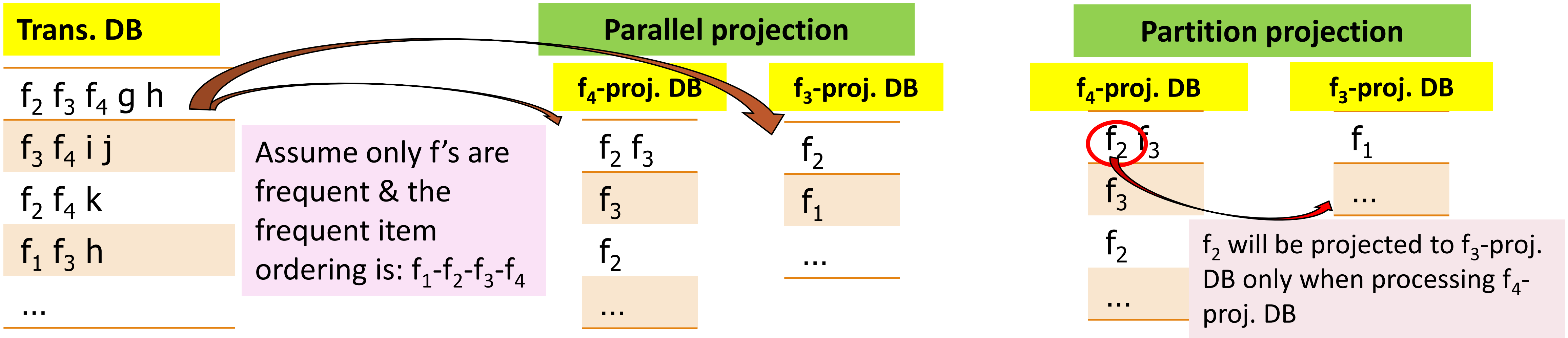
- Essence of frequent pattern growth (FPGrowth) methodology
  - Find frequent single items and partition the database based on each such single item pattern
  - Recursively grow frequent patterns by doing the above for each *partitioned database* (also called the pattern's *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed

FPGrowth: Mining Frequent Patterns by Pattern Growth

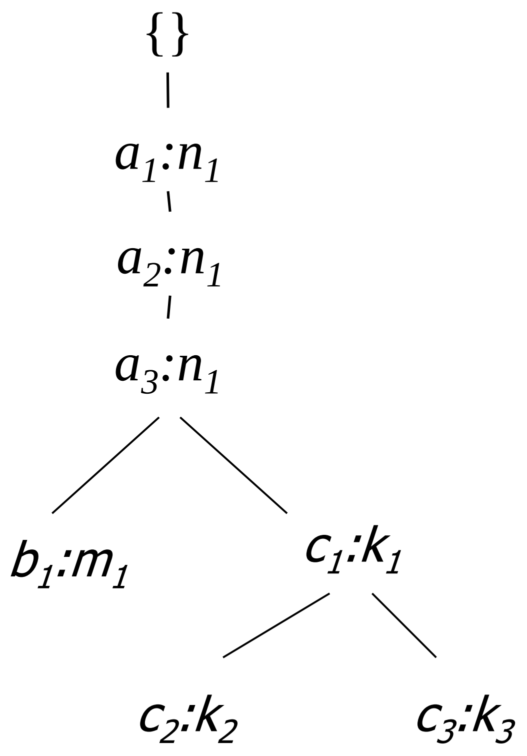
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Scaling FP-growth by Item-Based Data Projection

- What if FP-tree cannot fit in memory?—Do not construct FP-tree
  - "Project" the database based on frequent single items
  - Construct & mine FP-tree for each projected DB
- Parallel projection vs. partition projection
  - Parallel projection: Project the DB on each frequent item
    - Space costly, all partitions can be processed in parallel
  - Partition projection: Partition the DB in order
    - Passing the unprocessed parts to subsequent partitions



CLOSET+: Mining Closed Itemsets by Pattern-Growth



- Efficient, *direct* mining of closed itemsets
- Intuition:
  - If an FP-tree contains a single branch as shown left
  - "a_1,a_2, a_3" should be merged
- Itemset merging: If Y appears in every occurrence of X, then Y is merged with X
  - *d*-proj. db: {acef, acf} → *acfd*-proj. db: {e}
  - Final closed itemset: acfd:2
- There are many other tricks developed
  - For details, see J. Wang, et al,, "CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets", KDD'03

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2

a:3, c:3, d:2, e:3, f:3

F-List: a-c-e-f-d

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Pattern Evaluation

- Limitation of the Support-Confidence Framework
- Interestingness Measures: Lift and χ^2
- Null-Invariant Measures
- Comparison of Interestingness Measures

How to Judge if a Rule/Pattern Is Interesting?

- Pattern-mining will generate a large set of patterns/rules
  - Not all the generated patterns/rules are interesting
- Interestingness measures: Objective vs. subjective
  - Objective interestingness measures
    - Support, confidence, correlation, …
  - Subjective interestingness measures:
      - Different users may judge interestingness differently
    - Let a user specify
      - Query-based: Relevant to a user’s particular request
    - Judge against one’s knowledge-base
      - unexpected, freshness, timeliness

Limitation of the Support-Confidence Framework

- Are *s* and *c* interesting in association rules: "A $\Rightarrow$ B" [*s*, *c*]? Be careful!
- Example: Suppose one school may have the following statistics on # of students who may play basketball and/or eat cereal:

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

2-way contingency table

- Association rule mining may generate the following:
  - *play-basketball* $\Rightarrow$ *eat-cereal* [40%, 66.7%] (higher s & c)
- But this strong association rule is misleading: The overall % of students eating cereal is 75% > 66.7%, a more telling rule:
  - ¬ *play-basketball* $\Rightarrow$ *eat-cereal* [35%, 87.5%] (high s & c)

Interestingness Measure: Lift

- Measure of dependent/correlated events: **lift**

$$lift(B,C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

- Lift(B, C) may tell how B and C are correlated
  - Lift(B, C) = 1: B and C are independent
  - > 1: positively correlated
  - < 1: negatively correlated
- For our example,

$$lift(B,C) = \frac{400/1000}{600/1000 \times 750/1000} = 0.89$$

$$lift(B,\neg C) = \frac{200/1000}{600/1000 \times 250/1000} = 1.33$$

- Thus, B and C are negatively correlated since lift(B, C) < 1;
  - B and ¬C are positively correlated since lift(B, ¬C) > 1

Lift is more telling than s & c

	B	¬B	Σ_{row}
C	400	350	750
¬C	200	50	250
$\Sigma_{col.}$	600	400	1000

Interestingness Measure: χ^2

- Another measure to test correlated events: χ^2

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- For the table on the right,

$$c^2 = \frac{(400 - 450)^2}{450} + \frac{(350 - 300)^2}{300} + \frac{(200 - 150)^2}{150} + \frac{(50 - 100)^2}{100} = 55.56$$

	B	¬B	Σ_{row}
C	400 (450)	350 (300)	750
¬C	200 (150)	50 (100)	250
Σ_{col}	600	400	1000

Expected value

Observed value

- Lookup χ^2 distribution table => B, C are correlated
- χ^2-test shows B and C are negatively correlated since the expected value is 450 but the observed is only 400
- Thus, χ^2 is also more telling than the support-confidence framework

Lift and χ^2 : Are They Always Good Measures?

- Null transactions: Transactions that contain neither B nor C
- Let's examine the new dataset D
  - BC (100) is much rarer than B¬C (1000) and ¬BC (1000), but there are many ¬B¬C (100000)
  - Unlikely B & C will happen together!
- But, Lift(B, C) = 8.44 >> 1 (Lift shows B and C are strongly positively correlated!)
- χ^2 = 670: Observed(BC) >> expected value (11.85)
- *Too many null transactions may "spoil the soup"!*

	B	¬B	Σ_{row}
C	100	1000	1100
¬C	1000	100000	101000
$\Sigma_{col.}$	1100	101000	102100

null transactions

Contingency table with expected values added

	B	¬B	Σ_{row}
C	100 (11.85)	1000	1100
¬C	1000 (988.15)	100000	101000
$\Sigma_{col.}$	1100	101000	102100

Interestingness Measures & Null-Invariance

- *Null invariance:* Value does not change with the # of null-transactions
- A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant?
$\chi^2(A,B)$	$\sum_{i,j} \frac{(e(a_i,b_j)-o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0, \infty]$	No
$Lift(A,B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$Allconf(A,B)$	$\frac{s(A \cup B)}{max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A,B)$	$\frac{s(A \cup B)}{s(A)+s(B)-s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A,B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A,B)$	$\frac{1}{2}\left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)}\right)$	$[0, 1]$	Yes
$MaxConf(A,B)$	$max\{\frac{s(A \cup B)}{s(A)}, \frac{s(A \cup B)}{s(B)}\}$	$[0, 1]$	Yes

Let

$$p = \frac{s(A \cup B)}{s(A)} = P(B|A)$$

$$q = \frac{s(A \cup B)}{s(B)} = P(A|B)$$

p, q are null invariant

Essentially min, max, mean variants of p, q

Null Invariance: An Important Property

- Why is null invariance crucial for the analysis of massive transaction data?
  - Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	$milk$	$\neg milk$	Σ_{row}
$coffee$	mc	$\neg mc$	c
$\neg coffee$	$m\neg c$	$\neg m\neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

- Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!
- Many measures are not null-invariant!

Data set	mc	$\neg mc$	$m\neg c$	$\neg m\neg c$	χ^2	$Lift$
D_1	10,000	1,000	1,000	100,000	90557	9.26
D_2	10,000	1,000	1,000	100	0	1
D_3	100	1,000	1,000	100,000	670	8.44
D_4	1,000	1,000	1,000	100,000	24740	25.75
D_5	1,000	100	10,000	100,000	8173	9.18
D_6	1,000	10	100,000	100,000	965	1.97

Comparison of Null-Invariant Measures

- Not all null-invariant measures are created equal
- Which one is better?
  - D_4—D_6 differentiate the null-invariant measures
  - Kulc (Kulczynski 1927) holds firm and is in balance of both directional implications

2-variable contingency table

	$milk$	$\neg milk$	Σ_{row}
$coffee$	mc	$\neg mc$	c
$\neg coffee$	$m\neg c$	$\neg m\neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

All 5 are null-invariant

Data set	mc	$\neg mc$	$m\neg c$	$\neg m\neg c$	$AllConf$	Jaccard	$Cosine$	$Kulc$	$MaxConf$
D_1	10,000	1,000	1,000	100,000	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	0.01	0.01	0.10	0.5	0.99

Imbalance Ratio with Kulczynski Measure

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications:

$$IR(A,B) = \frac{|s(A) - s(B)|}{s(A) + s(B) - s(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
  - D_4 is neutral & balanced; D_5 is neutral but imbalanced
  - D_6 is neutral but very imbalanced

Data set	mc	$\neg mc$	$m\neg c$	$\neg m \neg c$	Jaccard	$Cosine$	$Kulc$	IR
D_1	10,000	1,000	1,000	100,000	0.83	0.91	0.91	0
D_2	10,000	1,000	1,000	100	0.83	0.91	0.91	0
D_3	100	1,000	1,000	100,000	0.05	0.09	0.09	0
D_4	1,000	1,000	1,000	100,000	0.33	0.5	0.5	0
D_5	1,000	100	10,000	100,000	0.09	0.29	0.5	0.89
D_6	1,000	10	100,000	100,000	0.01	0.10	0.5	0.99

Example: Analysis of DBLP Coauthor Relationships

- DBLP: Computer science research publication bibliographic database
  - > 3.8 million entries on authors, paper, venue, year, and other information

ID	Author A	Author B	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	*Cosine*	*Kulc*
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related?
  - Use Kulc to find Advisor-advisee, close collaborators

What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
  - Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers; ……
- *Null-invariance* is an important property
- Lift, χ^2 and cosine are good measures if null transactions are not predominant
  - Otherwise, *Kulczynski* + *Imbalance Ratio* should be used to judge the interestingness of a pattern

What Measures to Choose for Effective Pattern Evaluation?

- Exercise: Mining research collaborations from research bibliographic data
  - Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
  - Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
  - Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
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- Summary

Summary

- Basic Concepts
  - What Is Pattern Discovery?   Why Is It Important?
  - Basic Concepts: Frequent Patterns and Association Rules
  - Compressed Representation: Closed Patterns and Max-Patterns
- Efficient Pattern Mining Methods
  - The Downward Closure Property of Frequent Patterns
  - The Apriori Algorithm
  - Extensions or Improvements of Apriori
  - Mining Frequent Patterns by Exploring Vertical Data Format
  - FPGrowth: A Frequent Pattern-Growth Approach
  - Mining Closed Patterns
- Pattern Evaluation
  - Interestingness Measures in Pattern Mining
  - Interestingness Measures: Lift and χ^2
  - Null-Invariant Measures
  - Comparison of Interestingness Measures

Recommended Readings (Basic Concepts)

- R. Agrawal, T. Imielinski, and A. Swami, “Mining association rules between sets of items in large databases”, in Proc. of SIGMOD'93
- R. J. Bayardo, “Efficiently mining long patterns from databases”, in Proc. of SIGMOD'98
- N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, “Discovering frequent closed itemsets for association rules”, in Proc. of ICDT'99
- J. Han, H. Cheng, D. Xin, and X. Yan, “Frequent Pattern Mining: Current Status and Future Directions”, Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

Recommended Readings (Efficient Pattern Mining Methods)

- R. Agrawal and R. Srikant, “Fast algorithms for mining association rules”, VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, “An efficient algorithm for mining association rules in large databases”, VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, “An effective hash-based algorithm for mining association rules”, SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, “Integrating association rule mining with relational database systems: Alternatives and implications”, SIGMOD'98
- M. J. Zaki, S. Parthasarathy, M. Ogihara, and W. Li, “Parallel algorithm for discovery of association rules”, Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, “Mining frequent patterns without candidate generation”, SIGMOD’00
- M. J. Zaki and Hsiao, “CHARM: An Efficient Algorithm for Closed Itemset Mining”, SDM'02
- J. Wang, J. Han, and J. Pei, “CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets”, KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, “Frequent Pattern Mining Algorithms: A Survey”, in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

Recommended Readings (Pattern Evaluation)

- C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
- T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371-397, 2010