CS 412 Intro. to Data Mining

Chapter 2. Getting to Know Your Data

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Chapter 2. Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Correlation
- Summary

Types of Data Sets: (1) Record Data

- Relational records
  - Relational tables, highly structured
- Data matrix, e.g., numerical matrix, crosstabs

	China	England	France	Japan	USA	Total
Active Outdoors Crochet Glove		12.00	4.00	1.00	240.00	257.00
Active Outdoors Lycra Glove		10.00	6.00		323.00	339.00
InFlux Crochet Glove	3.00	6.00	8.00		132.00	149.00
InFlux Lycra Glove		2.00			143.00	145.00
Triumph Pro Helmet	3.00	1.00	7.00		333.00	344.00
Triumph Vertigo Helmet		3.00	22.00		474.00	499.00
Xtreme Adult Helmet	8.00	8.00	7.00	2.00	251.00	276.00
Xtreme Youth Helmet		1.00			76.00	77.00
Total	14.00	43.00	54.00	3.00	1,972.00	2,086.00

Person:

Pers_ID	Surname	First_Name	City
0	Miller	Paul	London
1	Ortega	Alvaro	Valencia
2	Huber	Urs	Zurich
3	Blanc	Gaston	Paris
4	Bertolini	Fabrizio	Rom

no relation

Car:

Car_ID	Model	Year	Value	Pers_ID
101	Bentley	1973	100000	0
102	Rolls Royce	1965	330000	0
103	Peugeot	1993	500	3
104	Ferrari	2005	150000	4
105	Renault	1998	2000	3
106	Renault	2001	7000	3
107	Smart	1999	2000	2

- Transaction data

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

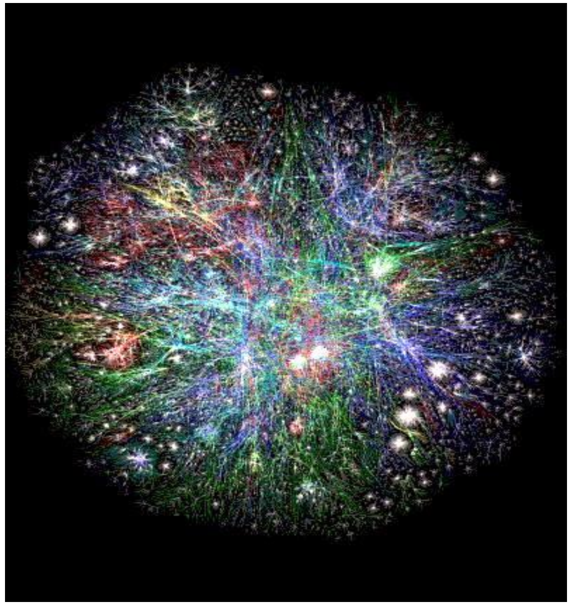
	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

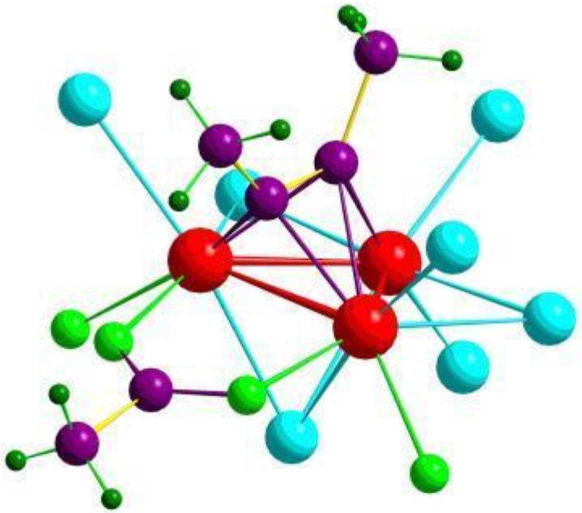
- Document data: Term-frequency vector (matrix) of text documents

Types of Data Sets: (2) Graphs and Networks

- Transportation network

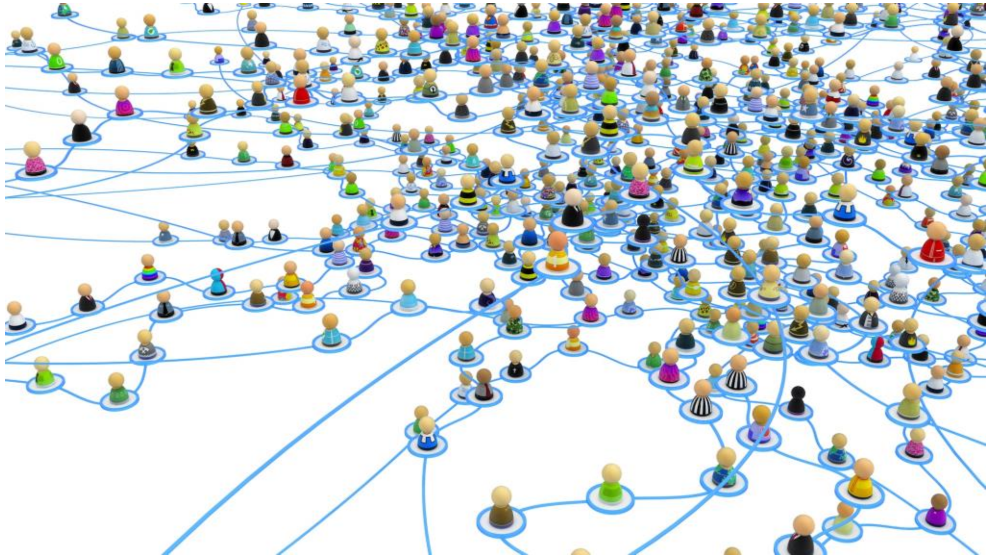
- World Wide Web





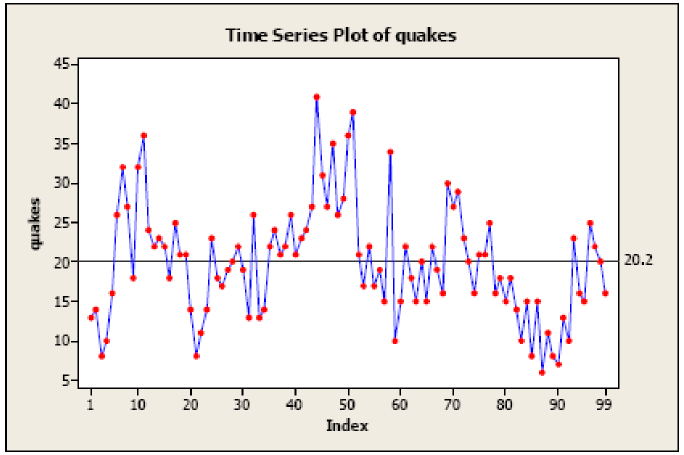
- Molecular Structures

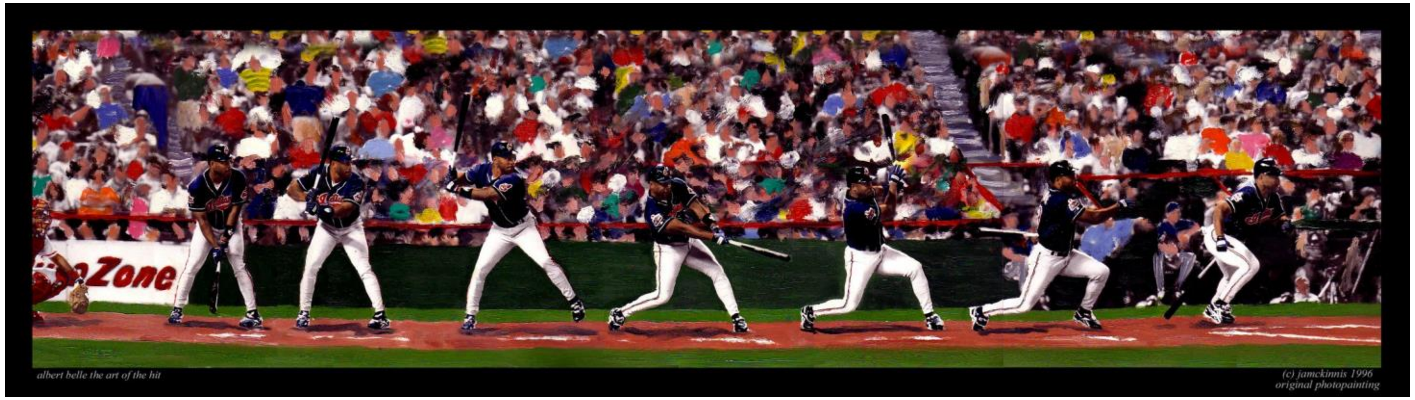
- Social or information networks



Types of Data Sets: (3) Ordered Data

- Video data: sequence of images
- Temporal data: time-series



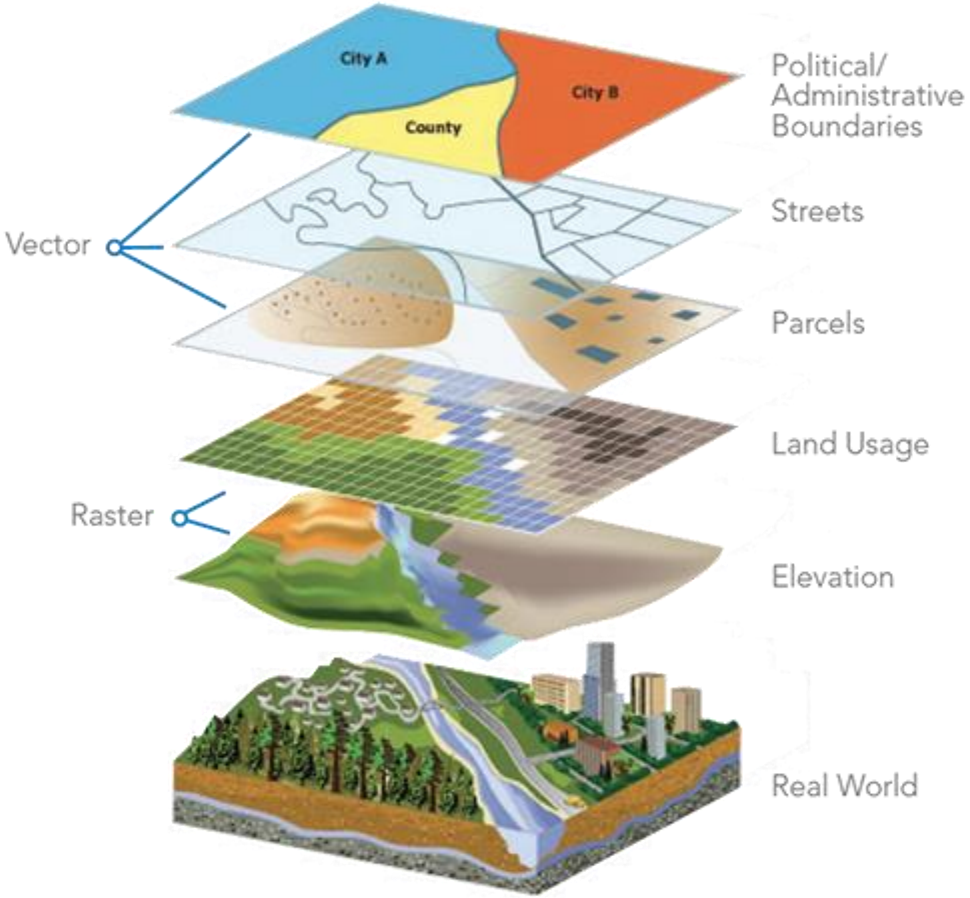


- Sequential Data: transaction sequences

- Genetic sequence data

Types of Data Sets: (4) Spatial, image and multimedia Data

- Spatial data: maps



- Image data:

- Video data:

Important Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

Data Objects

- Data sets are made up of data objects
- A **data object** represents an entity
- Also called *samples , examples, instances, data points, objects, tuples*

attributes

objects

id	name	gender	age
001	Bob	male	12
002	Jessica	female	23
003	Tim	male	56

Attributes or dimensions, features, variables

Name	Definition	Examples
Nominal	categories, states, or "names of things"	• Hair_color = {auburn, black, blond, brown, grey, red} • marital status, occupation, ID numbers, zip codes
Binary (0 or 1)	Symmetric: equally important	gender
	Asymmetric: not equally important	Medical test (negative & positive); assign 1 to most important outcome
Ordinal	Need order but no magnitude	Size = {small, medium, large}, grades, army rankings
Numeric	Interval: • equal-sized units; • ordered; • no true zero-point;	temperature in C˚or F˚, calendar dates
	Ratio: inherent zero-point; being an order of magnitude larger than the unit of measurement	temperature in Kelvin, length, counts, monetary quantities

Discrete vs. Continuous Attributes

Discrete Attribute	Continuous Attribute
only a finite /countably infinite, sometimes integer	real numbers
E.g., zip codes, profession, or the set of words in a collection of documents	E.g., temperature, height, or weight
special case : binary attributes	floating-point variables (practically with finite number of digits)

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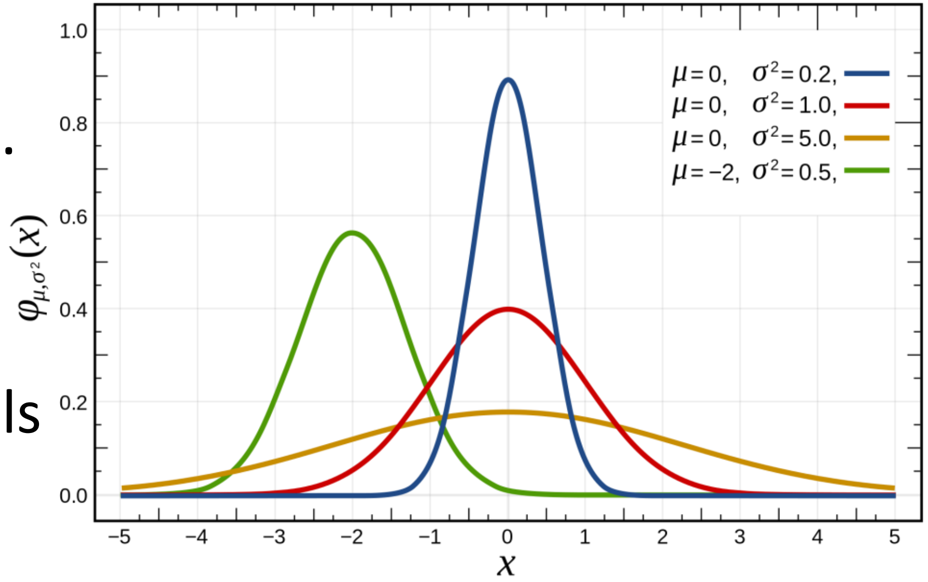
Basic Statistical Descriptions of Data

- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - Median, max, min, quantiles, outliers, variance, ...
  - Data dispersion:
    - Analyzed with multiple granularities of precision
- Numerical dimensions correspond to sorted intervals
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube



Measuring the Central Tendency: Mean, Median and Mode

- **Mean**: n->sample, N->population

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

Weighted arithmetic mean: $$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Trimmed mean: Chopping extreme values (e.g., Olympics gymnastics score computation)

- <u>Median</u>
  - Approximate median:

$$median = L_1 + \left(\frac{n/2 - (\sum freq)_l}{freq_{median}}\right) width$$

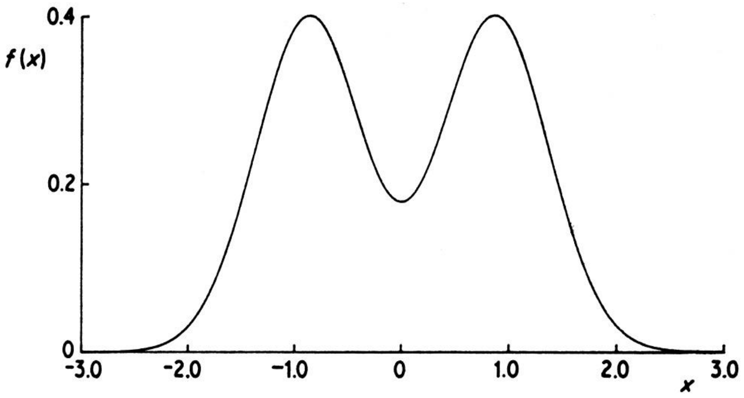
$\boldsymbol{L_1}$: $Low\ interval\ limit$

$\sum \boldsymbol{freq}$: sum before the median interval

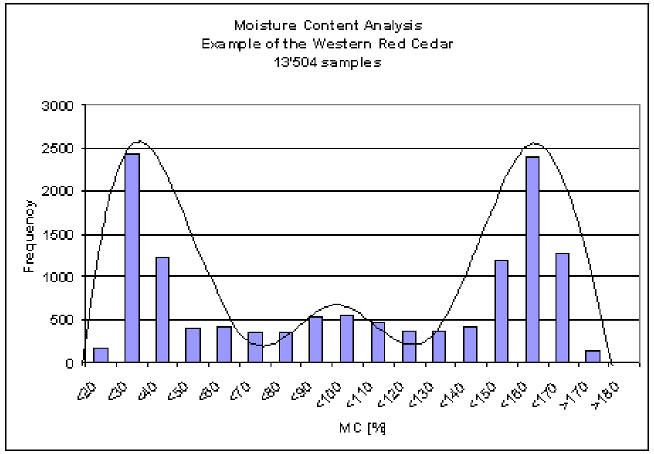
Width: interval width ($L_2 - L_1$)

- Mode:
  - Value that occurs most frequently in the data

Bimodal:

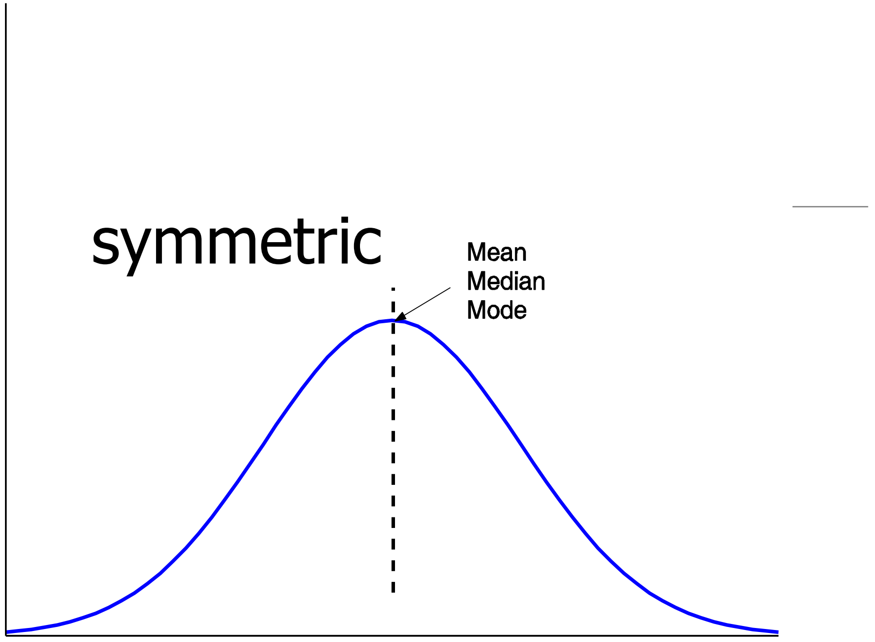


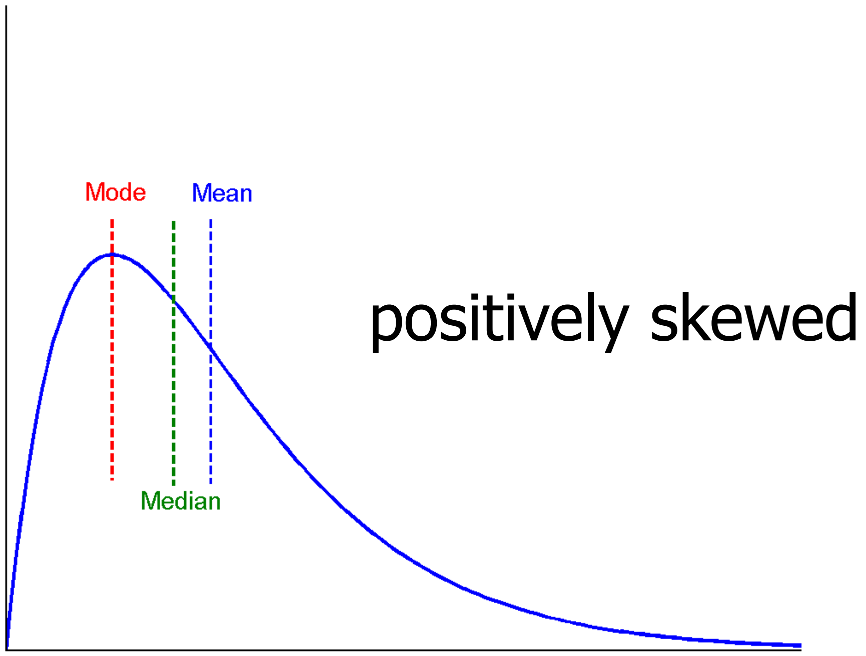
Trimodal:

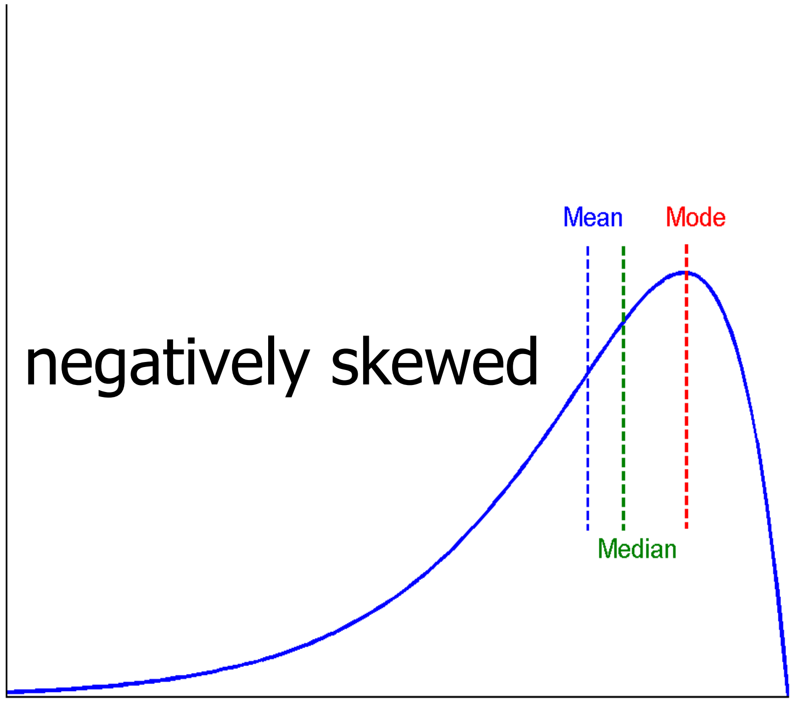


Symmetric vs. Skewed Data

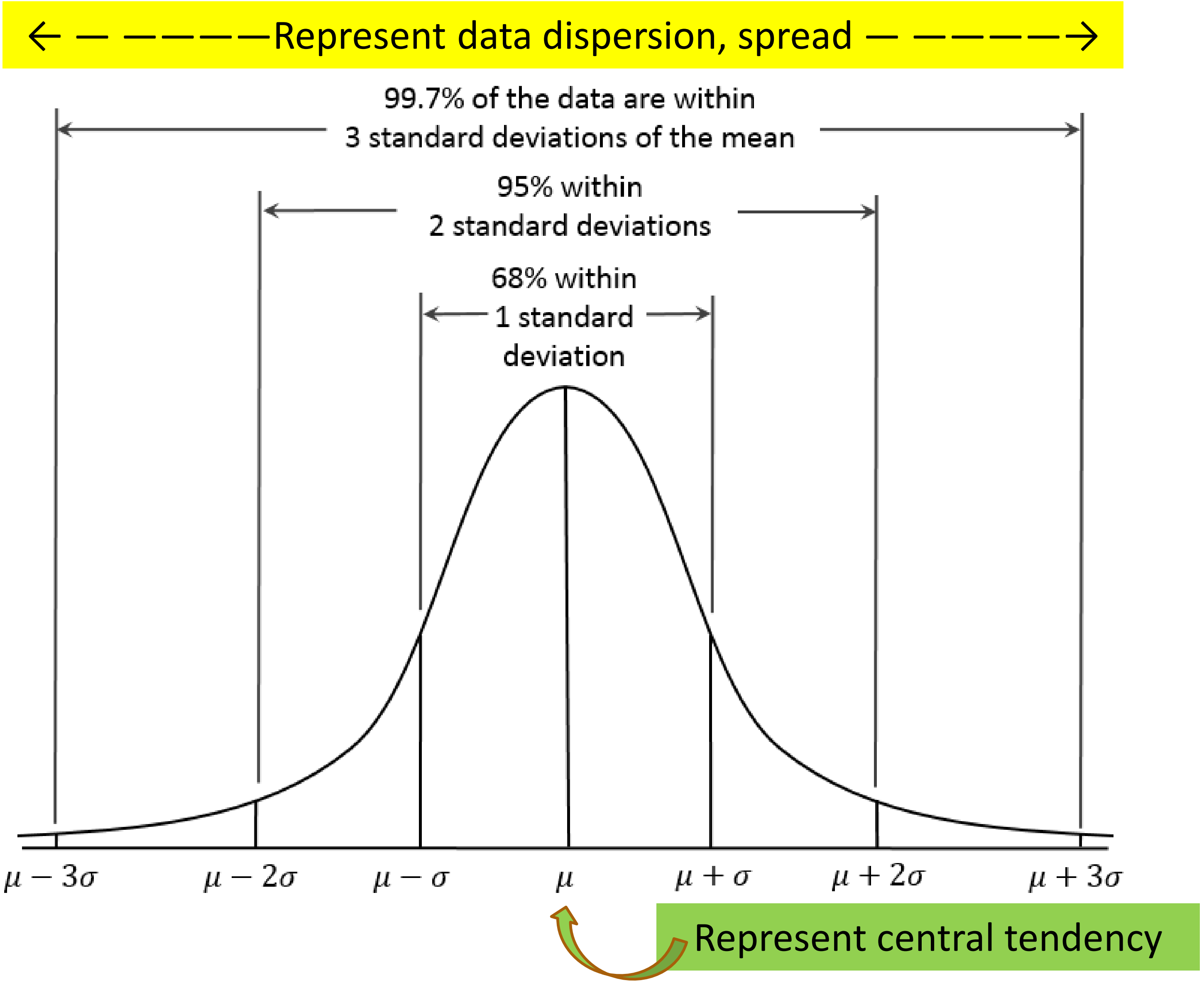
- Median, mean and mode of symmetric, positively and negatively skewed data







Properties of Normal Distribution Curve



Measures Data Distribution: Variance and Standard Deviation

- Variance and standard deviation (*sample: s, population: σ*)
  - **Variance**: (algebraic, scalable computation)
    - Q: Can you compute it incrementally and efficiently?

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2]$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{n} x_i^2 - \mu^2$$

Note: The subtle difference of formulae for sample vs. population
- n : the size of the sample
- N : the size of the population

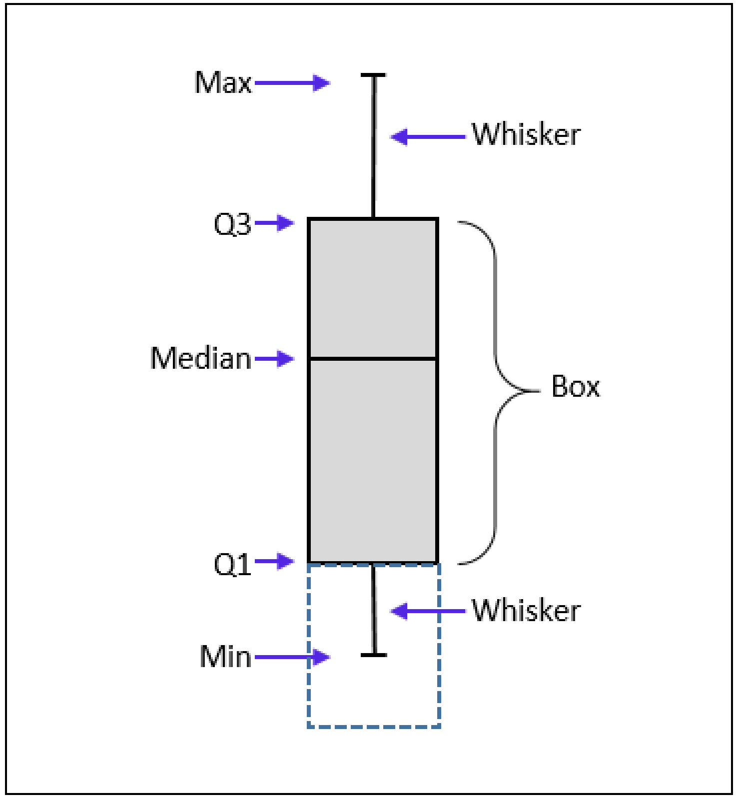
- **Standard deviation** *s (or σ)* is the square root of variance s^2 *(or σ^2)*

Graphic Displays of Basic Statistical Descriptions

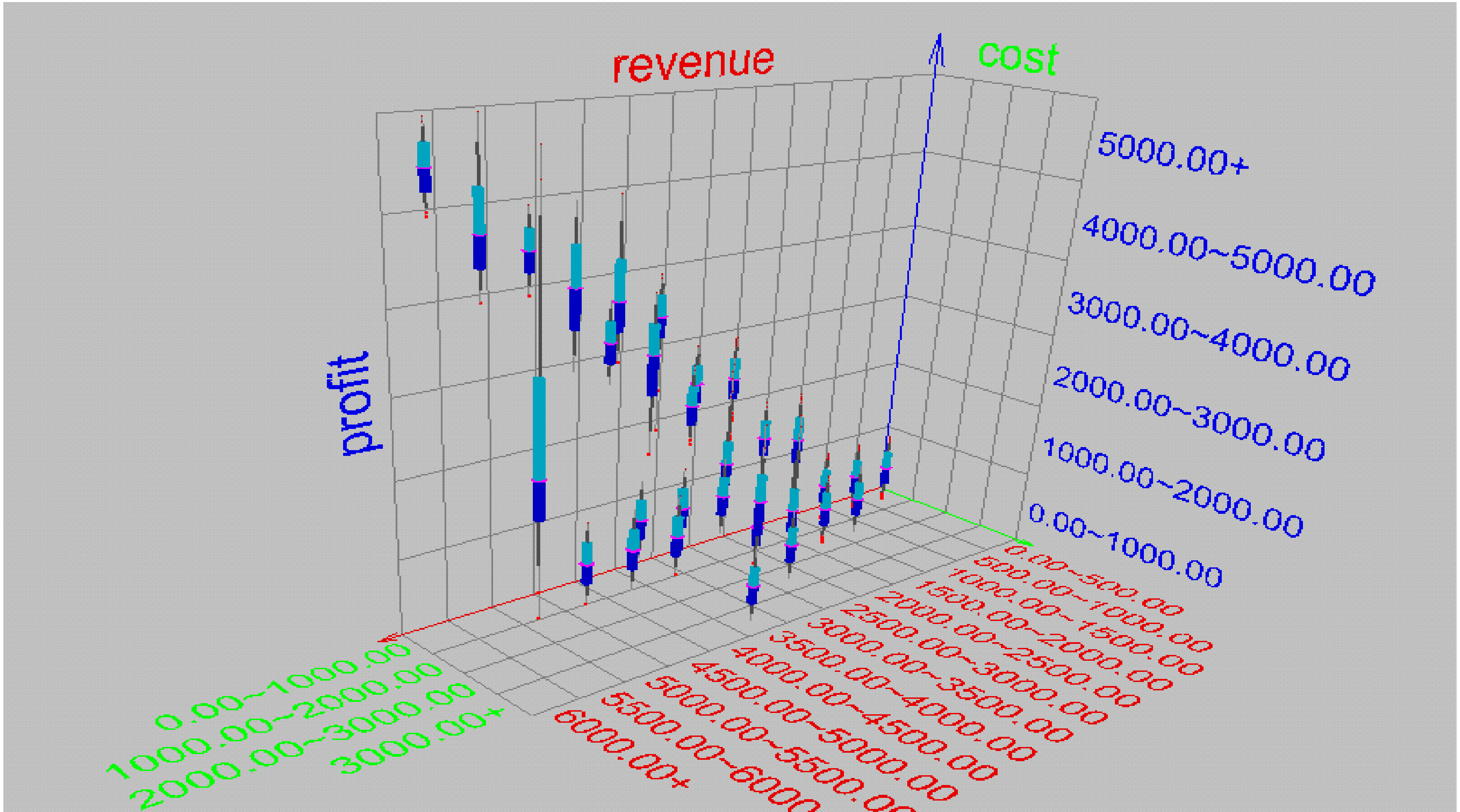
- **Boxplot**: five-number summary
- **Histogram**: values and frequencies
- **Quantile plot**: each value x_i is paired with f_i indicating that approximately 100 f_i % of data are $\leq x_i$
- **Quantile-quantile (q-q) plot**: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- **Scatter plot**: data plotted as points

Measuring the Dispersion of Data: Quartiles & Boxplots

- **Quartiles**: Q_1 (25th percentile), Q_3 (75th percentile)
- **Inter-quartile range**: IQR = $Q_3 – Q_1$
- **Five number summary**: min, Q_1, median, Q_3, max
- **Boxplot**:
  - Outliers: points beyond a specified outlier threshold, plotted individually
    - **Outlier**: usually, a value higher/lower than 1.5 x IQR



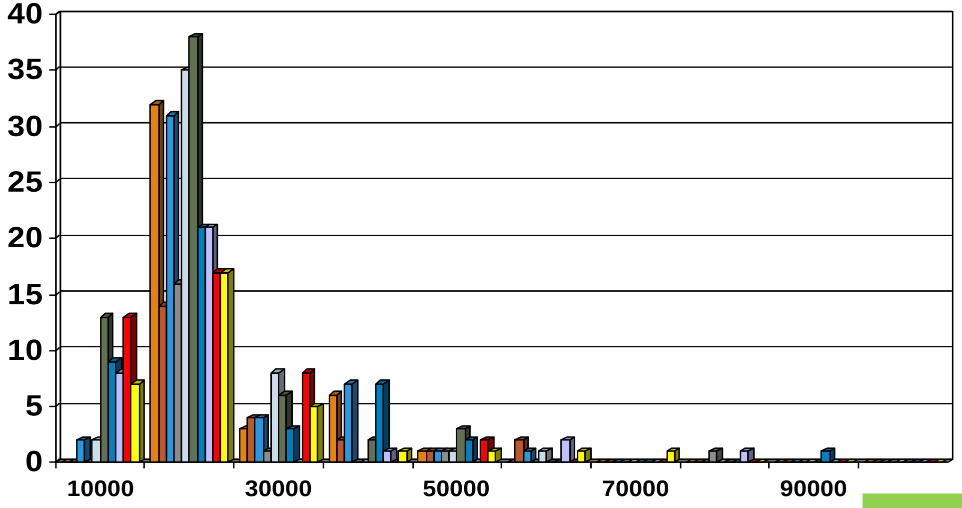
Visualization of Data Dispersion: 3-D Boxplots



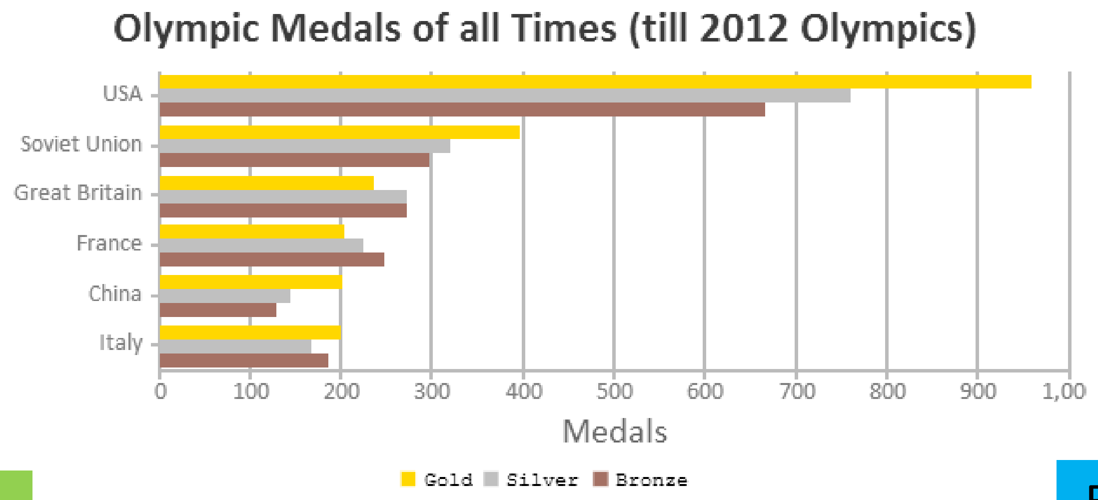
Histogram Analysis

- Histogram: tabulated frequencies, shown as bars

Histogram	Bar charts
distributions of variables	compare variables
quantitative data	categorical data
Value: area of the bar	Value: height of the bar (a crucial distinction when the categories are not of uniform width)
Order matters	Can be reordered

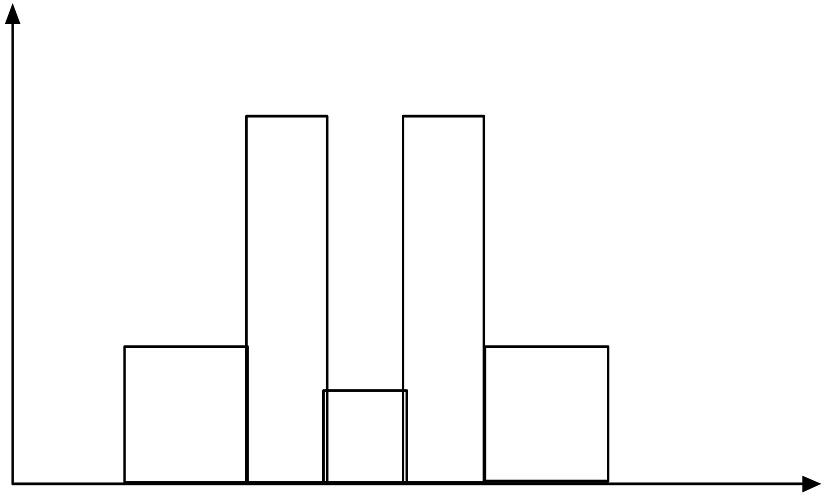


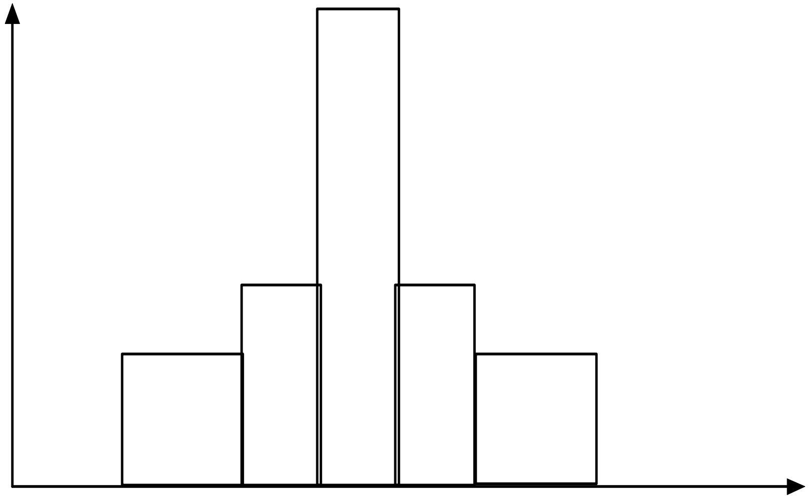
Histogram



Bar chart

Histograms Often Tell More than Boxplots

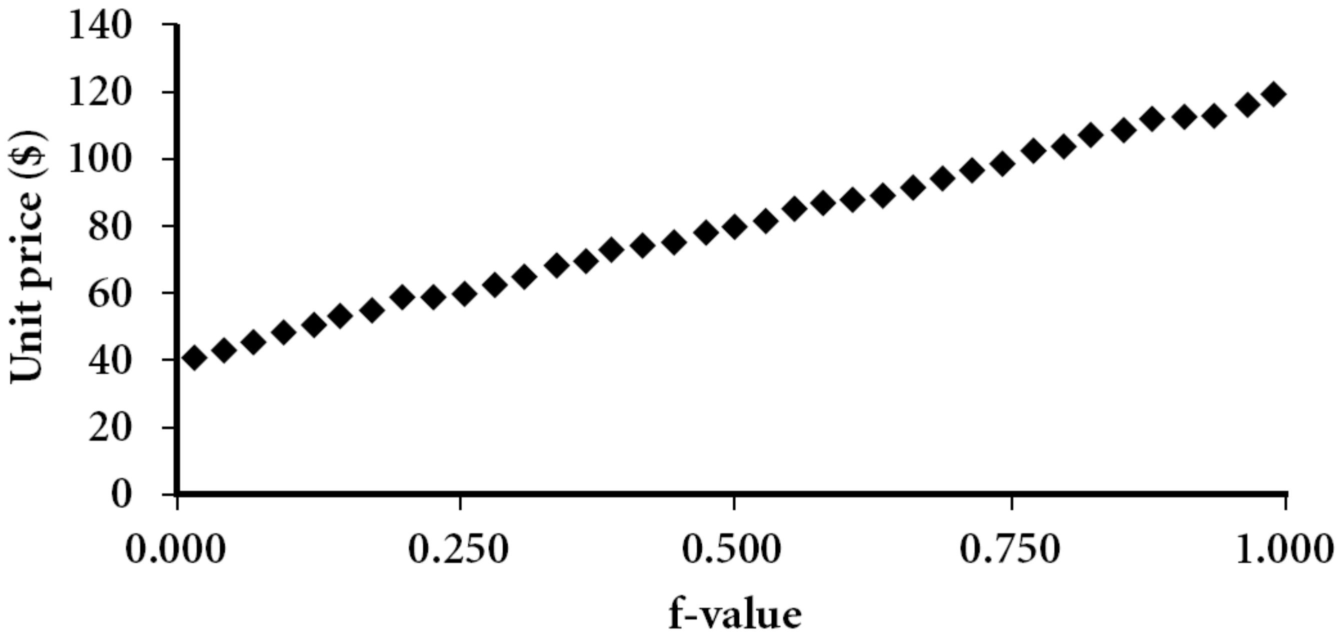




- **Same** boxplot representation
  - The same min, Q1, median, Q3, max
- **Different** data distributions

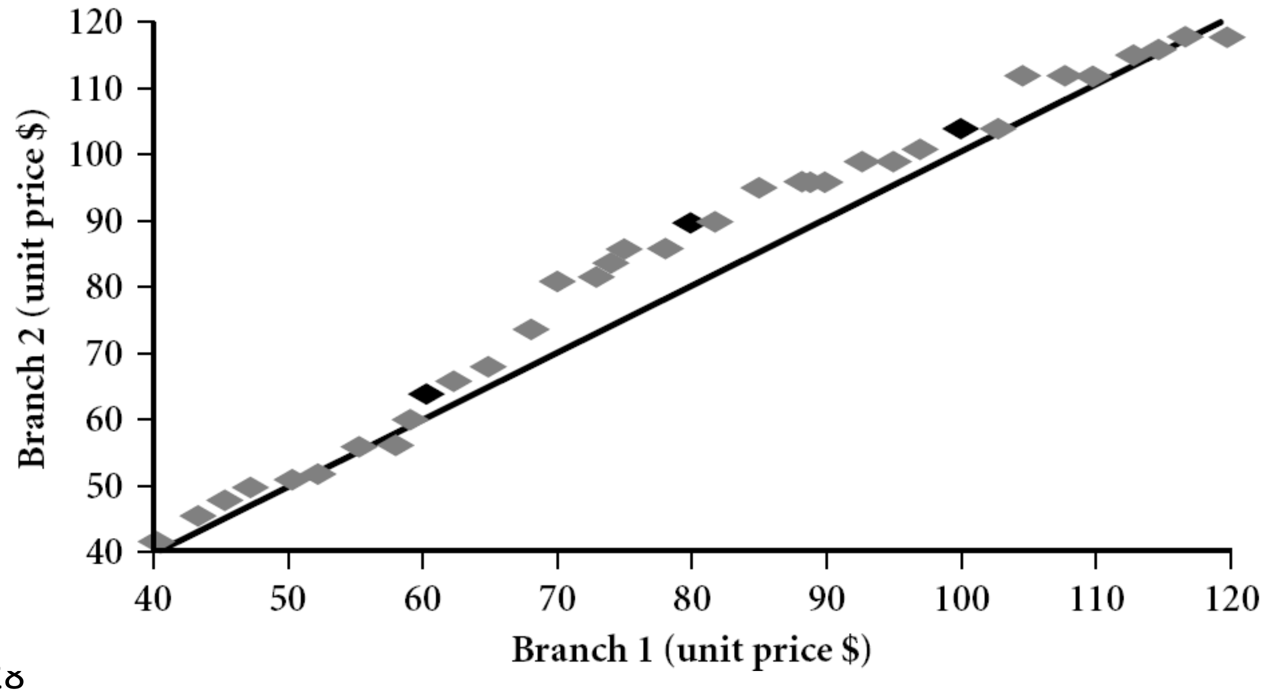
Quantile Plot

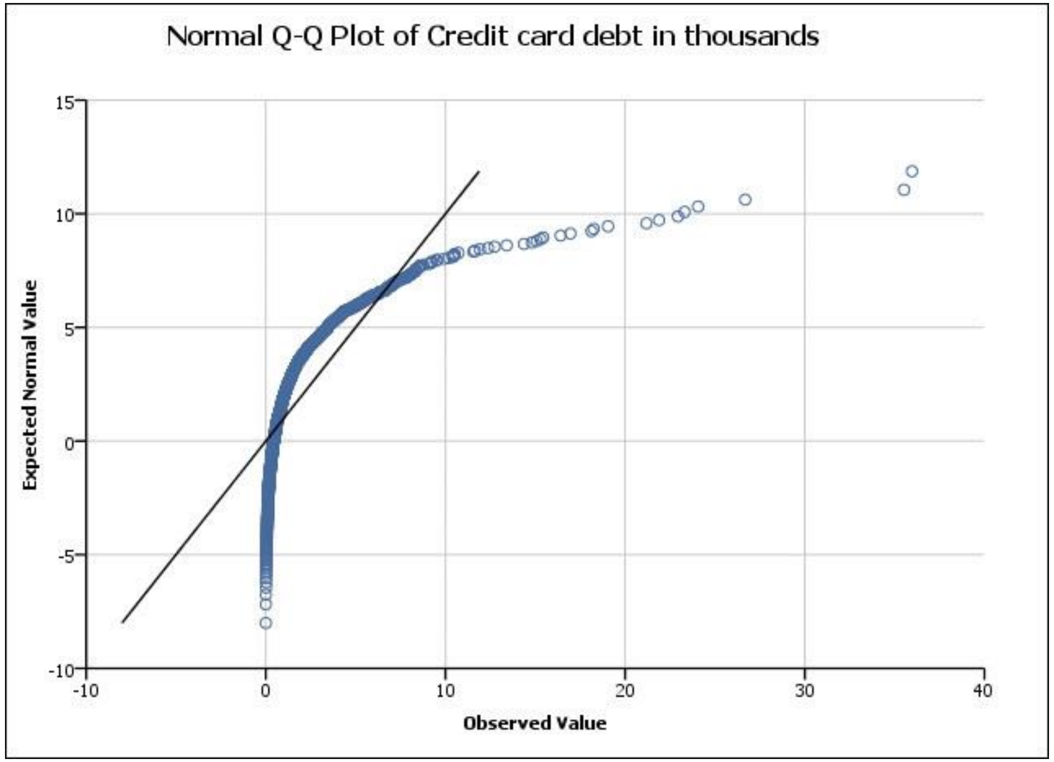
- Displays all of the data
  - **overall** behavior and **unusual** occurrences
- Plots **quantile** information
  - For a data x_i data sorted in increasing order, f_i indicates that approximately 100 f_i% of the data are below or equal to the value x_i



Quantile-Quantile (Q-Q) Plot

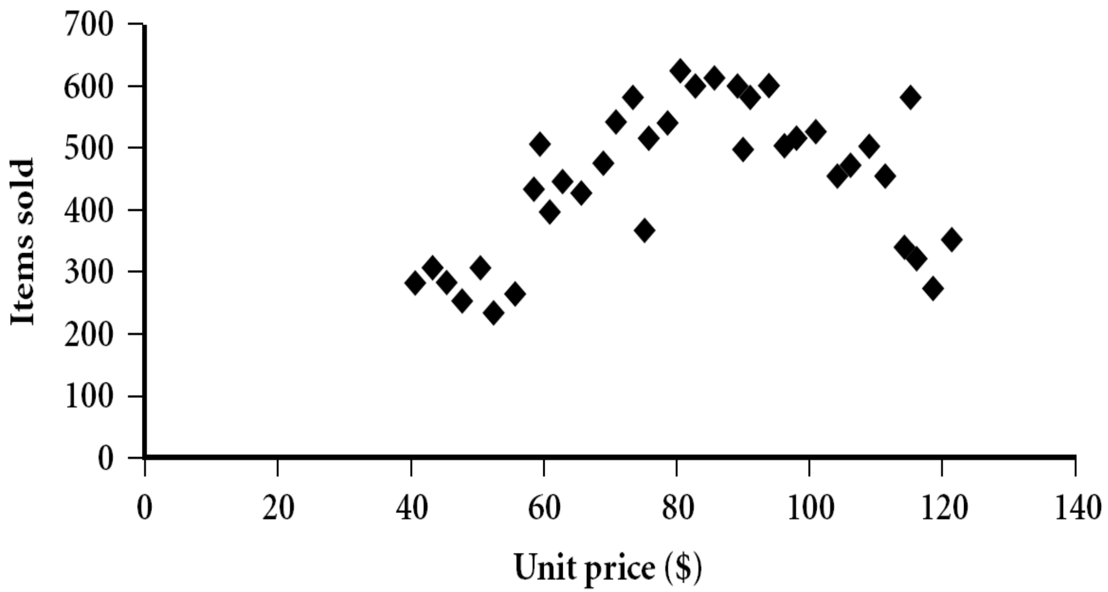
- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2

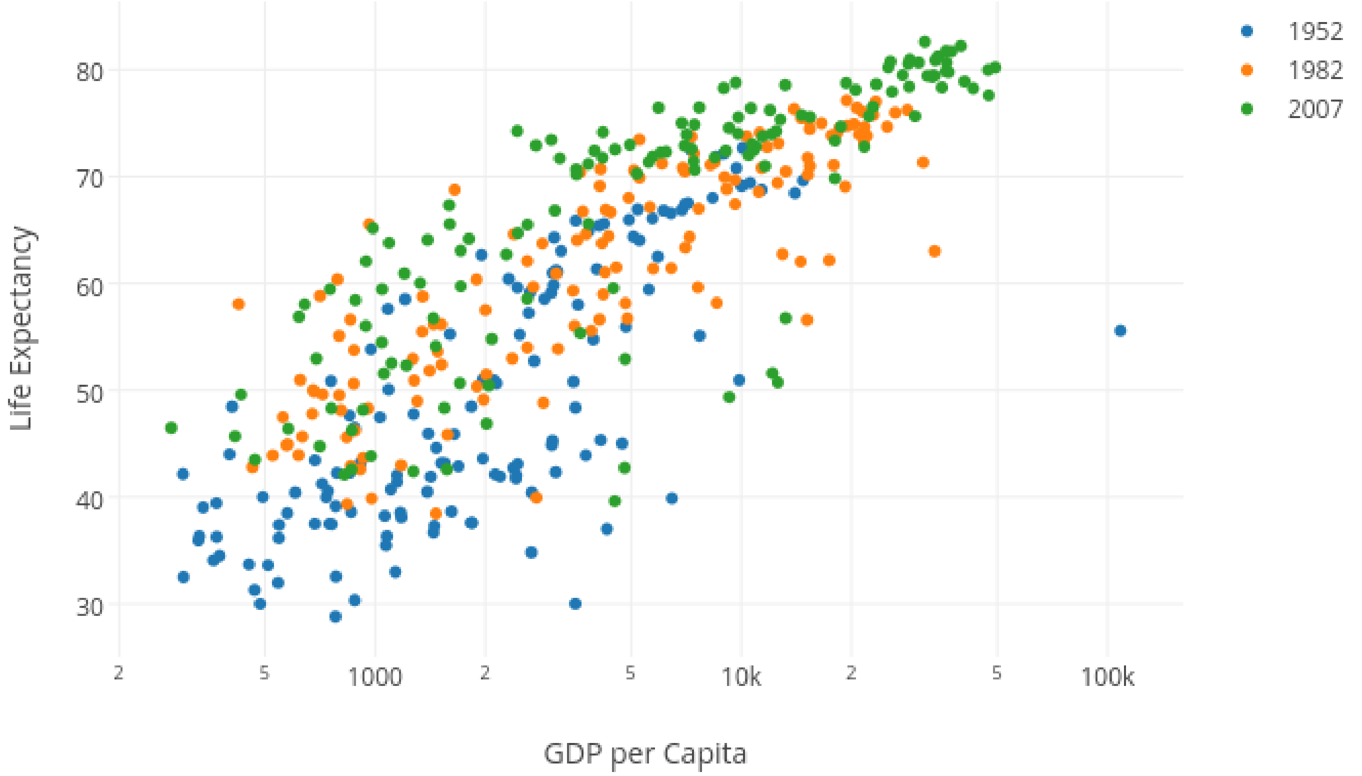




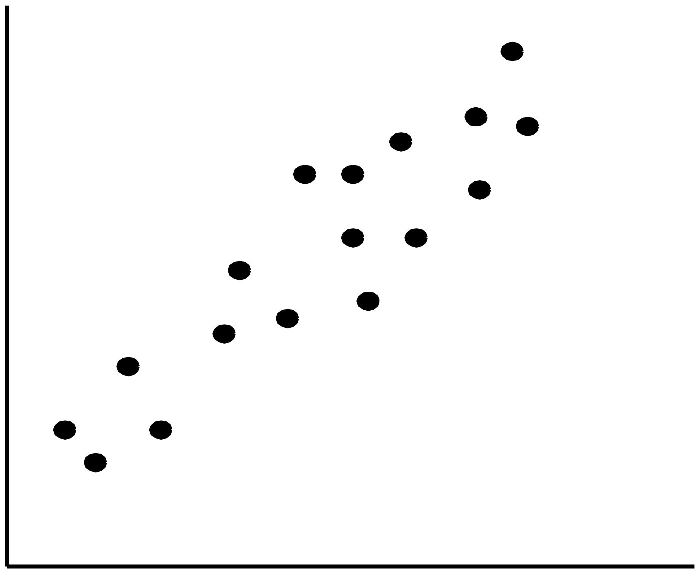
Scatter plot

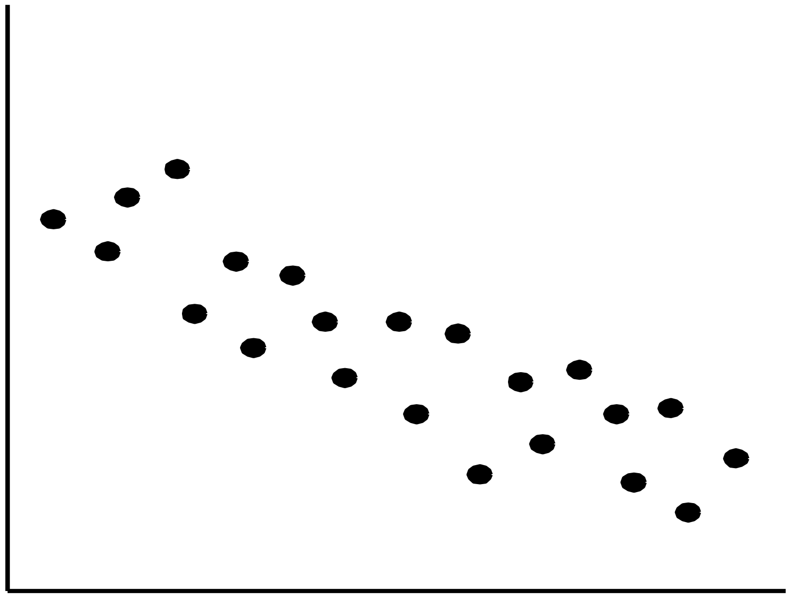
- Provides a first look at **bivariate** data to see clusters of points, outliers, etc.

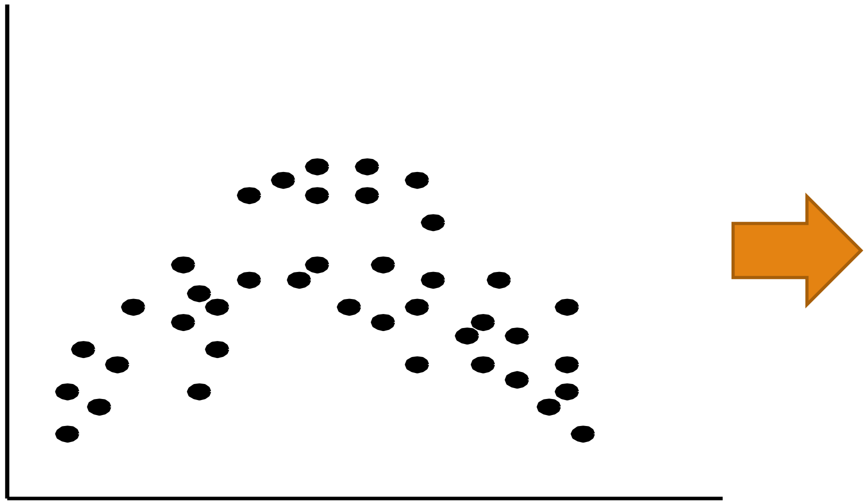




Positively and Negatively Correlated Data

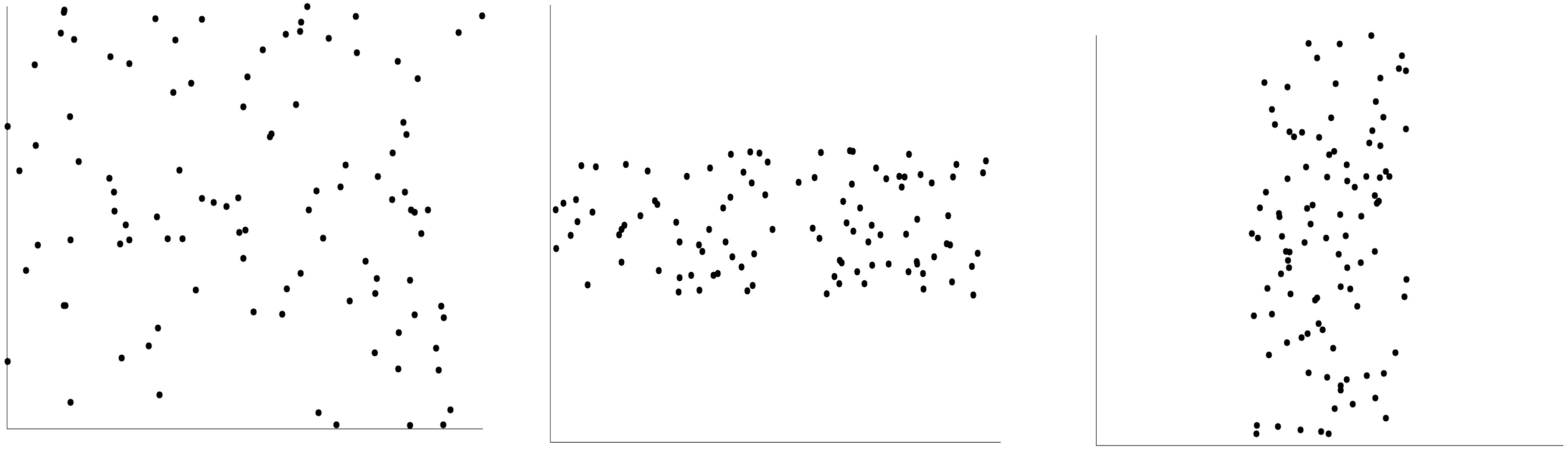






- The left half fragment is positively correlated
- The right half is negative correlated

Uncorrelated Data



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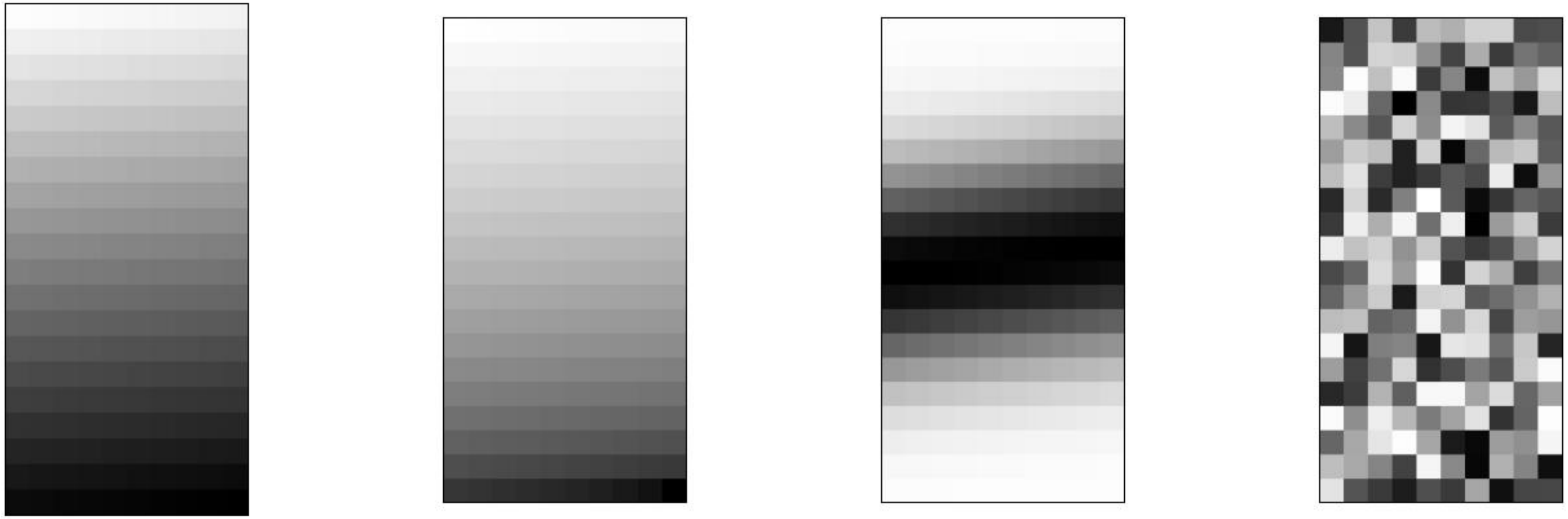
Data Visualization

- Why data visualization?
  - Gain insight into an information space by mapping data onto graphical primitives
  - Provide qualitative overview of large data sets
  - Search for patterns, trends, structure, irregularities, relationships among data
  - Help find interesting regions and suitable parameters for further quantitative analysis
  - Provide a visual proof of computer representations derived
- Categorization of visualization methods:
  - Pixel-oriented visualization techniques
  - Geometric projection visualization techniques
  - Icon-based visualization techniques
  - Hierarchical visualization techniques
  - Visualizing complex data and relations

Pixel-Oriented Visualization Techniques

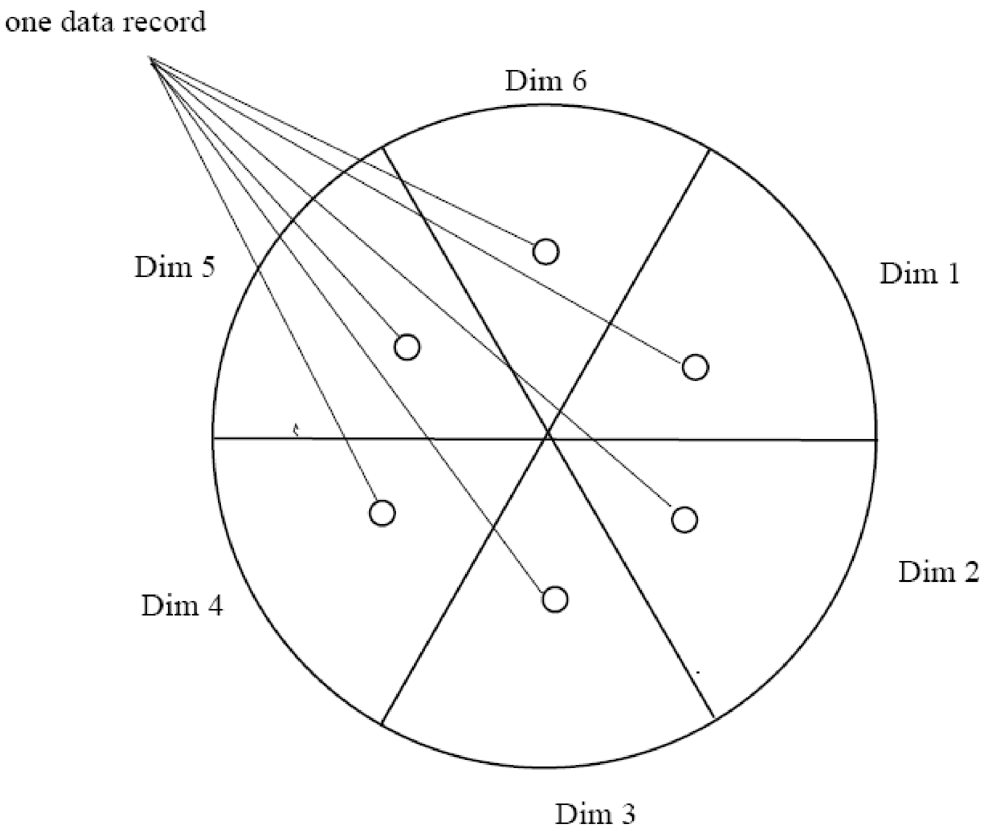
- For a data set of *m* dimensions, visualization has *m* windows, one for each dimension
- The *m* dimension values of a record are mapped to *m* pixels at the corresponding positions in the windows
- The colors of the pixels reflect the corresponding values

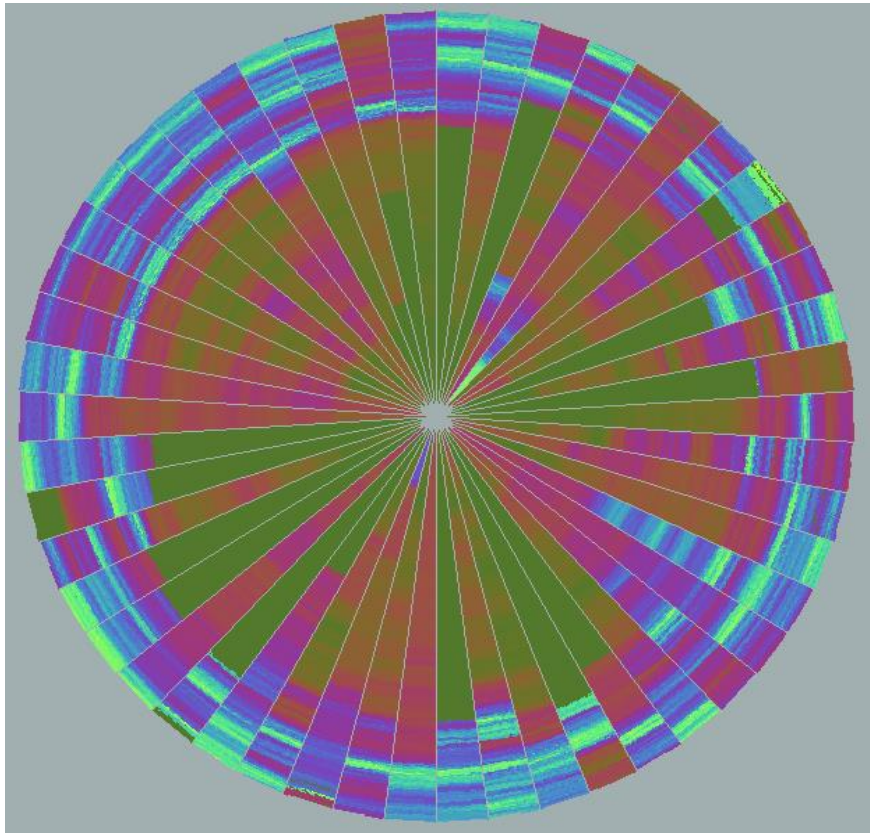


(a) Income (b) Credit Limit (c) transaction volume (d) age

Laying Out Pixels in Circle Segments

- Good for datasets with many dimensions
- Segments that look similar represent correlated dimensions

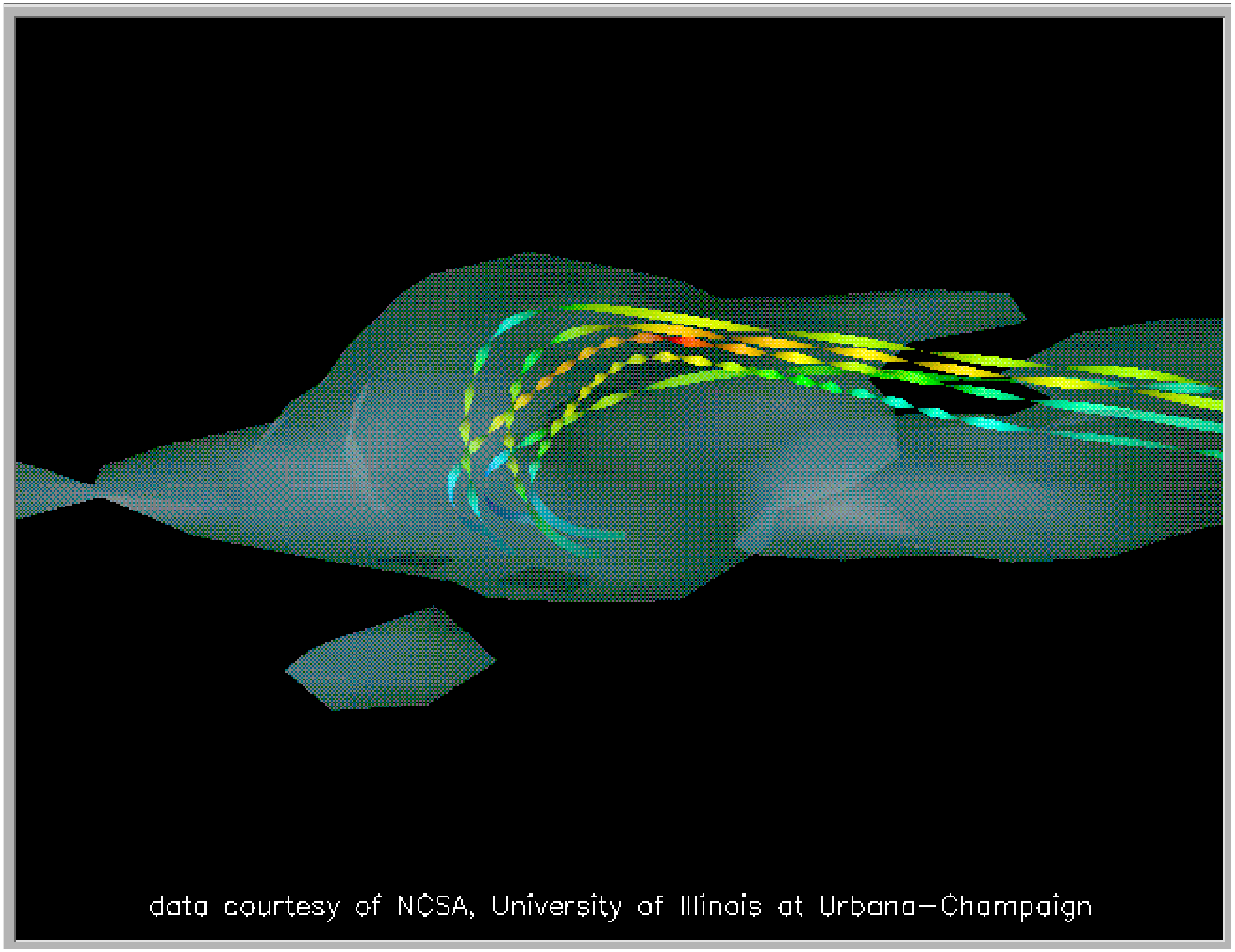


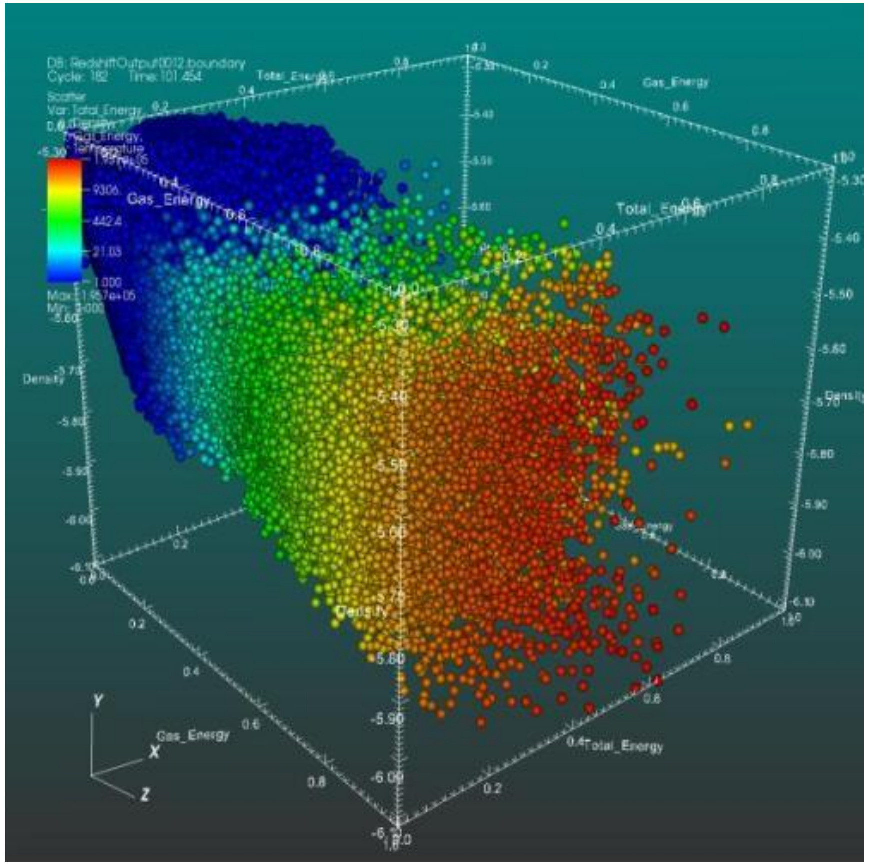


Representing about 265,000 50-dimensional Data Items with the 'Circle Segments' Technique

Direct Data Visualization

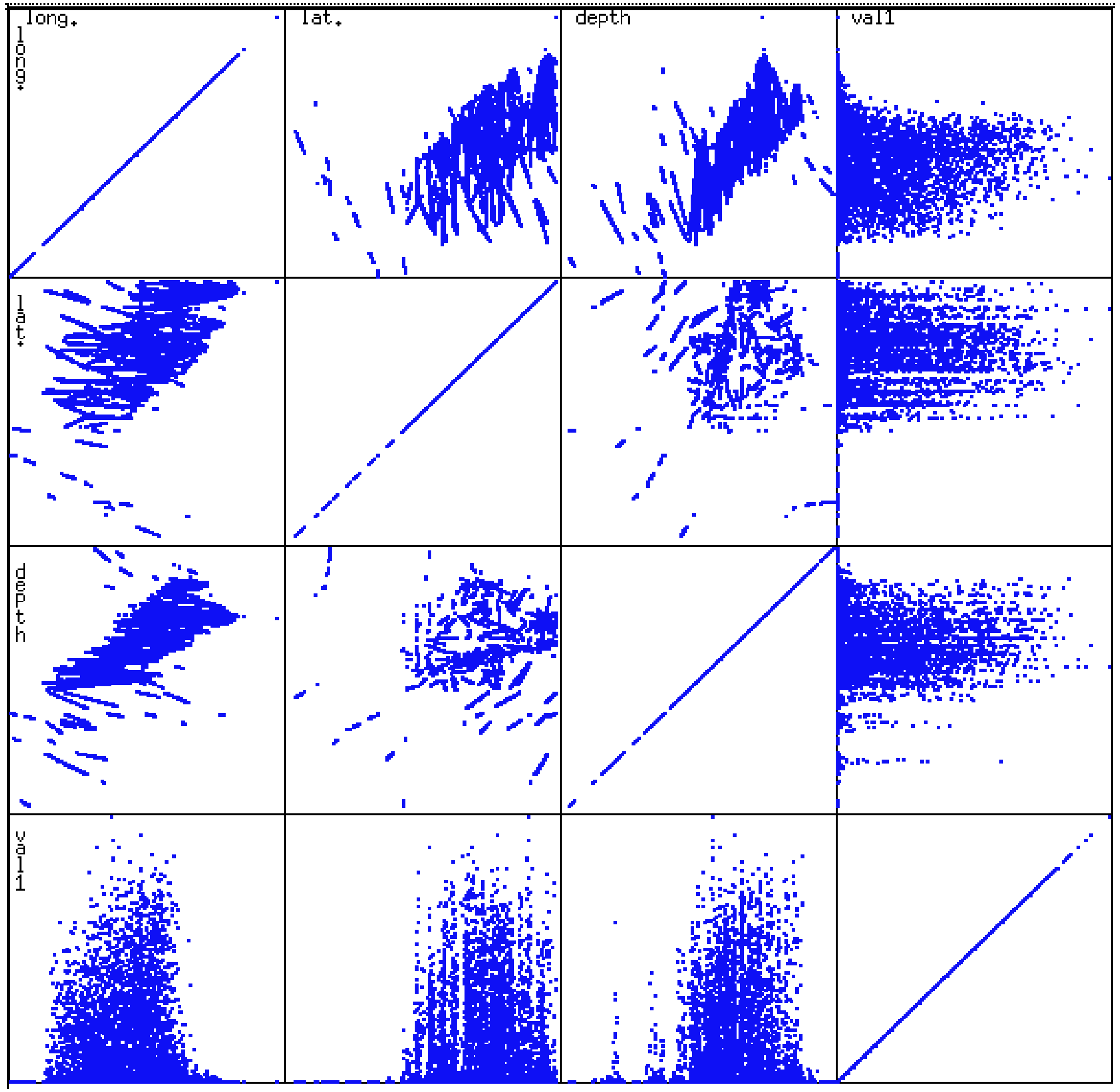
Ribbons with Twists Based on Vorticity





From Wiki: Scatter plot: A 3D scatter plot to visualize multivariate data

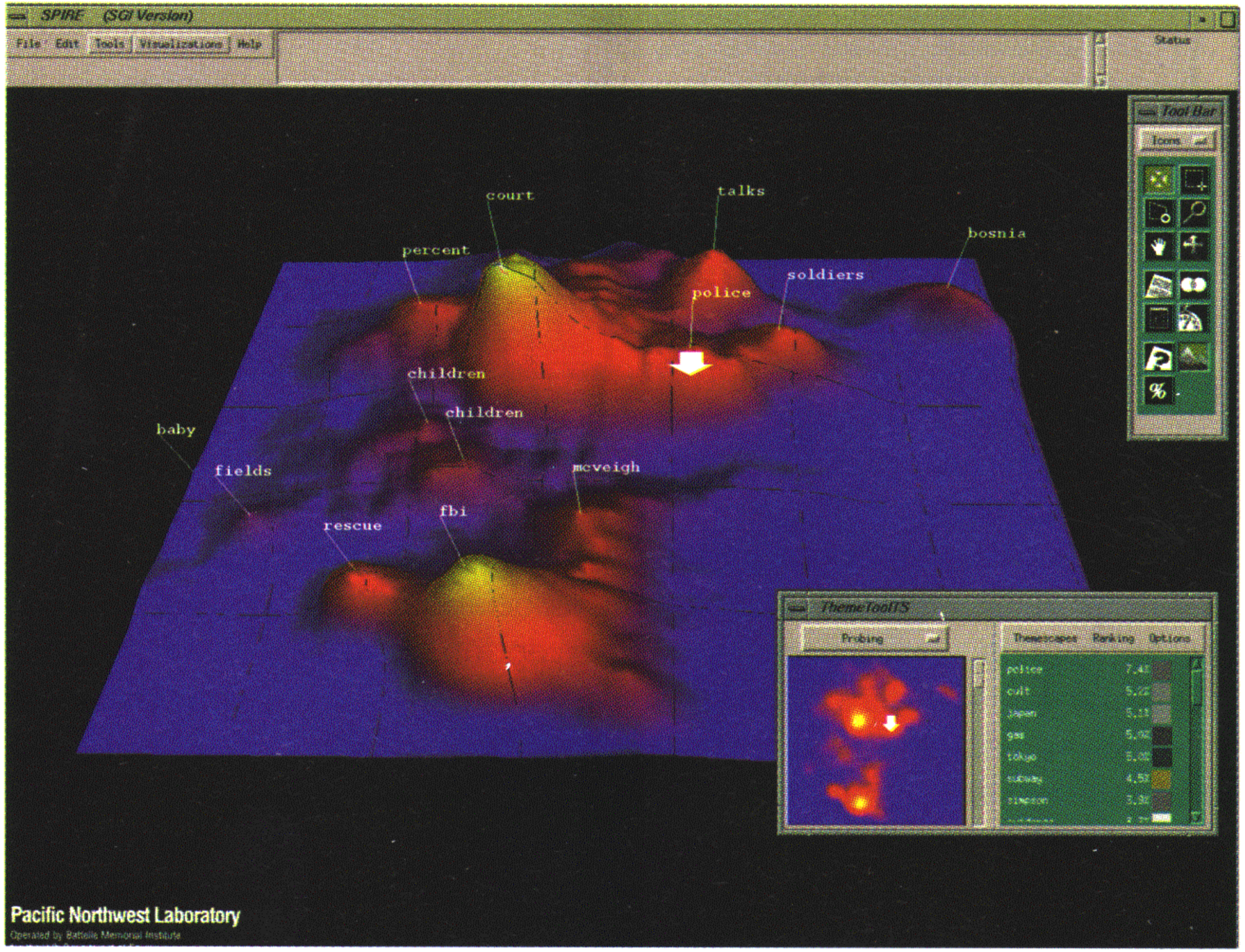
Scatterplot Matrices



Used by ermission of M. Ward, Worcester Polytechnic Institute

- Matrix of scatterplots (x-y-diagrams) of the k-dim. data
- A total of k(k-1)/2 distinct scatterplots
- Good for understanding whether two variables are correlated
- Not as helpful for high-dimensional data

Landscapes

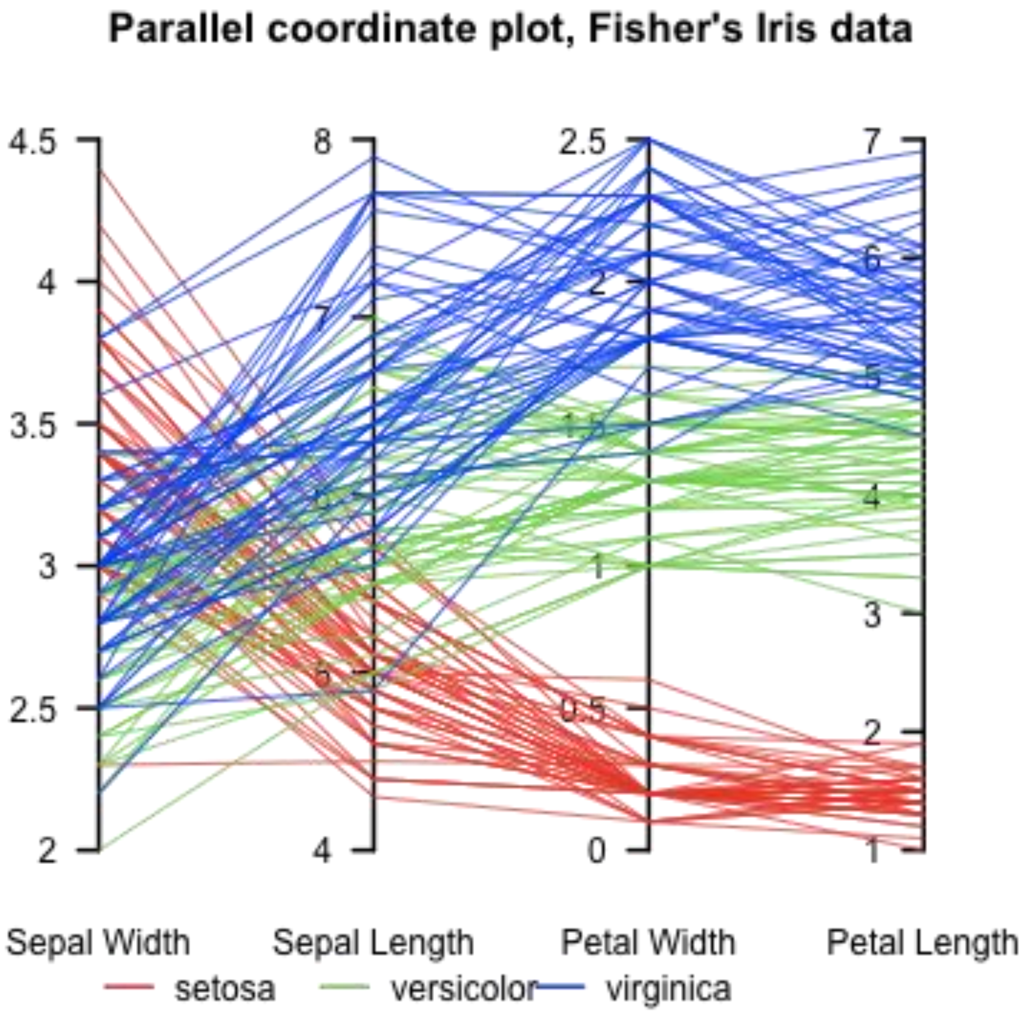


news articles visualized as a landscape

- Visualization of the data as perspective landscape
- Color indicates range of specific variables
- More advanced technique, requires in-depth understanding of the data to know how to transform data into a 2D spatial representation in a meaningful way

Parallel Coordinates

- n equidistant axes which correspond to the attributes of the data set
- Each data item corresponds to a line which intersects the axes at the point which corresponds to the value for the attribute
- Good for determining which attributes are most important for distinguishing between categories (e.g., Petal Length here)

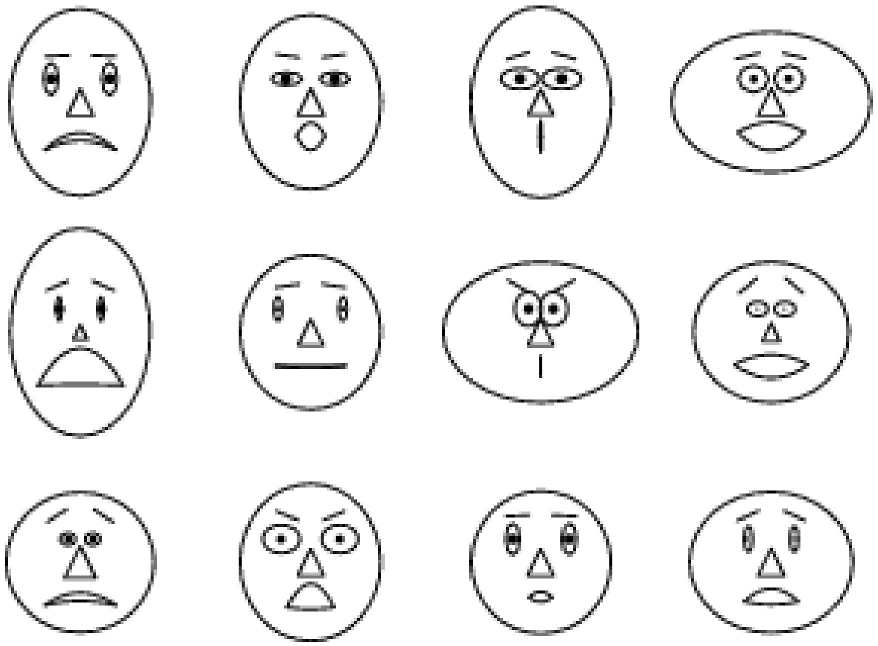


Icon-Based Visualization Techniques

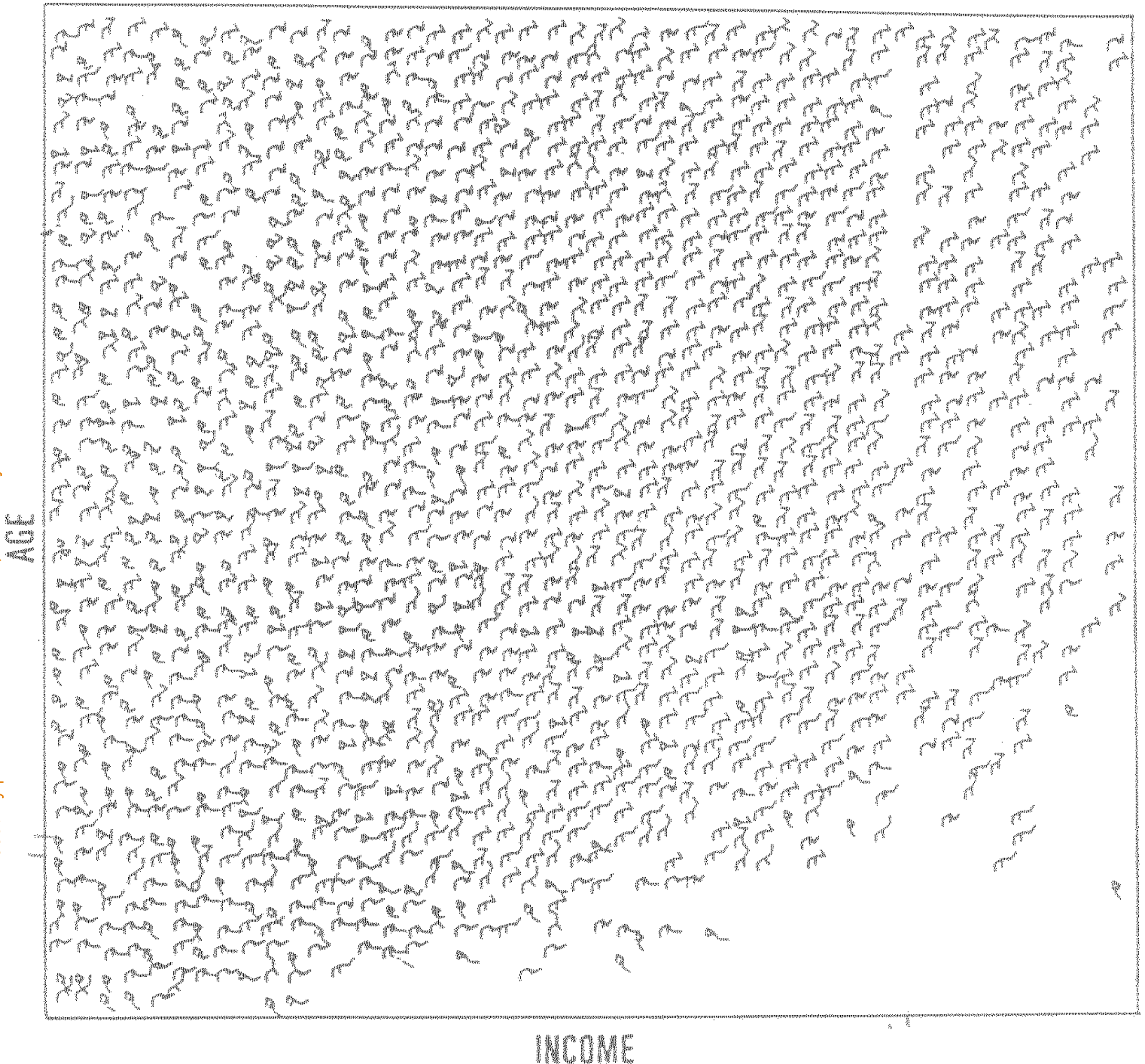
- Visualization of the data values as features of icons
- Typical visualization methods
  - Chernoff Faces
  - Stick Figures
- General techniques
  - Shape coding: Use shape to represent certain information encoding
  - Color icons: Use color icons to encode more information
  - Tile bars: Use small icons to represent the relevant feature vectors in document retrieval

Chernoff Faces

- A way to display variables on a two-dimensional surface, e.g., let x be eyebrow slant, y be eye size, z be nose length, etc.
- The figure shows faces produced using 10 characteristics--head eccentricity, eye size, eye spacing, eye eccentricity, pupil size, eyebrow slant, nose size, mouth shape, mouth size, and mouth opening): Each assigned one of 10 possible values, generated using *Mathematica* (S. Dickson)
- Humans are good at distinguishing differences in faces
- Can be difficult to implement (need a good way to map variables to facial features)



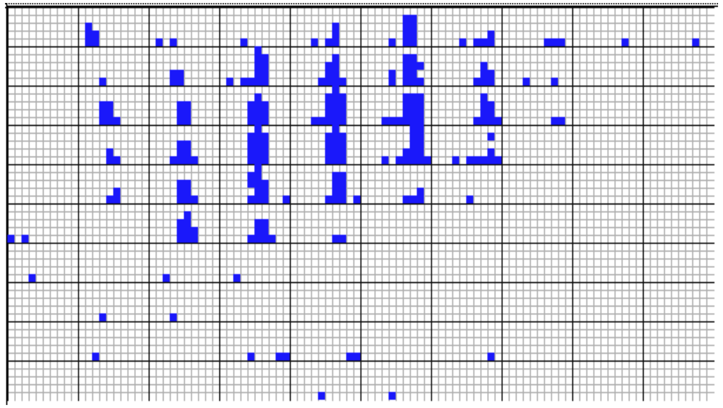
Stick Figure

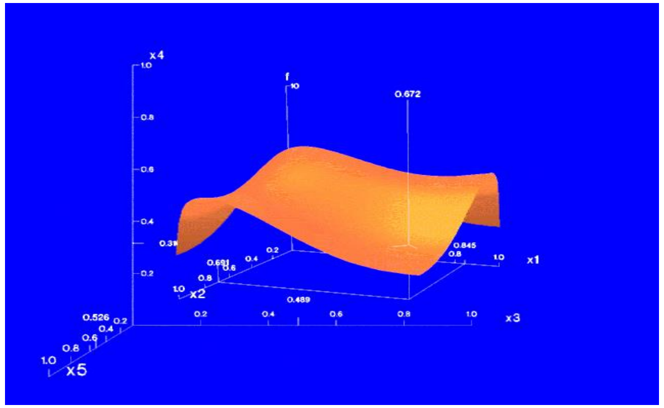


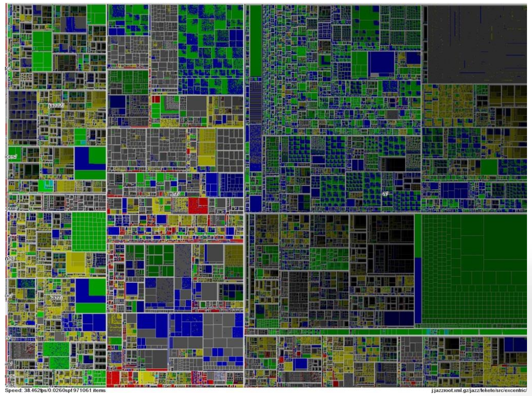
- A census data figure showing age, income, gender, education, etc.
- A 5-piece stick figure (1 body and 4 limbs w. different angle/length)
- Uses smaller number of features than Chernoff Faces
- Also requires careful design to make visualization meaningful

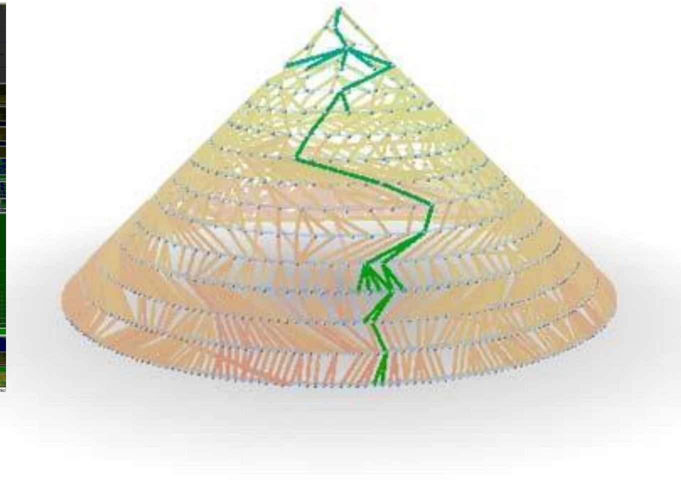
Hierarchical Visualization Techniques

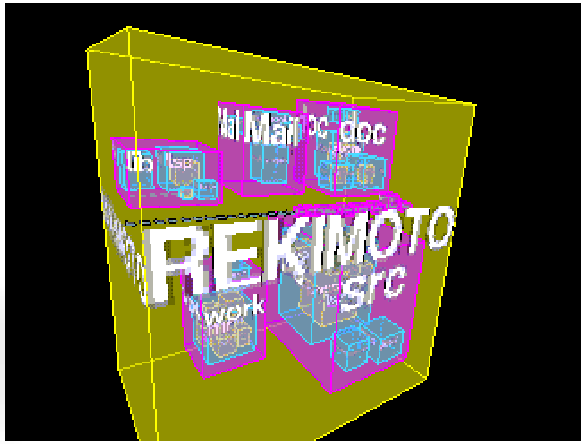
- Visualization of the data using a hierarchical partitioning into subspaces
- Methods
  - Dimensional Stacking
  - Worlds-within-Worlds
  - Tree-Map
  - Cone Trees
  - InfoCube



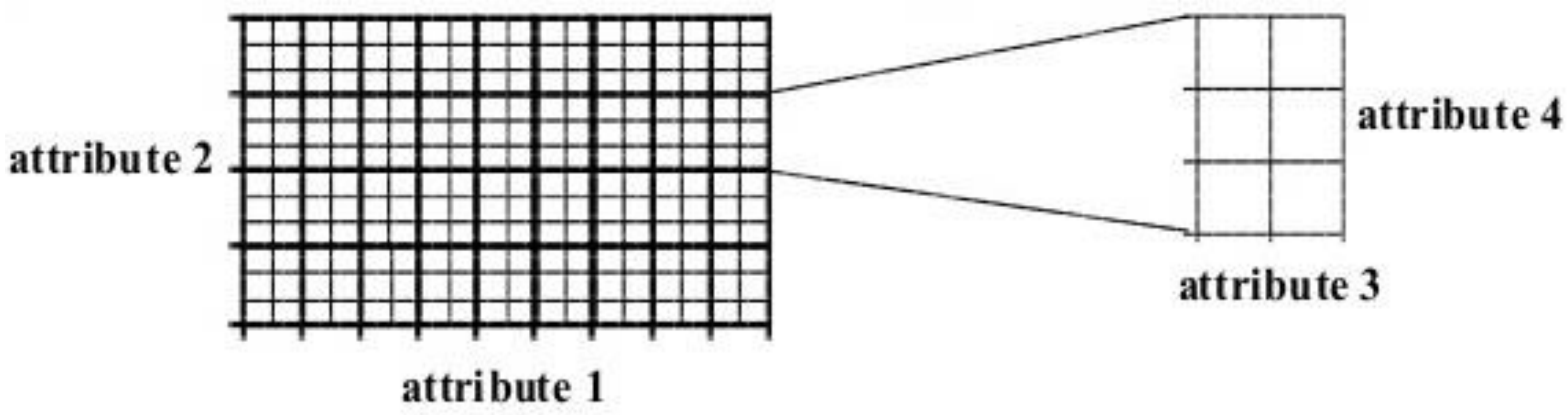








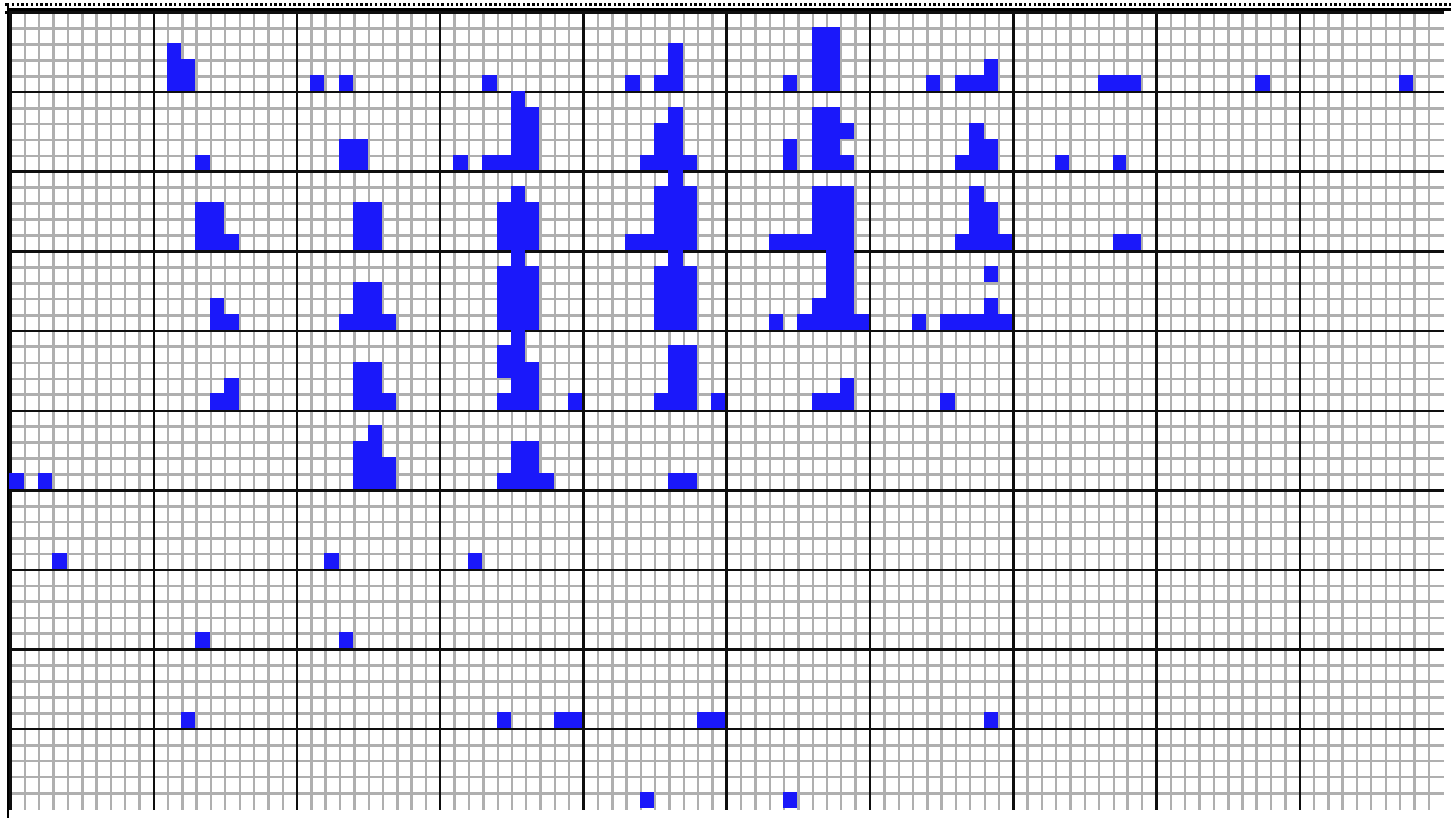
Dimensional Stacking



- Partitioning of the n-dimensional attribute space in 2-D subspaces, which are 'stacked' into each other
- Partitioning of the attribute value ranges into classes. The important attributes should be used on the outer levels.
- Adequate for data with ordinal attributes of low cardinality
- But, difficult to display more than nine dimensions
- Important to map dimensions appropriately

Dimensional Stacking

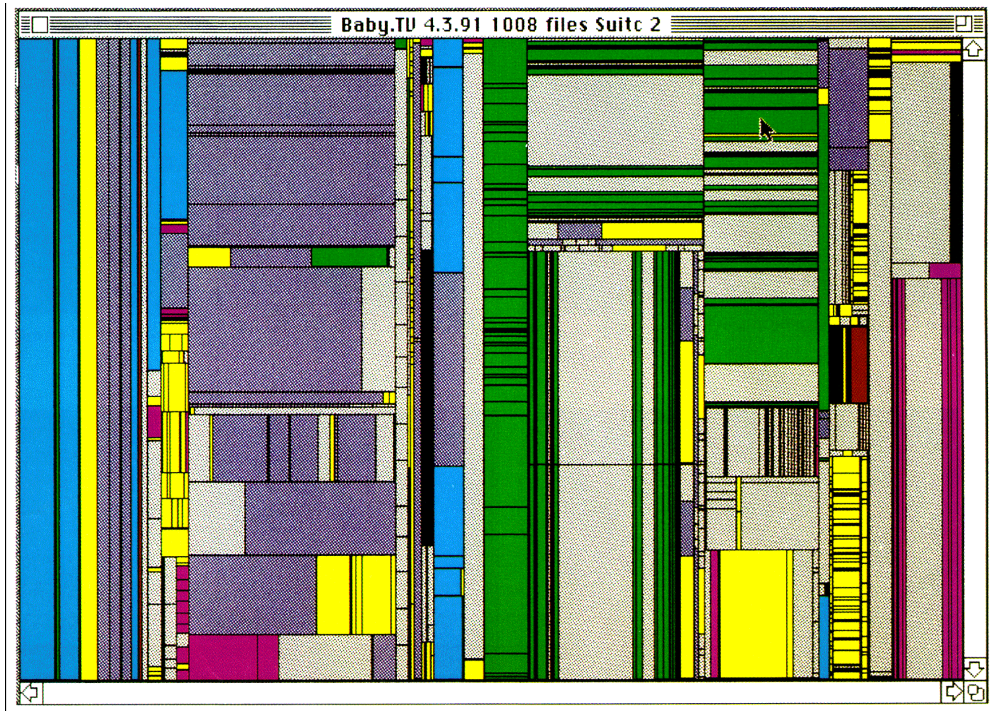
Used by permission of M. Ward, Worcester Polytechnic Institute



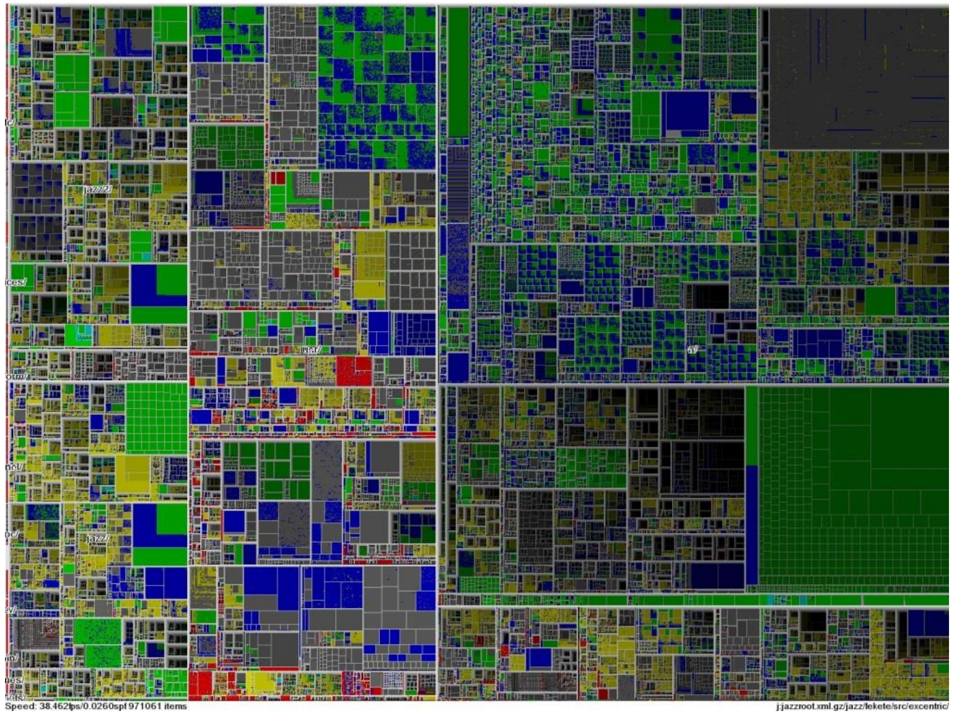
Visualization of oil mining data with longitude and latitude mapped to the outer x-, y-axes and ore grade and depth mapped to the inner x-, y-axes

Tree-Map

- Screen-filling method which uses a hierarchical partitioning of the screen into regions depending on the attribute values (e.g., size)
- Great for data that is naturally hierarchical (e.g., file systems)



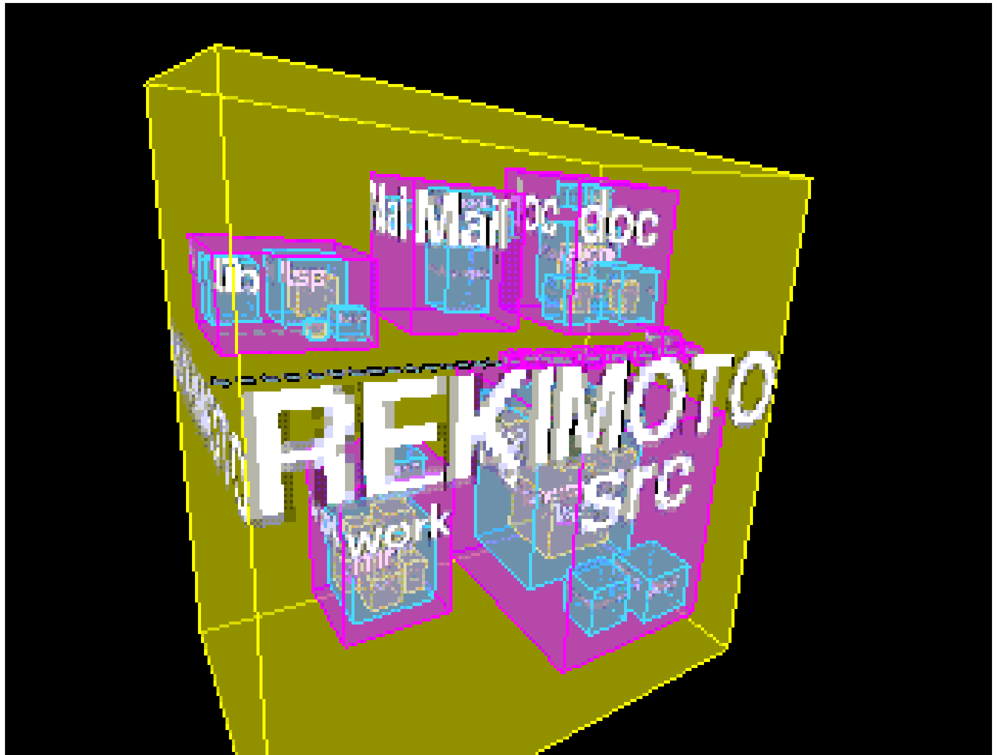
Schneiderman@UMD: Tree-Map of a File System



Schneiderman@UMD: Tree-Map to support large data sets of a million items

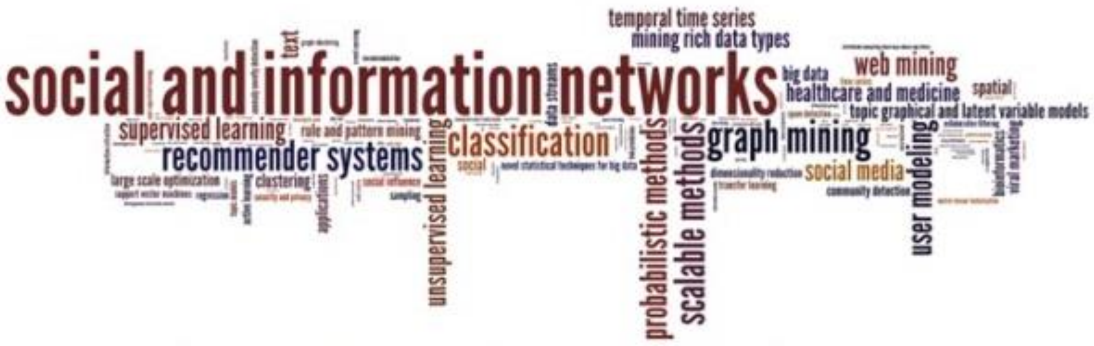
InfoCube

- A 3-D visualization technique where hierarchical information is displayed as nested semi-transparent cubes
  - Similar to Tree-Map, but in 3-D
- The outermost cubes correspond to the top level data, while the subnodes or the lower level data are represented as smaller cubes inside the outermost cubes, etc.

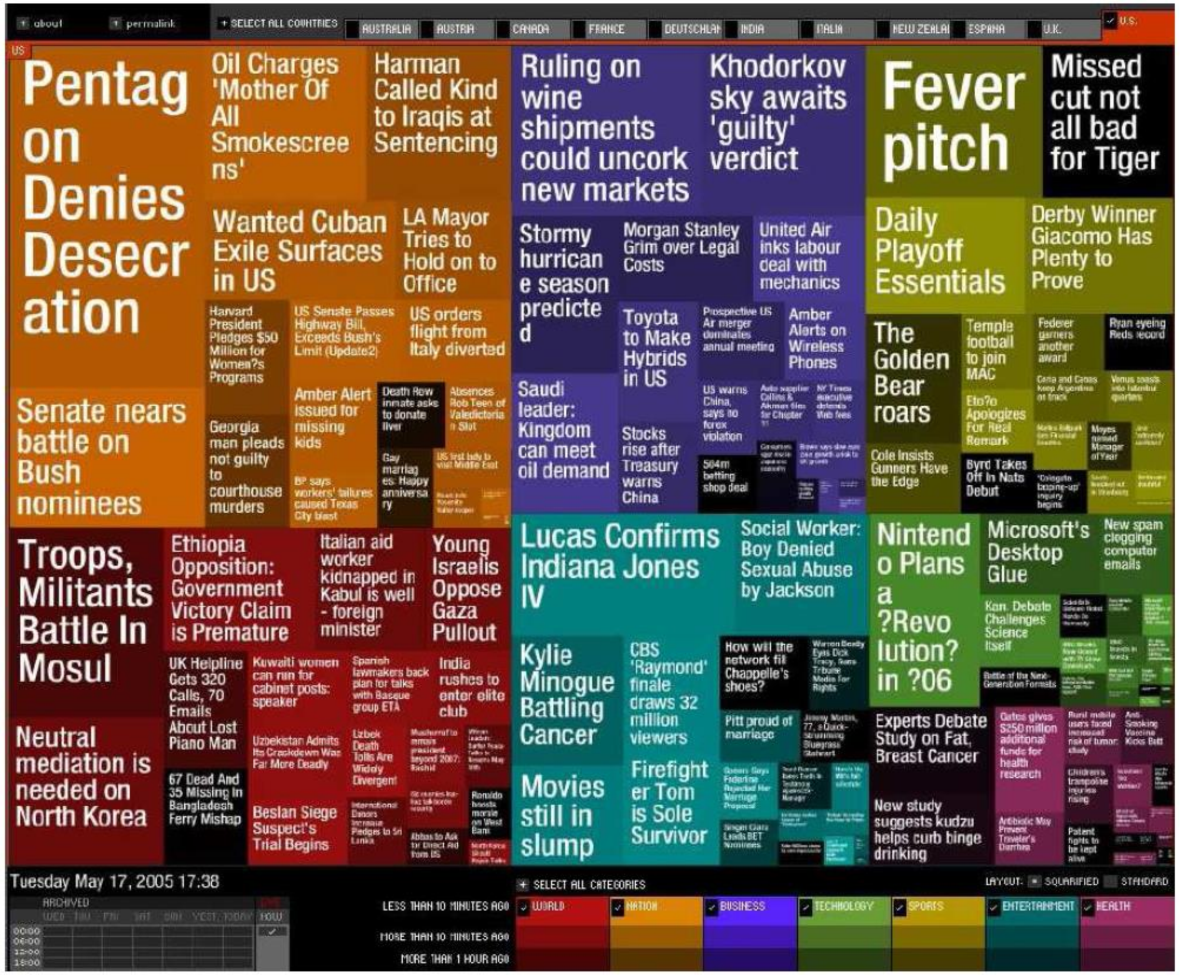


Visualizing Complex Data and Relations: Tag Cloud

- Tag cloud: Visualizing text data (e.g., user-generated tags)
  - The importance/frequency of tag is represented by font size/color
  - Popularly used to visualize word/phrase distributions



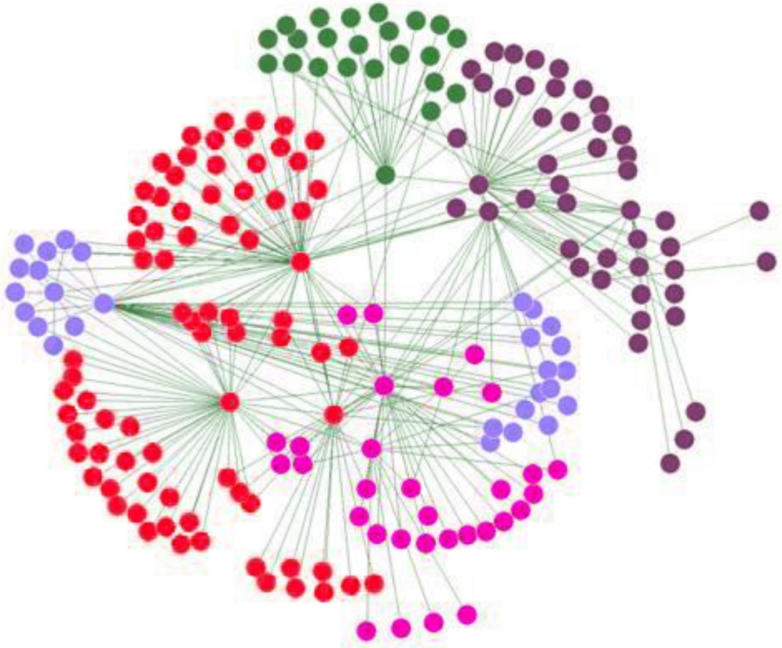
KDD 2013 Research Paper Title Tag Cloud



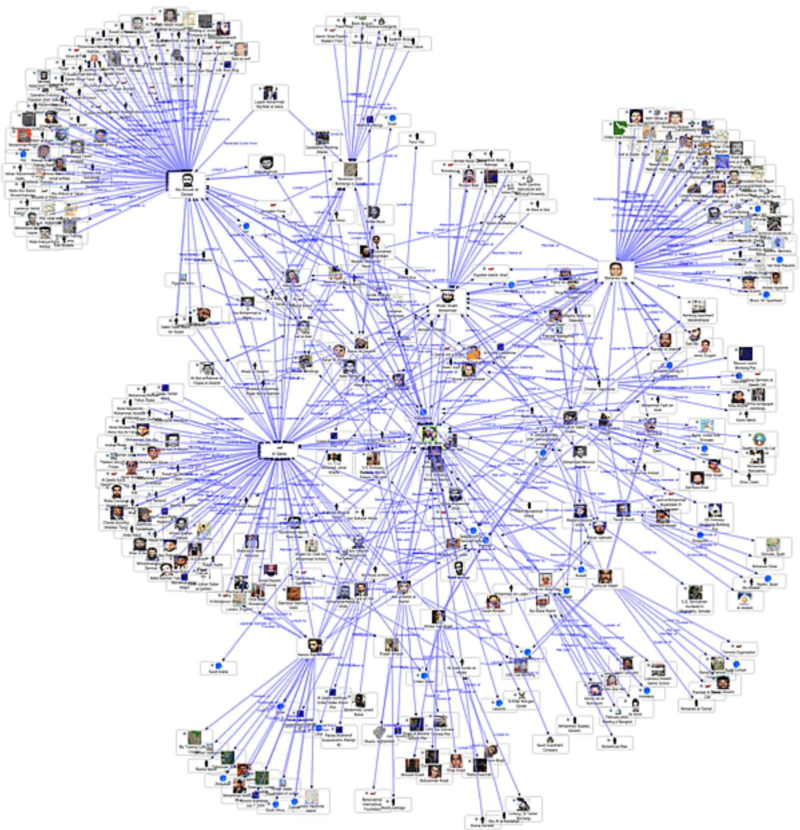
Newsmap: Google News Stories in 2005

Visualizing Complex Data and Relations: Social Networks

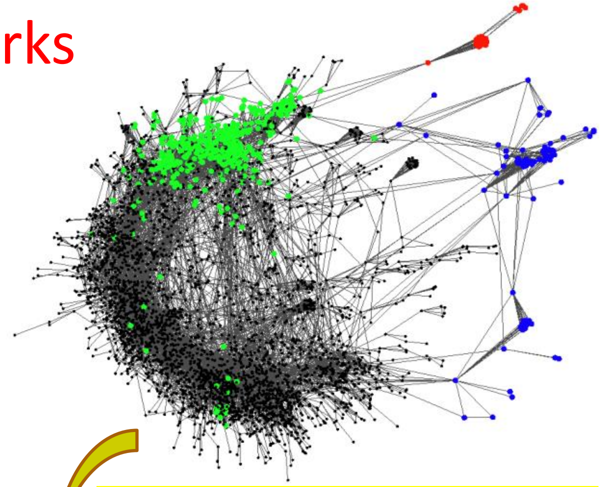
- Visualizing non-numerical data: social and information networks



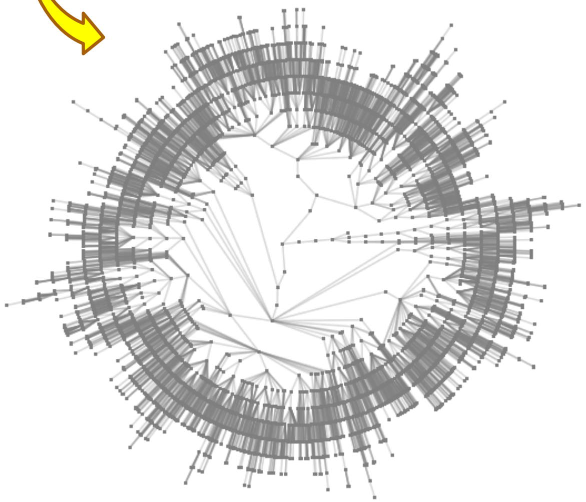
A typical network structure



A social network



organizing information networks



Chapter 2. Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Correlation
- Summary

Similarity, Dissimilarity, and Proximity

- **Similarity measure**
  - Similarity between two objects
  - The higher value, the more alike
  - Often falls in the range [0,1]: 0: no similarity; 1: completely similar
- **Dissimilarity** (or **distance**) measure
  - How different two data objects are
  - The lower, the more alike
  - Minimum dissimilarity is often 0 (i.e., completely similar)
- **Proximity** usually refers to either similarity or dissimilarity

Data Matrix and Dissimilarity Matrix

- Data matrix
  - Compare each row of data matrix.
- Dissimilarity (distance) matrix
  - *Distance of x*(*i, j*) is same as distance of *x(j,i)*
  - Distance functions (d) are usually different for real, boolean, categorical, ordinal, ratio, and vector variables
  - Weights can be associated with different variables based on applications and data semantics

$$D = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1l} \\ x_{21} & x_{22} & \cdots & x_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nl} \end{pmatrix}$$

$$\begin{pmatrix} 0 & & & \\ d(2,1) & 0 & & \\ \vdots & \vdots & \ddots & \\ d(n,1) & d(n,2) & \cdots & 0 \end{pmatrix}$$

Standardizing Numeric Data

- Z-score: $z = \frac{x - \mu}{\sigma}$
  - X: raw score to be standardized, μ: mean of the population, σ: standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, "+" when above
- An alternative way: Calculate the mean absolute deviation

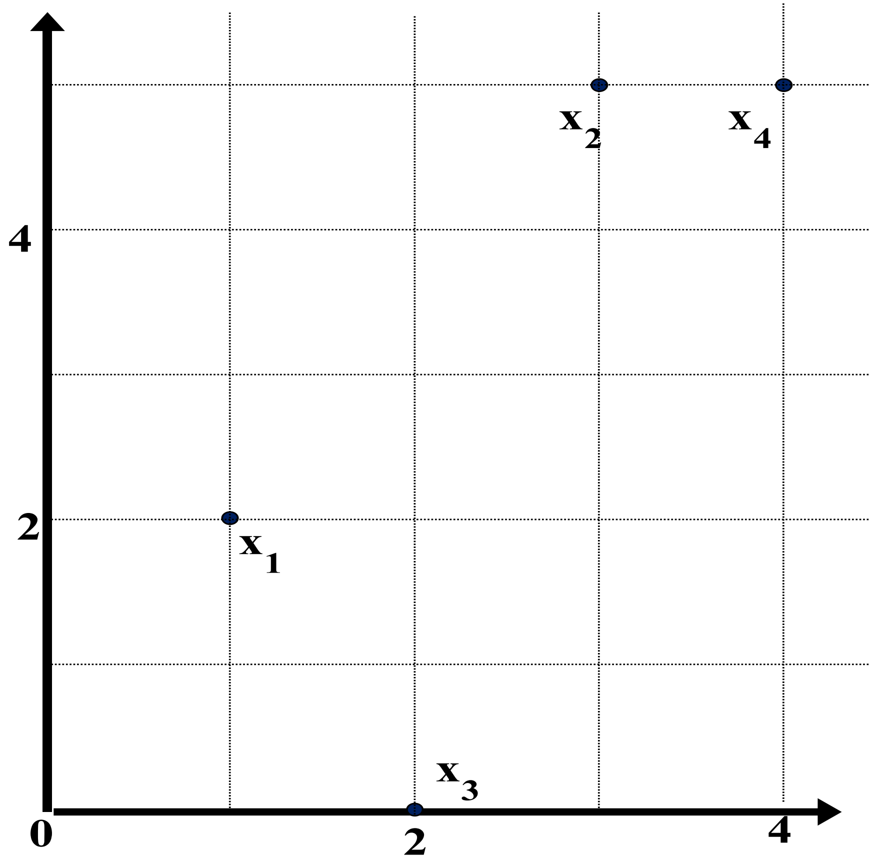
$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + ... + |x_{nf} - m_f|)$$

where

$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + ... + x_{nf}).$$

  - standardized measure (*z-score*): $z_{if} = \frac{x_{if} - m_f}{s_f}$
- Using mean absolute deviation is more robust than using standard deviation

Example: Data Matrix and Dissimilarity Matrix



Data Matrix

point	attribute1	attribute2
x1	1	2
x2	3	5
x3	2	0
x4	4	5

Dissimilarity Matrix (by Euclidean Distance)

	x1	*x2*	*x3*	*x4*
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Distance on Numeric Data: Minkowski Distance

- Minkowski distance: A popular distance measure

$$d(i, j) = \sqrt[p]{|x_{i1} - x_{j1}|^p + |x_{i2} - x_{j2}|^p + \cdots + |x_{il} - x_{jl}|^p}$$

  where $i = (x_{i1}, x_{i2}, \ldots, x_{il})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jl})$ are two l-dimensional data objects, and p is the order (the distance so defined is also called L-p norm)

- Properties
  - d(i, j) > 0 if i ≠ j, and d(i, i) = 0 (Positivity)
  - d(i, j) = d(j, i) (Symmetry)
  - d(i, j) ≤ d(i, k) + d(k, j) (Triangle Inequality)
- A distance that satisfies these properties is a metric
- Note: There are nonmetric dissimilarities, e.g., set differences

Special Cases of Minkowski Distance

- p = 1: (L_1 norm) Manhattan (or city block) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{il} - x_{jl}|$$

- p = 2: (L_2 norm) Euclidean distance

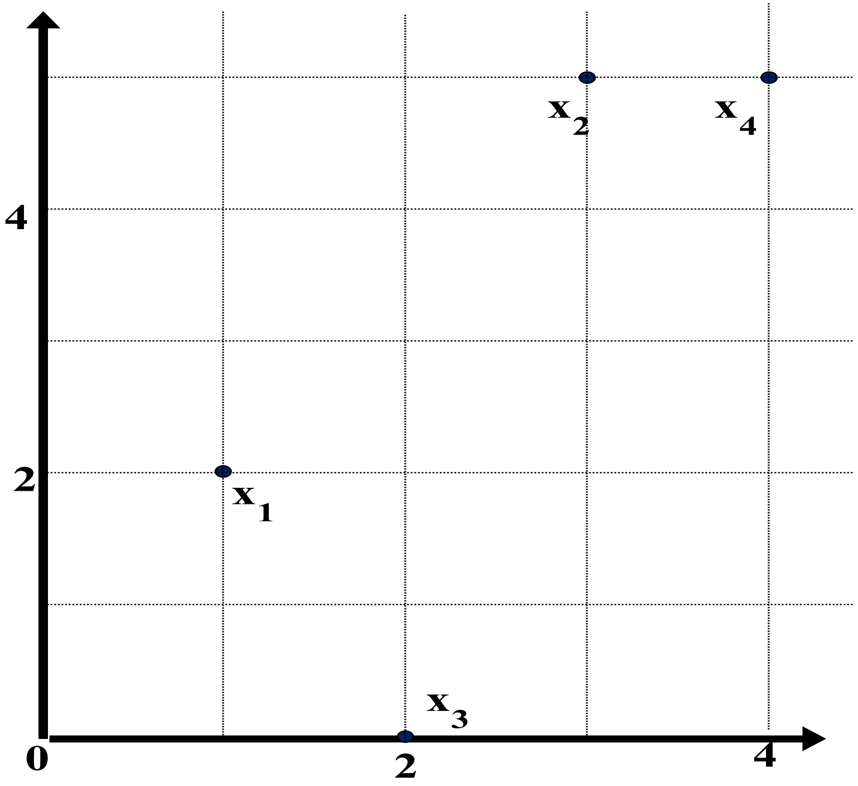
$$d(i,j) = \sqrt{|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \cdots + |x_{il} - x_{jl}|^2}$$

- $p \to \infty$: (L_{max} norm, L_∞ norm) "supremum" distance
  - The maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{p \to \infty} \sqrt[p]{|x_{i1} - x_{j1}|^p + |x_{i2} - x_{j2}|^p + \cdots + |x_{il} - x_{jl}|^p} = \max_{f=1}^{l} |x_{if} - x_{jf}|$$

Example: Minkowski Distance at Special Cases

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



Manhattan (L_1)

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Euclidean (L_2)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Supremum (L_∞)

L_∞	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

Proximity Measure for Binary Attributes

- A contingency table for binary data

		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- Distance measure for symmetric binary variables $$d(i, j) = \frac{r+s}{q+r+s+t}$$

- Distance measure for asymmetric binary variables: $$d(i, j) = \frac{r+s}{q+r+s}$$

- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables): $$sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$$

- Note: Jaccard coefficient is the same as (a concept discussed in Pattern Discovery)

$$coherence(i, j) = \frac{sup(i, j)}{sup(i) + sup(j) - sup(i, j)} = \frac{q}{(q+r) + (q+s) - q}$$

Example: Dissimilarity between Asymmetric Binary Variables

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- Gender is a symmetric attribute (not counted in)
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N be 0
- Distance: $d(i, j) = \frac{r + s}{q + r + s}$

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

Jack (rows) vs. Mary (columns):

	1	0	Σ_{row}
1	2	0	2
0	1	3	4
Σ_{col}	3	3	6

Jack (rows) vs. Jim (columns):

	1	0	Σ_{row}
1	1	1	2
0	1	3	4
Σ_{col}	2	4	6

Jim (rows) vs. Mary (columns):

	1	0	Σ_{row}
1	1	1	2
0	2	2	4
Σ_{col}	3	3	6

Proximity Measure for Categorical Attributes

- Categorical data, also called nominal attributes
  - Example: Color (red, yellow, blue, green), profession, etc.
- Method 1: Simple matching
  - *m*: # of matches, *p*: total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- Method 2: Use a large number of binary attributes
  - Creating a new binary attribute for each of the *M* nominal states

Ordinal Variables

- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank (e.g., freshman, sophomore, junior, senior)
- Can be treated like interval-scaled
  - Replace *an ordinal variable value* by its rank: $r_{if} \in \{1, ..., M_f\}$
  - Map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by $z_{if} = \frac{r_{if} - 1}{M_f - 1}$
    - Example: freshman: 0; sophomore: 1/3; junior: 2/3; senior 1
      - Then distance: d(freshman, senior) = 1, d(junior, senior) = 1/3
  - Compute the dissimilarity using methods for interval-scaled variables

Attributes of Mixed Type

- A dataset may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, and ordinal
- One may use a weighted formula to combine their effects:

$$d(i, j) = \frac{\sum_{f=1}^{p} w_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} w_{ij}^{(f)}}$$

  - If f is numeric: Use the normalized distance
  - If f is binary or nominal: $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$; or $d_{ij}^{(f)} = 1$ otherwise
  - If f is ordinal
    - Compute ranks z_{if} (where $z_{if} = \frac{r_{if} - 1}{M_f - 1}$)
    - Treat z_{if} as interval-scaled

Cosine Similarity of Two Vectors

- A **document** can be represented by a bag of terms or a long vector, with each attribute recording the *frequency* of a particular term (such as word, keyword, or phrase) in the document

Document	*team*	*coach*	*hockey*	*baseball*	*soccer*	*penalty*	*score*	*win*	*loss*	*season*
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

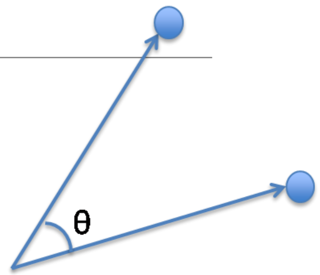
- Other vector objects: Gene features in micro-arrays
- Applications: Information retrieval, biologic taxonomy, gene feature mapping, etc.
- Cosine measure: If d_1 and d_2 are two vectors (e.g., term-frequency vectors), then

$$cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\| d_1 \| \times \| d_2 \|}$$

where • indicates vector dot product, $||d||$: the length of vector d

Example: Calculating Cosine Similarity

- Calculating Cosine Similarity: $cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\| d_1 \| \times \| d_2 \|}$ $sim(A,B) = \cos(\theta) = \frac{A \cdot B}{\|A\|\|B\|}$



  where • indicates vector dot product, $||d||$: the length of vector d

- Ex: Find the **similarity** between documents 1 and 2.

  d_1 = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0) d_2 = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)

  - First, calculate vector dot product

    $d_1 \bullet d_2$ = 5 X 3 + 0 X 0 + 3 X 2 + 0 X 0 + 2 X 1 + 0 X 1 + 0 X 1 + 2 X 1 + 0 X 0 + 0 X 1 = 25

  - Then, calculate $||d_1||$ and $||d_2||$

$$\| d_1 \| = \sqrt{5\times5+0\times0+3\times3+0\times0+2\times2+0\times0+0\times0+2\times2+0\times0+0\times0} = 6.481$$

$$\| d_2 \| = \sqrt{3\times3+0\times0+2\times2+0\times0+1\times1+1\times1+0\times0+1\times1+0\times0+1\times1} = 4.12$$

  - Calculate cosine similarity: cos(d_1, d_2) = 25/ (6.481 X 4.12) = 0.94

Correlation Analysis (for Categorical Data)

- X^2 **(chi-square) test:**

observed

$$\chi^2 = \sum_{i}^{n} \frac{(O_i - E_i)^2}{E_i}$$

expected

- Null hypothesis: The two distributions are independent
- The cells that contribute the most to the X^2 value are those whose actual count is very different from the expected count
  - The larger the X^2 value, the more likely the variables are related
- Note: Correlation does not imply causality
  - # of hospitals and # of car-theft in a city are correlated
  - Both are causally linked to the third variable: population

Chi-Square Calculation: An Example

	Play chess	Not play chess	Sum (row)
Like science fiction	250 (X1)	200 (X2)	450
Not like science fiction	50 (X3)	1000 (X4)	1050
Sum(col.)	300	1200	1500

- Null hypothesis: The two distributions are independent
  - What does that mean?
  - The ratio between people who play chess vs not play chess is the same for both groups of like science fiction and not like science fiction
  - X1:X2=X3:X4=300:1200
  - X1:X3=X2:X4=450:1050
  - X1+X2=450 X3+X4=1050
  - X1+X3=300 X2+X4=1200

Chi-Square Calculation: An Example

	Play chess	Not play chess	Sum (row)
Like science fiction	250 (90)	200 (360)	450
Not like science fiction	50 (210)	1000 (840)	1050
Sum(col.)	300	1200	1500

How to derive 90?
450/1500 * 300 = 90

- X^2 (chi-square) calculation (numbers in parenthesis are expected counts calculated based on the data distribution in the two categories)

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93$$

We can reject the null hypothesis of independence at a confidence level of 0.001

- It shows that like_science_fiction and play_chess are correlated in the group

Chi-Square Calculation: An Example

	A	B	C	D	Sum (row)
1					200
0					1000
Sum(col.)	300	300	300	300	1200

- Degree of freedom
  - (#categories_in_variable_A -1)((#categories_in_variable_B -1)
  - number of values that are free to vary

Chi-Square Calculation: An Example

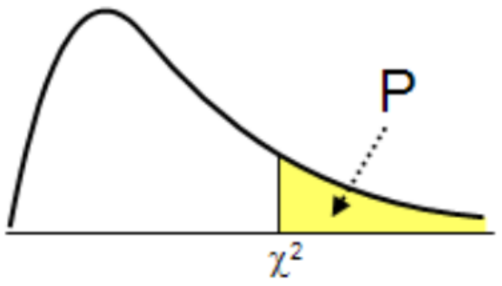
	Play chess	Not play chess	Sum (row)
Like science fiction	250 (90)	200 (360)	450
Not like science fiction	50 (210)	1000 (840)	1050
Sum(col.)	300	1200	1500

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93$$

We can reject the null hypothesis of independence at a confidence level of 0.001

- Degree of freedom =?

Values of the Chi-squared distribution



	P										
DF	**0.995**	**0.975**	**0.20**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.002**	**0.001**
1	0.0000393	0.000982	1.642	2.706	3.841	5.024	5.412	6.635	7.879	9.550	10.828
2	0.0100	0.0506	3.219	4.605	5.991	7.378	7.824	9.210	10.597	12.429	13.816
3	0.0717	0.216	4.642	6.251	7.815	9.348	9.837	11.345	12.838	14.796	16.266
4	0.207	0.484	5.989	7.779	9.488	11.143	11.668	13.277	14.860	16.924	18.467
5	0.412	0.831	7.289	9.236	11.070	12.833	13.388	15.086	16.750	18.907	20.515
6	0.676	1.237	8.558	10.645	12.592	14.449	15.033	16.812	18.548	20.791	22.458

Variance for Single Variable (Numerical Data)

- The variance of a random variable *X* provides a measure of how much the value of *X* deviates from the mean or expected value of *X:*

$$\sigma^2 = \text{var}(X) = E[(X-\mu)^2] = \begin{cases} \sum_x (x-\mu)^2 f(x) & \text{if } X \text{ is discrete} \\ \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx & \text{if } X \text{ is continuous} \end{cases}$$

  - where σ^2 is the variance of X, σ is called *standard deviation*

    μ is the mean, and μ = E[X] is the expected value of X
  - That is, variance is the expected value of the square deviation from the mean
  - It can also be written as: $\sigma^2 = \text{var}(X) = E[(X-\mu)^2] = E[X^2] - \mu^2 = E[X^2] - [E(x)]^2$
- Sample variance

$$s^2 = \frac{1}{n}\sum_i^n (x_i - \hat{\mu})^2 \qquad s^2 = \frac{1}{n-1}\sum_i^n (x_i - \hat{\mu})^2$$

Covariance for Two Variables

- Covariance between two variables X_1 and X_2

$$\sigma_{12} = E[(X_1 - \mu_1)(X_2 - \mu_2)] = E[X_1 X_2] - \mu_1\mu_2 = E[X_1 X_2] - E[X_1]E[X_2]$$

where $\mu_1 = E[X_1]$ is the respective mean or **expected value** of X_1; similarly for μ_2

- Sample covariance between X_1 and X_2: $\hat{\sigma}_{12} = \frac{1}{n}\sum_{i=1}^{n}(x_{i1} - \widehat{\mu_1})(x_{i2} - \widehat{\mu_2})$
- Sample covariance is a generalization of the sample variance:

$$\hat{\sigma}_{11} = \frac{1}{n}\sum_{i=1}^{n}(x_{i1} - \widehat{\mu_1})(x_{i1} - \widehat{\mu_1})$$

- **Positive covariance:** If $\sigma_{12} > 0$
- **Negative covariance**: If $\sigma_{12} < 0$

Covariance for Two Variables

- **Independence**: If X_1 and X_2 are independent, $\sigma_{12} = 0$ but the reverse is not true
  - Some pairs of random variables may have a covariance 0 but are not independent
  - Only under some additional assumptions (e.g., the data follow multivariate normal distributions) does a covariance of 0 imply independence
- Example:

X_1	1	-1	
X_2	0	1	-1

$$\sigma_{12} = E[(X_1 - \mu_1)(X_2 - \mu_2)] = E[X_1 X_2] - \mu_1\mu_2 = E[X_1 X_2] - E[X_1]E[X_2]$$

$\mathrm{E}(X_1)$=?

$\mathrm{E}(X_2)$=?

$\mathrm{E}(X_1 X_2)$=?

Example: Calculation of Covariance

- Suppose two stocks X_1 and X_2 have the following values in one week:
  - (2, 5), (3, 8), (5, 10), (4, 11), (6, 14)
- Question: If the stocks are affected by the same industry trends, will their prices rise or fall together?
- Covariance formula

$$\sigma_{12} = E[(X_1 - \mu_1)(X_2 - \mu_2)] = E[X_1 X_2] - \mu_1 \mu_2 = E[X_1 X_2] - E[X_1]E[X_2]$$

- Its computation can be simplified as: $\sigma_{12} = E[X_1 X_2] - E[X_1]E[X_2]$
  - $E(X_1) = (2 + 3 + 5 + 4 + 6)/ 5 = 20/5 = 4$
  - $E(X_2) = (5 + 8 + 10 + 11 + 14) /5 = 48/5 = 9.6$
  - $\sigma_{12} = (2\times5 + 3\times8 + 5\times10 + 4\times11 + 6\times14)/5 - 4 \times 9.6 = 4$
- Thus, X_1 and X_2 rise together since $\sigma_{12} > 0$

Correlation between Two Numerical Variables

- **Correlation** between two variables X_1 and X_2 is the standard covariance, obtained by normalizing the covariance with the standard deviation of each variable

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{\sigma_{12}}{\sqrt{\sigma_1^{\,2} \sigma_2^{\,2}}}$$

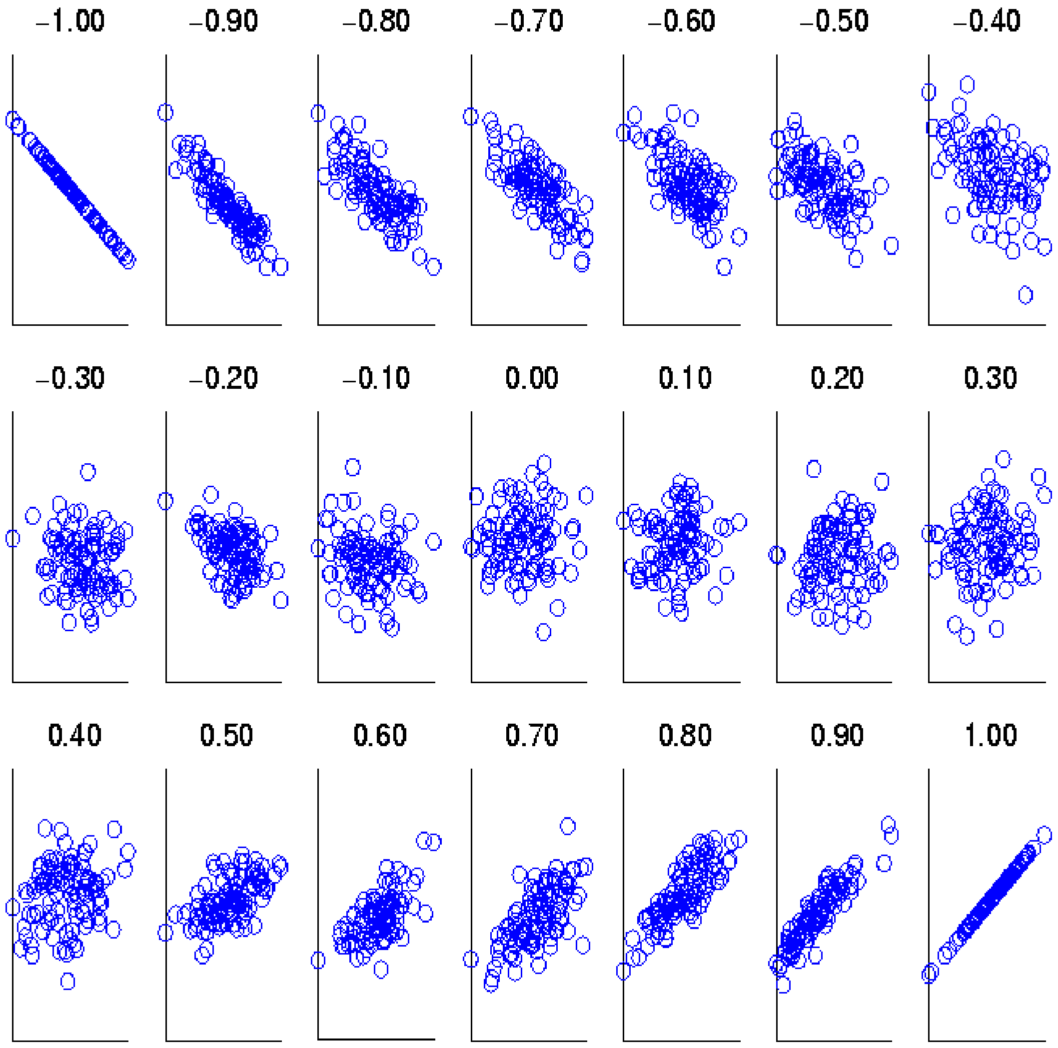
- **Sample correlation** for two attributes X_1 and X_2:

$$\hat{\rho}_{12} = \frac{\hat{\sigma}_{12}}{\hat{\sigma}_1 \hat{\sigma}_2} = \frac{\sum_{i=1}^{n} (x_{i1} - \hat{\mu}_1)(x_{i2} - \hat{\mu}_2)}{\sqrt{\sum_{i=1}^{n} (x_{i1} - \hat{\mu}_1)^2 \sum_{i=1}^{n} (x_{i2} - \hat{\mu}_2)^2}}$$

  where n is the number of tuples, μ_1 and μ_2 are the respective means of X_1 and X_2 , σ_1 and σ_2 are the respective standard deviation of X_1 and X_2

- If $\rho_{12} > 0$: A and B are positively correlated (X_1's values increase as X_2's)
  - The higher, the stronger correlation
- If $\rho_{12} = 0$: independent (under the same assumption as discussed in co-variance)
- If $\rho_{12} < 0$: negatively correlated

Visualizing Changes of Correlation Coefficient



- Correlation coefficient value range: [–1, 1]
- A set of scatter plots shows sets of points and their correlation coefficients changing from –1 to 1

Covariance Matrix

- The variance and covariance information for the two variables X_1 and X_2 can be summarized as 2 X 2 covariance matrix as

$$\Sigma = E[(\mathbf{X}-\mu)(\mathbf{X}-\mu)^T] = E[\begin{pmatrix} X_1-\mu_1 \\ X_2-\mu_2 \end{pmatrix}(X_1-\mu_1 \quad X_2-\mu_2)]$$

$$= \begin{pmatrix} E[(X_1-\mu_1)(X_1-\mu_1)] & E[(X_1-\mu_1)(X_2-\mu_2)] \\ E[(X_2-\mu_2)(X_1-\mu_1)] & E[(X_2-\mu_2)(X_2-\mu_2)] \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}$$

- Generalizing it to *d* dimensions, we have,

$$D = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ x_{21} & x_{22} & \cdots & x_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ x_{d1} & x_{d2} & \cdots & x_{dd} \end{pmatrix} \quad \mathbf{\Sigma} = E[(\mathbf{X}-\mu)(\mathbf{X}-\mu)^T] = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1d} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{d1} & \sigma_{d2} & \cdots & \sigma_d^2 \end{pmatrix}$$

KL Divergence: Comparing Two Probability Distributions

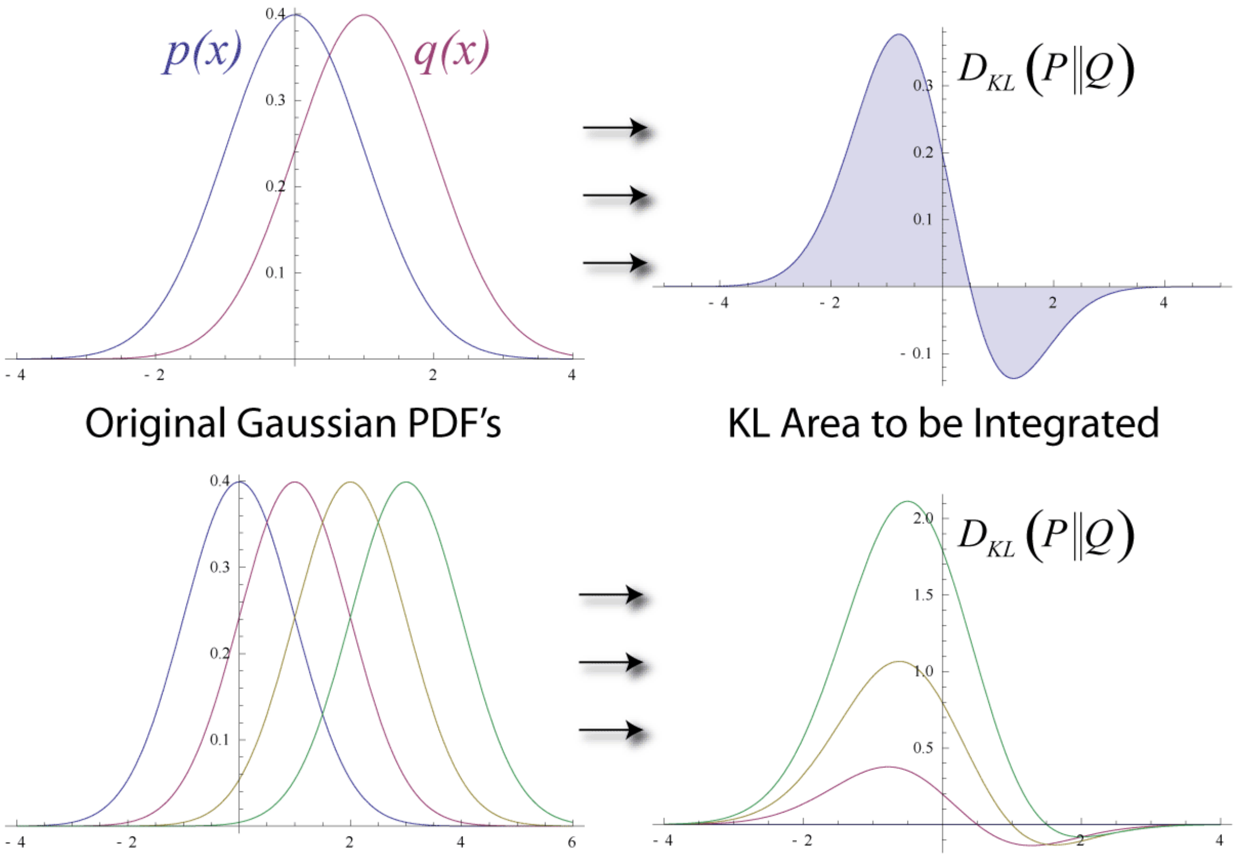
- *The Kullback-Leibler (KL) divergence:* Measure the difference between two probability distributions over the same variable *x*
  - From information theory, closely related to *relative entropy*, *information divergence*, and *information for discrimination*
- $D_{KL}(p(x) \,||\, q(x))$: divergence of $q(x)$ from $p(x)$, measuring the information lost when $q(x)$ is used to approximate $p(x)$

$$D_{KL}(p(x)||q(x)) = \sum_{x \in X} p(x) \ln \frac{p(x)}{q(x)}$$

Discrete form

$$D_{KL}(p(x)||q(x)) = \int_{-\infty}^{\infty} p(x) \ln \frac{p(x)}{q(x)} dx$$

Continuous form



Ack.: Wikipedia entry: *The Kullback-Leibler (KL) divergence*

Normalization

Measure	Input	Range	Pros/Cons
Z-Score	$z = \frac{x - \mu}{\sigma}$	$[-\infty, +\infty]$ But scores outside $[-3, 3]$ are likely to be outliers	• Pros: • Easy to calculate • Good for outlier detection • Cons: • Small data sets skew the results
Mean Absolute Deviance	$\frac{\sum_{i=1}^{n} \lvert x_i - \overline{x} \rvert}{n}$	$[0, +\infty]$	
Min/Max Normalization	$v'_I = \frac{v_I - \min_A}{\max_A - \min_A} \times (nw\_max_A - nw\_min_A) + nw\_min_A$	$nw\_min_A \rightarrow nw\_max_A$	• Pros: • Allows for custom range of data

Distance Measures

Measure	Input	Range	Pros/Cons
Minkowski	$\left(\sum_{l=1}^{n} \lvert x_{il} - x_{jl} \rvert^p\right)^{1/p}$	$0 \rightarrow \infty$	• Pros: • Most commonly used distance for numerical data • Positivity/Symmetry/Triangle Inequality
Manhattan	$Minkowski, p = 1$ $\sum_{l=1}^{n} \lvert x_{il} - y_{jl} \rvert$	$0 \rightarrow \infty$	• Pros: • Not sensitive to outliers. • Cons: • Non differentiable
Euclidean	$Minkowski, p = 2$ $\left(\sum_{l=1}^{n} \lvert x_{il} - x_{jl} \rvert^2\right)^{1/2}$	$0 \rightarrow \infty$	• Pros: • differentiable • Cons • Sensitive to outliers
Supremum	$Minkowski, p \rightarrow \infty$ $\max_{f=1}^{l} \lvert x_{if} - x_{jf} \rvert$	$0 \rightarrow \infty$	

Similarity/Dissimilarity Measures (Binary)

Measure	Input	Range	Pros/Cons
Symmetric Binary Variable	$\frac{r+s}{q+r+s+t}$	[0, 1]	• Null variant • if 0 and 1 are equally important
Asymmetric Binary Variable	$\frac{r+s}{q+r+s}$	[0, 1]	• Null invariant • If 0 is not important (such as meaning did not appear, too common in data, …)
Jaccard Coefficient / Coherence	$\frac{q}{(q+r)+(q+s)-q}$	[0, 1]	• This is a similarity measure • The higher the value, the more similar the two vector

	1	0	sum
1	q	r	$q+r$
0	s	t	$s+t$
sum	$q+s$	$r+t$	p

Measures – More

Measure	Input	Range	Pros/Cons
Cosine Similarity	$cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\| d_1 \| \times \| d_2 \|}$	$[-1, 1]$ In many applications, d_i are all positive, then [0, 1]	Commonly used in text mining 1-> similar 0-> irrelevant -1-> opposite
Chi-Squared Test	$\chi^2 = \sum_i^n \frac{(O_i - E_i)^2}{E_i}$	$[0, +\infty]$	Correlation measure for categorical data Higher value->strong correlation
Variance / Covariance	$\sigma_{12} = E\left[(X_1 - \mu_1)(X_2 - \mu_2)\right]$	$[-\infty, +\infty]$	Correlation measure for continuous data High positive value->strong positive correlation Very negative value->strong negative correlation
Correlation coefficient	$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{\sigma_{12}}{\sqrt{\sigma_1^2 \sigma_2^2}}$	$[-1, 1]$	Correlation measure for continuous data High positive value->strong positive correlation Very negative value->strong negative correlation

Chapter 2. Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Correlation
- Summary

Summary

- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity and correlation
- Above steps are the beginning of data preprocessing
- Many methods have been developed but still an active area of research

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