

CS 412 Intro. to Data Mining Chapter 5. Data Cube Technology

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Chapter 5: Data Cube Technology

location

time

product

data cell

Data Cube Computation: Basic Concepts

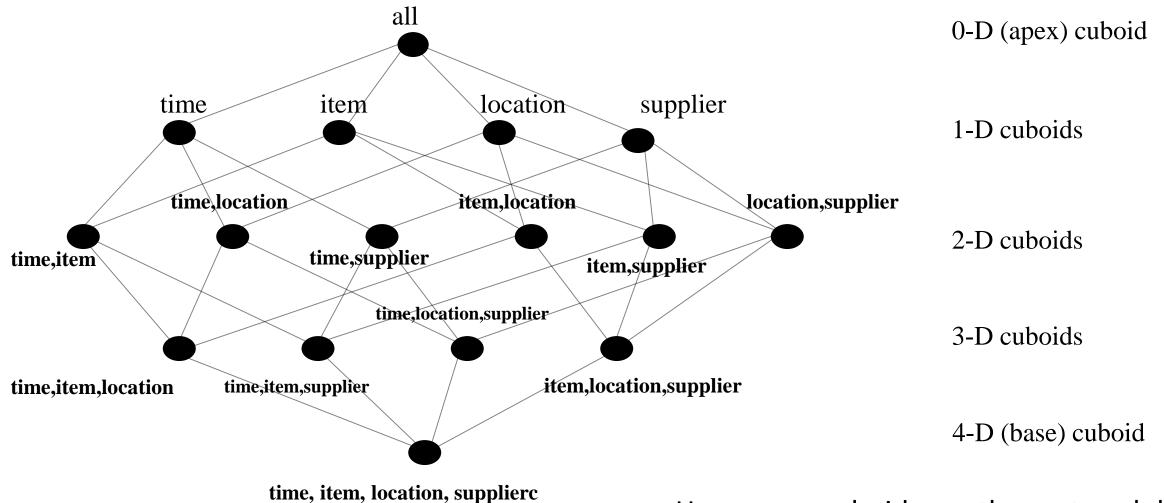
Data Cube Computation Methods

Processing Advanced Queries with Data Cube Technology

Multidimensional Data Analysis in Cube Space

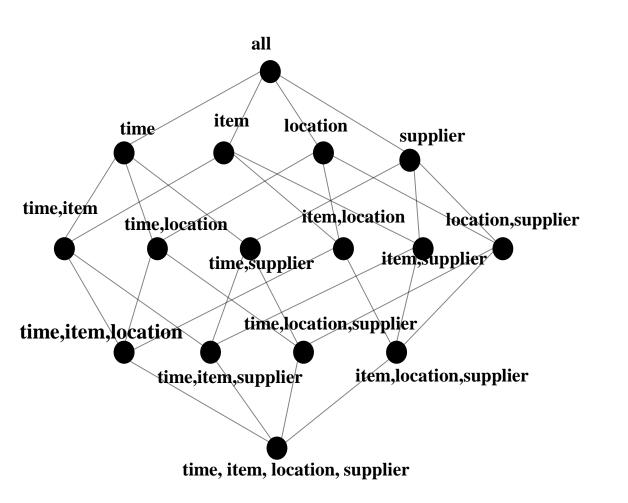
Summary

Data Cube: A Lattice of Cuboids



How many cuboids are there at each level?

Data Cube: A Lattice of Cuboids



Base vs. aggregate cells Ancestor vs. descendant cells Parent vs. child cells □ (*,*,*,*) ~ 0-D (agg) □ (*, milk, *, *) 1-D (agg) `□ (*, milk, Urbana, *) < 2-D (agg) 🗀 (*, milk, Chicago, *) 2-D (agg) □ (9/15, milk, Urbana, *) -3-D (agg) (9/15, milk, Urbana, Dairy land) 4-D (base)

Cube Materialization: Full Cube vs. Iceberg Cube

□ Full cube vs. iceberg cube

compute cube sales_iceberg as SELECT month, city, customer_group, COUNT(*) FROM salesInfo CUBE BY month, city, customer_group HAVING count(*) >= min support





- Compute only the cells whose measure satisfies the iceberg condition
 - **Ex.:** Show only those cells whose count is at least 100
- Only a small portion of cells may be "above the water" in a sparse cube

Why Iceberg Cube?

- No need to save nor show those cells whose value is below the threshold (iceberg condition)
- Efficient methods may even avoid computing the un-needed, intermediate cells
- Avoid explosive growth

Example

Example: A cube with 100 dimensions

- □ Suppose it contains only 2 base cells: {(a₁, a₂, a₃, ..., a₁₀₀), (a₁, a₂, b₃, ..., b₁₀₀)}
- □ How many aggregate cells if "having count >= 1"?

□ Answer: (2¹⁰¹ – 2) – 4 (Why?!)

Example

Example: A cube with 100 dimensions

- □ Suppose it contains only 2 base cells: {(a₁, a₂, a₃, ..., a₁₀₀), (a₁, a₂, b₃, ..., b₁₀₀)}
- What about the iceberg cells, (i,e., with condition: "having count >= 2")?
 Answer: 4 (Why?!)

Is Iceberg Cube Good Enough? Closed Cube & Cube Shell

- Let cube P have only 2 base cells: $\{(a_1, a_2, a_3 \dots, a_{100}): 10, (a_1, a_2, b_3, \dots, b_{100}): 10\}$
 - □ How many cells will the iceberg cube contain if "having count(*) \ge 10"?
 - □ Answer: $2^{101} 4$ (still too big!)
- **Closed cube:**
 - A cell c is *closed* if there exists no cell d, such that d is a descendant of c, and d has the same measure value as c
 - Ex. The same cube P has only 3 closed cells:
 - $\Box \{(a_1, a_2, *, ..., *): 20, (a_1, a_2, a_3 ..., a_{100}): 10, (a_1, a_2, b_3, ..., b_{100}): 10\}$
 - □ A *closed cube* is a cube consisting of only closed cells
- **Cube Shell:** The cuboids involving only a small # of dimensions, e.g., 2
 - Idea: Only compute cube shells, other dimension combinations can be computed on the fly

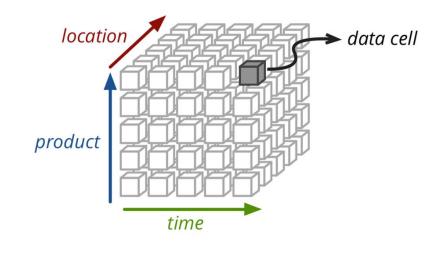
Chapter 5: Data Cube Technology

Data Cube Computation: Basic Concepts

Data Cube Computation Methods

Multidimensional Data Analysis in Cube Space

Summary



Roadmap for Efficient Computation

- General computation heuristics ^[1]
- Computing full/iceberg cubes: 3 methodologies
 - Bottom-Up:
 - □ Multi-Way array aggregation ^[2]
 - Top-down:
 - □ BUC ^[3]
- High-dimensional OLAP:
 - □ A Shell-Fragment Approach ^[4]
- Computing alternative kinds of cubes:
 - Partial cube, closed cube, approximate cube,

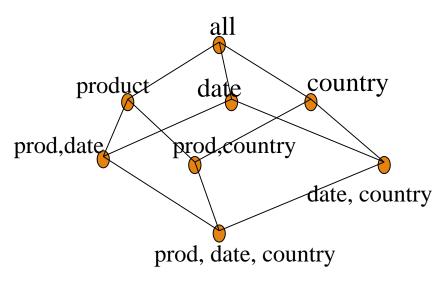
- 1. (Agarwal et al.'96)
- 2. (Zhao, Deshpande & Naughton, SIGMOD'97)
- 3. (Beyer & Ramarkrishnan, SIGMOD'99)
- 4. (Li, et al. VLDB'04)

Efficient Data Cube Computation: General Heuristics

- Sorting, hashing, and grouping operations are applied
 - **Share-sorts**
 - Share-partitions

Reuse

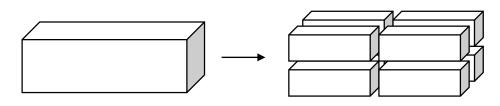
- Smallest-child: computing a cuboid from the smallest, previously computed cuboid
- Cache-results: caching results of a cuboid from which other cuboids are computed to reduce disk I/Os
- Amortize-scans: computing as many as possible cuboids at the same time to amortize disk reads



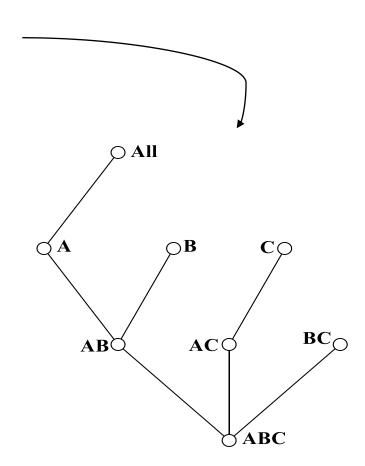
S. Agarwal, R. Agrawal, P. M. Deshpande, A. Gupta, J. F. Naughton, R. Ramakrishnan, S. Sarawagi. On the computation of multidimensional aggregates. VLDB'96

Multi-Way Array Aggregation (MOLAP)

- How can I efficiently calculate all group-by cell aggregations? Full cube computation
- Fundamental Concept: AB, AC, and BC can be computed from ABC. A, B, and C can be computed from AB/AC/BC.
- Common Practice with limited memory: Do not load the entire dimension (in array form) into memory at once. Use Chunks:



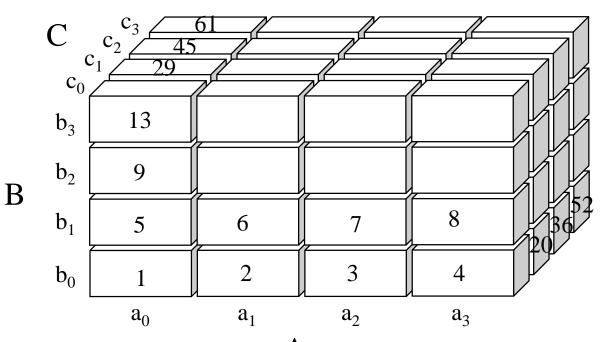
http://pages.cs.wisc.edu/~nil/764/DADS/38_zhao97array based.pdf - Zhao et al. '97



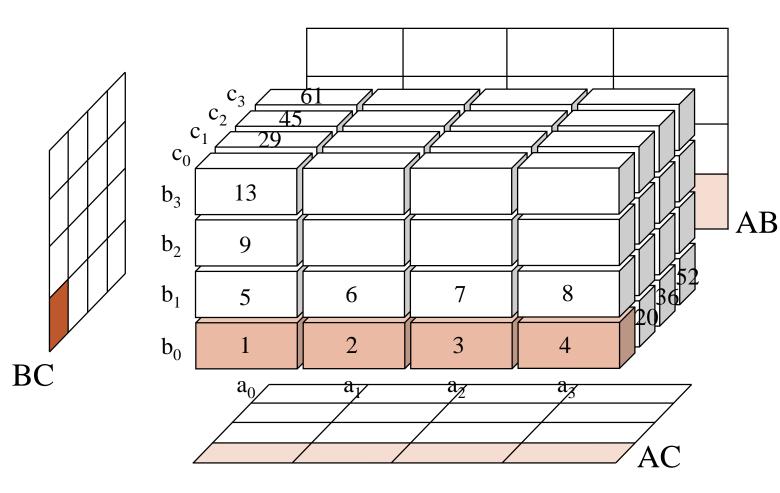
Multi-Way Array Aggregation (MOLAP)

- Chunk and store as(chunk_id, offset)
 - Tells which cells in the chunk have data
- Goal: Read chunk only once in memory
 - BC /AB only once
- **Example: Student Record Data Warehouse**
- count(A) > count (B) > count(C)
- What is best order to put the chunks in order to calculate the aggregation?

Example:
A: 4000, B: 400, C: 40
Chunk:
1000 x 100 x 10

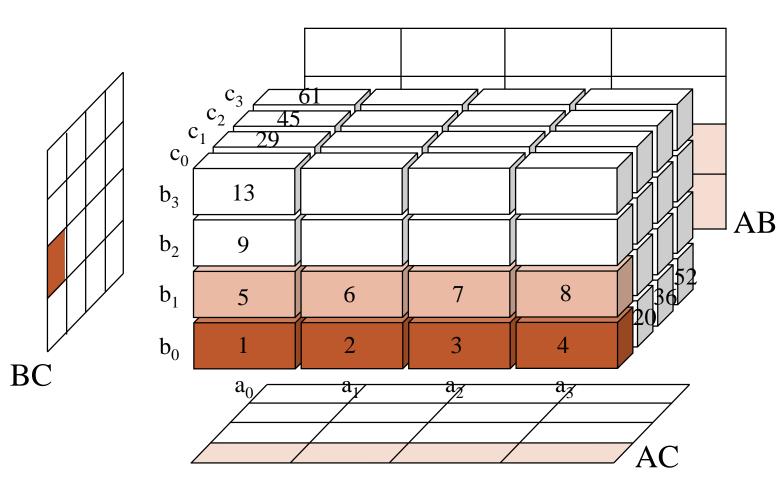


- □ Scan Order: 1 2 3 4 5 6 ...
- Goal: Fully compute chunk only once



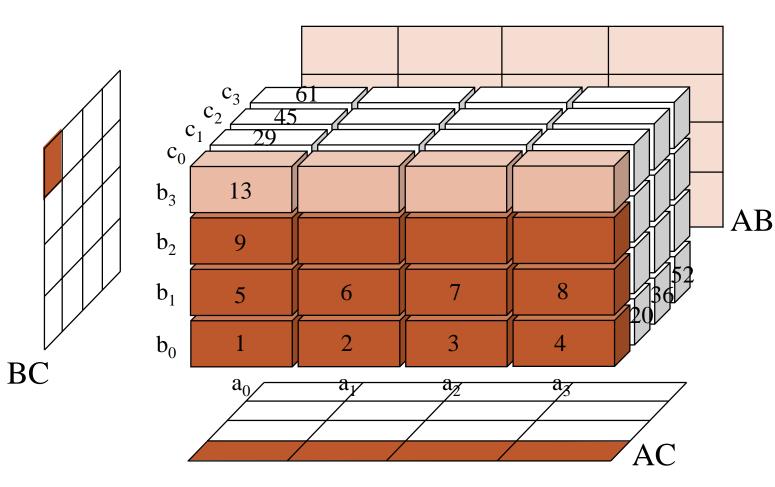
- □ While we scan through 1..4
 - One row of AC plane is partially computed
 - One chunk of BC plane is fully computed (write to file)
 - One row in AB plane is partially computed
- now scan through 5...8

- □ Scan Order: 1 2 3 4 5 6 ...
- Goal: Fully compute chunk only once



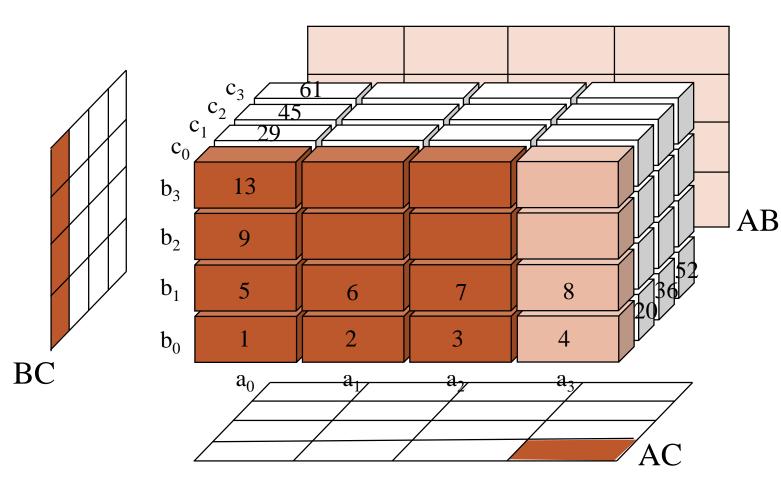
- □ While we scan through 5...8
 - Same **row** of AC plane is updated
 - Another chunk of BC plane is fully computed (reuse the same place in memory)
 - another row in AB plane is partially computed
- Continue on 9...12
- Continue on 13...16

- □ Scan Order: 1 2 3 4 5 6 ...
- Goal: Fully compute chunk only once

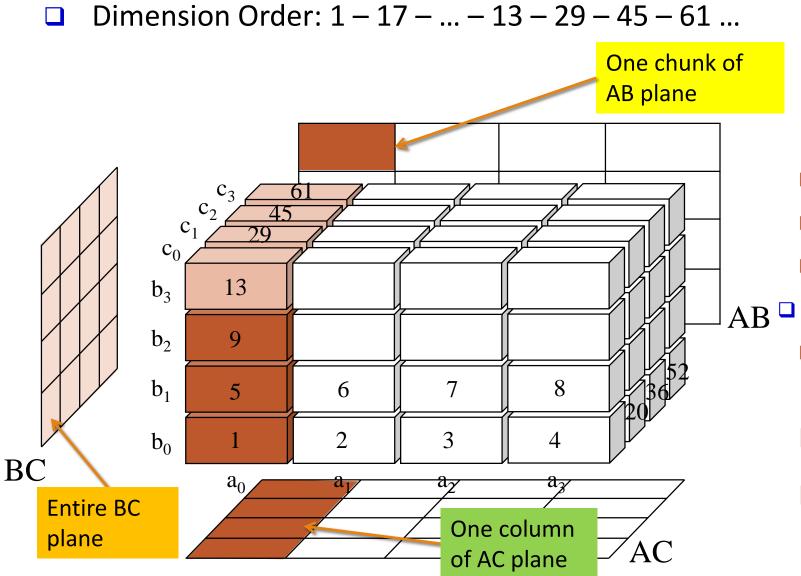


- □ While we scan through 13...16
 - One **row** of AC plane is fully computed (write to file)
 - Another chunk of BC plane is fully computed (reuse the same place in memory)
 - Whole AB plane is partially computed
- Memory requirement:
 - 4000 x 10 (AC) + 100 x 10 (BC) +
 4000 x 400 (AB) = 1,641,000 units

□ Dimension Order: 1 – 5 – 9 – 13 – 2 – 6 – ...



- One column of BC plane is fully computed (write to file)
- Another chunk of AC plane is fully computed (reuse the same place in memory)
- Whole AB plane is partially computed
- Memory:
 - 400 x 10 (BC) + 100 x 10 (BC) + + 4000 x 400 (AB)
 - **1,605,000** units



- One row of AC plane
- One chunk of AB plane
- All chunks in BC plane
- Memory:
 - 1000 x 40 (AC) + 1000 x 100 (AB) + 400 x 40 (AB)
 - □ 156,000 units
 - The best order

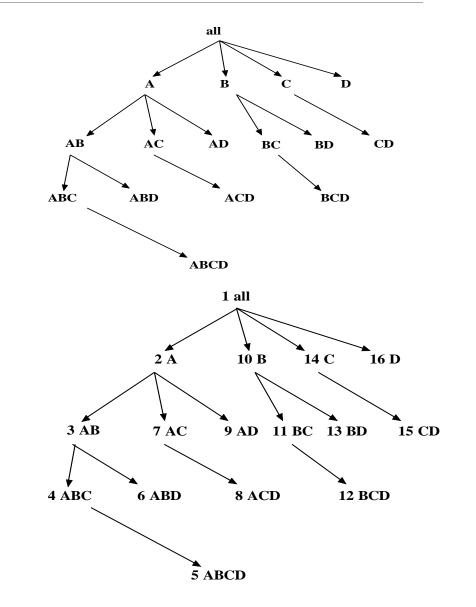
- Main Goal of Multi-Way: Reducing memory and I/O
 - □ How?
 - □ Keep the smallest plane in main memory
 - Fetch and compute only one chunk at a time for the largest plane
 - The planes should be sorted and computed according to their size in ascending order
 - □ Suppose A>B>C>...

for a in A: for b in B: for c in C: ...

- Pros and Cons of Multi-Way
 - **Pro:** Efficient for computing the **full cube** for a small number of dimensions
 - **Con:** Can not calculate iceberg cube.
 - i.e: If there are a large number of dimensions, "top-down" computation and iceberg cube computation methods (e.g., BUC) should be used

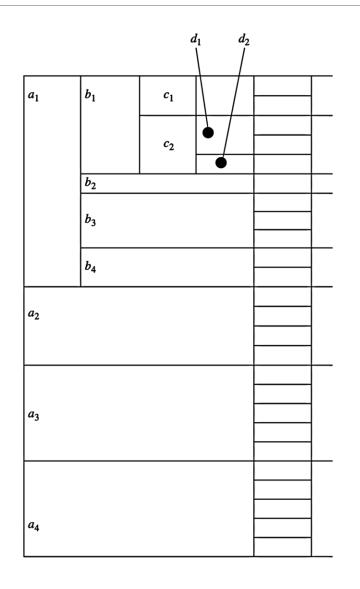
Cube Computation: Computing in Reverse Order

- Iceberg cube computation
- BUC (Beyer & Ramakrishnan, SIGMOD'99)
 - Bottom-Up (cube) Computation
 - "top-down" in our view since we put Apex cuboid on the top!
- Divides dimensions into partitions and facilitates iceberg pruning
 - Prune if not satisfy min_sup
 - □ If *minsup* = 1 ▷ compute full CUBE!
- No simultaneous aggregation



BUC: Partitioning and Aggregating

- Cannot fit in main memory
 - Sort *distinct* values and partition to fit
 - Aggregation when sorting
 - Continue processing
- Iceberg cube
 - □ If count of (a1, b1, *, *, *) < min_support
 - No need to sort on C

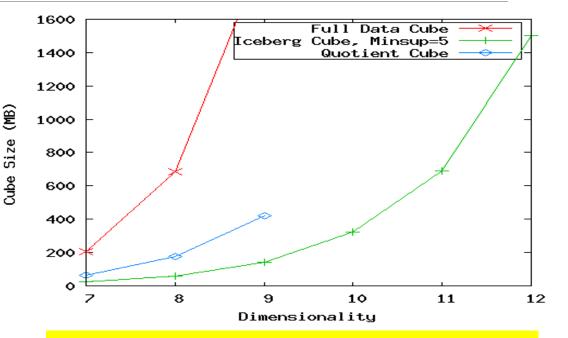


MultiWay VS BUC

	multiway	BUC
Input format	Multi-dimensional array	Relational database
Good for	Full cube	Iceberg cube
Key idea	Simultaneously Aggregation	Partition and sort
Calculation direction	A B C B C B C B C B C B C B C B C B C B	$\begin{array}{c} 1 \text{ all} \\ 2 \text{ A} & 10 \text{ B} & 14 \text{ C} & 16 \text{ D} \\ 2 \text{ A} & 10 \text{ B} & 14 \text{ C} & 16 \text{ D} \\ 3 \text{ AB} & 7 \text{ AC} & 9 \text{ AD} & 11 \text{ BC} & 13 \text{ BD} & 15 \text{ CD} \\ 4 \text{ ABC} & 6 \text{ ABD} & 8 \text{ ACD} & 12 \text{ BCD} \\ \hline 5 \text{ ABCD} \end{array}$

High-Dimensional OLAP?—The Curse of Dimensionality

- □ High-D OLAP Applications:
 - **E**.g. bio-data analysis, statistical surveys
- None of the previous cubing method can handle high dimensionality!
 - Iceberg cube and compressed cubes: only delay the inevitable explosion
 - Full materialization: still significant overhead in accessing results on disk
- A shell-fragment approach: X. Li, J. Han, and H. Gonzalez, High-Dimensional OLAP: A Minimal Cubing Approach, VLDB'04

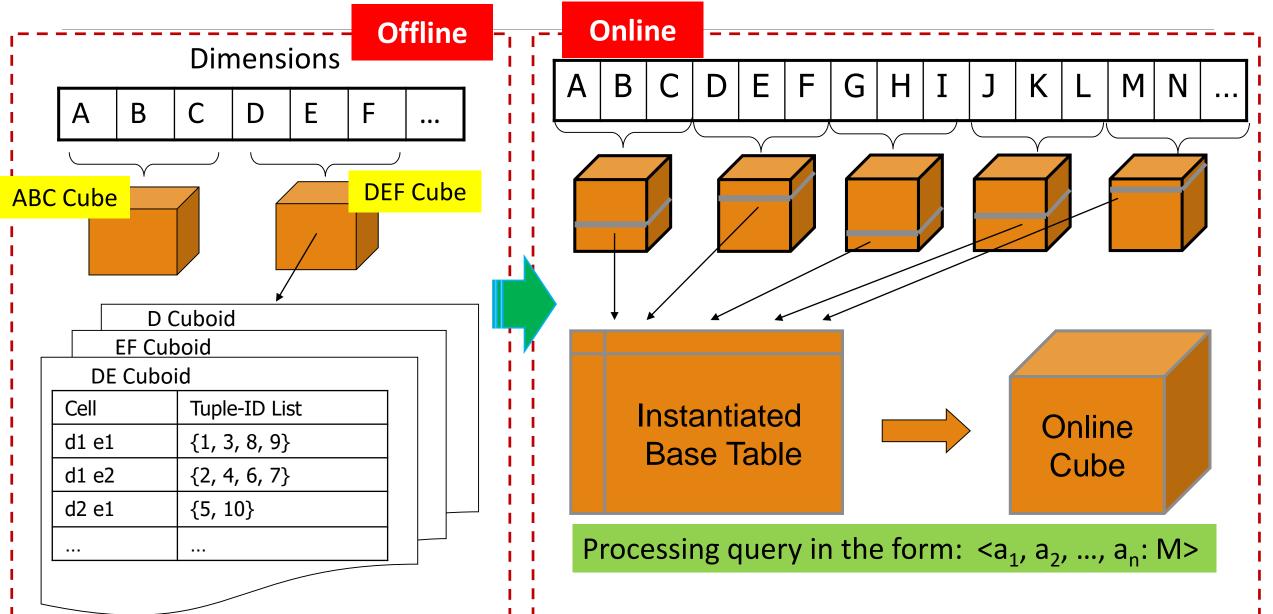


A curse of dimensionality: A database of 600k tuples. Each dimension has cardinality of 100 and *zipf* of 2.

Fast High-D OLAP with Minimal Cubing

- Observation: OLAP occurs only on a small subset of dimensions at a time
- Semi-Online Computational Model
 - Partition the set of dimensions into shell fragments
 - Compute data cubes for each shell fragment while retaining inverted indices or value-list indices
 - Given the pre-computed **fragment cubes**, dynamically compute cube cells of the high-dimensional data cube *online*
- Major idea: Tradeoff between the amount of pre-computation and the speed of online computation
 - Reducing computing high-dimensional cube into precomputing a set of lower dimensional cubes
 - Online re-construction of original high-dimensional space
 - Lossless reduction

Use Frag-Shells for Online OLAP Query Computation



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Computing a 5-D Cube with 2-Shell Fragments

Example: Let the cube aggregation function be count

TID	Α	В	С	D	Ε
1	al	b1	c1	d1	e1
2	al	b2	c1	d2	e1
3	al	b2	c1	d1	e2
4	a2	b1	c1	d1	e2
5	a2	b1	c1	d1	e3

- Divide the 5-D table into 2 shell fragments:
 - □ (A, B, C) and (D, E)
- Build traditional invert index or RID list (1-D)

Attribute Value	TID List	List Size
a1	123	3
a2	4 5	2
b1	145	3
b2	2 3	2
c1	12345	5
d1	1345	4
d2	2	1
e1	12	2
e2	34	2
e3	5	1

Shell Fragment Cubes: Ideas

- Generalize the 1-D inverted indices to multi-**dimensional** ones in the data cube sense
- Compute all cuboids for data cubes ABC and DE while retaining the inverted indices
 - Ex. shell fragment cube ABC contains 7 cuboids:
 - □ A, B, C; AB, AC, BC; ABC
- This completes the offline computation
- ID Measure Table
 - If measures other than count are present, store in ID measure table separate from the shell fragments

	tid	count	sum
	1	5	70
	2	3	10
2	3	8	20
	4	5	40
	5	2	30

			ат		2.3	3	
and D		مانم	a2	4 5	5	2	
		me	b1		15	3	
			b2	23	3	2	
cuboi	ids:		c1	12	2345	5	
			d1	13	3 4 5	4	
			d2	2		1	
	_		e1	12	2	2	
Shell-1	ragm	ent AB	e2 34		ł	2	
			e3 5			1	
sum		Call	Interesti		TID List	1:-+ 0	
70		Cell	Intersectio	n	TID LISU	List S	bize
10		a1 b1	$123 \cap 14$	5	1	1	
20		a1 b2	123 023		23	2	
40		a2 b1	45∩145		4 5	2	
30		a2 b2	45∩23		φ	0	

TID List

123

List

Size

З

Attribute

Value

a1

Shell Fragment Cubes: Size and Design

- Given a database of T tuples, D dimensions, and F shell fragment size, the fragment cubes' space requirement is: $O\left(T\left[\frac{D}{F}\right](2^F-1)\right)$
 - □ For F < 5, the growth is sub-linear
- Fragment groupings can be arbitrary to allow for maximum online performance
 - Known common combinations (e.g.,<city, state>) should be grouped together
- Shell fragment sizes can be adjusted for optimal balance between offline and online computation

Attribute Value	TID List	List Size
al	123	3
a2	4 5	2
b1	145	3
b2	2 3	2
c1	12345	5
dl	1345	4
d2	2	1
e1	12	2
e2	3 4	2
e3	5	1

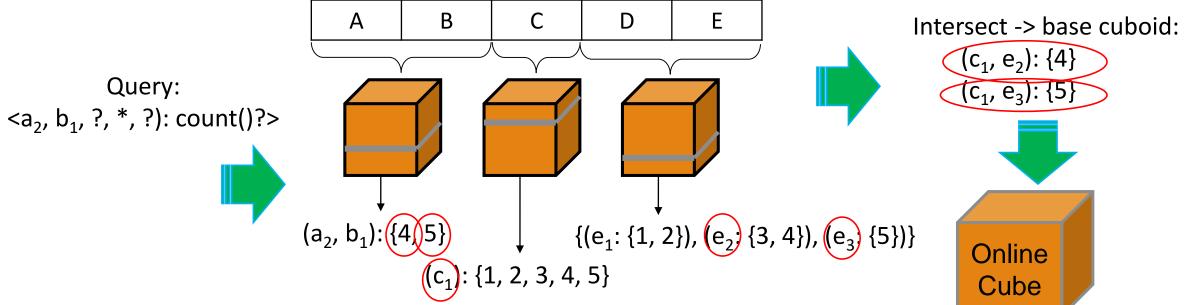
Cell	Intersection	TID List	List Size
a1 b1	123 \cap 145	1	1
a1 b2	123 023	23	2
a2 b1	45∩145	4 5	2
a2 b2	45∩23	φ	0

Online Query Computation with Shell-Fragments

- □ A query has the general form: $<a_1, a_2, ..., a_n$: M>
- Each a_i has 3 possible values
 - Instantiated value— this is what we want to look at
 - □ Inquire ? Function want to analyze these dimensions
 - □ Aggregate * function don't care about these dimensions
 - Ex: Suppose we want to query student data for junior (year 3) students and want to compare scores for different genders and ages, but don't care about what high school they attended.
 - <3, ?, ?, *, 1: count>

Online Query Computation with Shell-Fragments

- Method: Given the materialized fragment cubes, process a query as follows
 - Divide the query into fragments, same as the shell-fragment
 - □ Fetch the corresponding TID list for each fragment from the fragment cube
 - Intersect the TID lists from each fragment to construct **instantiated base table**
 - Compute the data cube using the base table with any cubing algorithm



Chapter 5: Data Cube Technology

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Data Cube Computation Methods

Multidimensional Data Analysis in Cube Space

product time

Summary

Data Mining in Cube Space

- Data cube is already aggregated
- Reports generated from a Data Cube can easily by drilled into through query in a drilldown fashion.

Complex Aggregation at Multiple Granularities: Multi-Feature Cubes

- Multi-feature cubes (Ross, et al. 1998): Compute complex queries involving multiple dependent aggregates at multiple granularities
- Ex. Grouping by all subsets of {item, region, month}, find the maximum price in 2019 for each group, and the total sales among all maximum price tuples

```
select item, region, month, max(price), sum(R.sales)
```

from purchases

```
where year = 2010
```

cube by item, region, month: R

```
such that R.price = max(price)
```

Discovery-Driven Exploration of Data Cubes

- Discovery-driven exploration of huge cube space (Sarawagi, et al.'98), suggested way to highlight data:
 - Pre-compute measures indicating exceptions.
 - □ i.e: significantly different from the value anticipated
 - Visual cues such as background color can be used to show the degree of exception of each cell

Examples: Discovery-Driven Data Cubes

item all region all

Sum of sales	mont	h										
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Total		1%	-1%	0%	1%	3%	- 1	-9%	-1%	2%	-4%	3%

Avg sales	топ	th										
item	Jan	Feb	Mar	Арг	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sony b/w printer		9%	-8%	2%	-5%	14%	-4%	0%	41%	-13%	-15%	-11%
Sony color printer		0%	0%	3%	2%	4%	-10%	-13%	0%	4%	6%	4%
HP b/w printer		-2%	1%	2%	3%	8%	0%	-12%	-9%	3%	-3%	6%
HP color printer		0%	0%	-2%	1%	0%	-1%	-7%	-2%	1%	-5%	1%
IBM home computer		1%	-2%	-1%	-1%	3%	3%	-10%	4%	1%	-4%	-1%
IBM laptop computer		0%	0%	-1%	3%	4%	2%	-10%	-2%	0%	-9%	3%
Toshiba home computer		-2%	-5%	1%	1%	-1%	1%	5%	-3%	-5%	-1%	-1%
Toshiba laptop computer		1%	0%	3%	0%	-2%	-2%	-5%	3%	2%	-1%	0%
Logitech mouse		3%	-2%	-1%	0%	4%	6%	-11%	2%	1%	-4%	0%
Ergo-way mouse		0%	0%	2%	3%	1%	-2%	-2%	-5%	0%	-5%	8%

item	IBI	IBM home computer											
Avg sales	mon	month											
region	Jan	Feb	Mar	Арг	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	
North South East West		-1% -1% -1% 4%	-3% 1% -2% 0 %	-1% -9% 2% -1%	0% 6% -3% -3%	3% -1% 1% 5%	4% -39% 18% 1%	-7% 9% -2% -18%	1% -34% 11% 8%	0% 4% -3% 5%	-3% 1% -2% -8%	-3% 7% -1% 1%	

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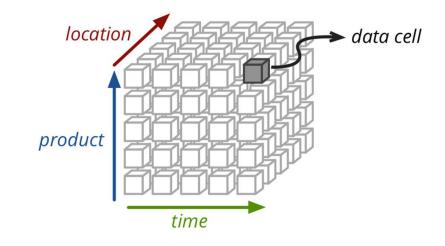
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Data Cube Computation Methods

Multidimensional Data Analysis in Cube Space







Data Cube Technology: Summary

- Data Cube Computation: Cuboids; iceberg cube; closed cube and cube shell
- Data Cube Computation Methods
 - MultiWay Array Aggregation
 - BUC
 - High-Dimensional OLAP with Shell-Fragments
- Multidimensional Data Analysis in Cube Space
 - Multi-feature Cubes
 - Discovery-Driven Exploration of Data Cubes

Text Cube



Data Cube Technology: References (I)

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