Lecture 1: Motivation for linear algebra

Admin: Textbook, syllabus, homework, midterms, final, grading, office hours, ...

MOTIVATION FOR LINEAR ALGEBRA

- Linear transformations are everywhere!
  - Calculus: integration and differentiation are linear
  - Signals: Fourier transform is linear
  - Quantum physics: time evolution is linear
- Many applications
  - Solving systems of linear equations

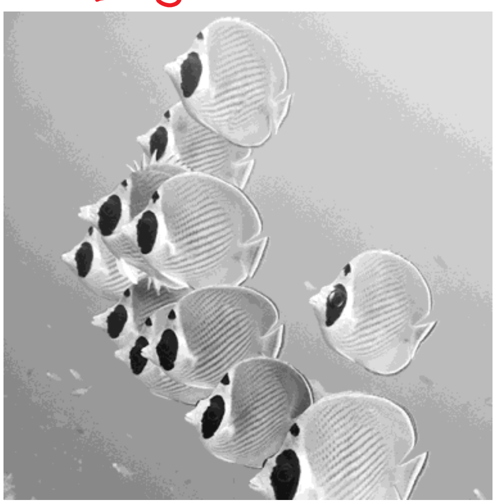
$$\begin{cases} 2x - y = 3 \\ -x + y = -2 \end{cases}$$

  - Solving differential equations and recursions

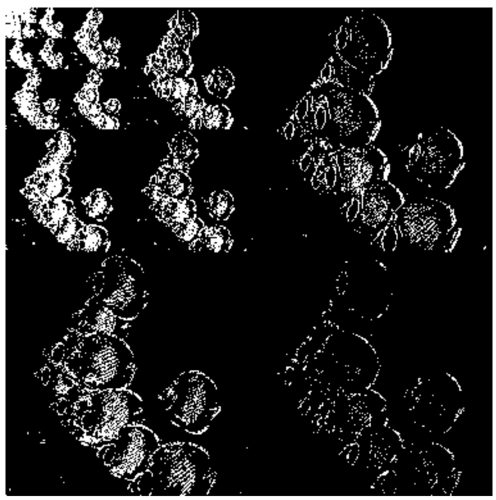
$$\ddot{y} = \dot{y} + y \qquad x_n = x_{n-1} + x_{n-2}$$

  - Image compression

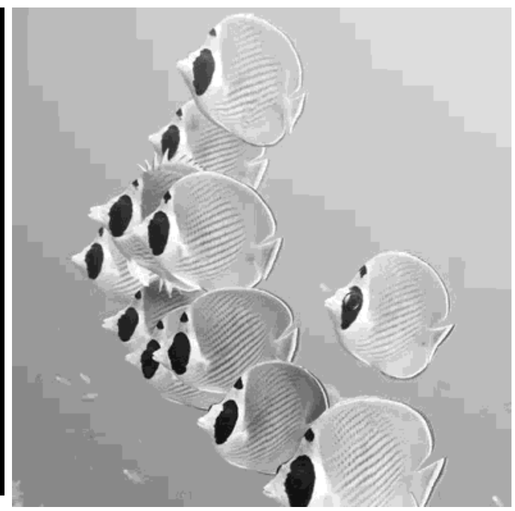
Original



Keep 10% of coeffs

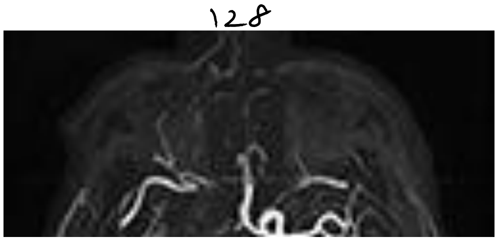


Result



  - Compressed sensing

128

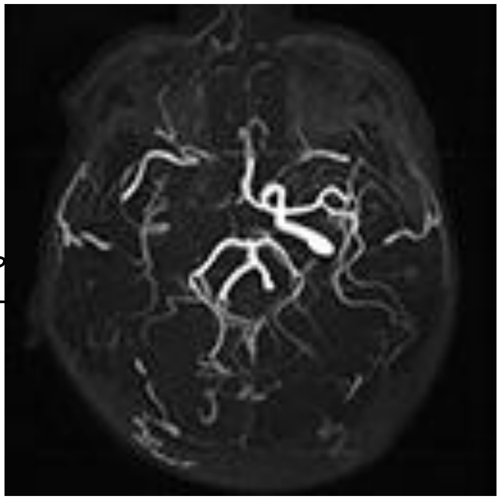


\# of variables

$N = 128^2 = 16384$

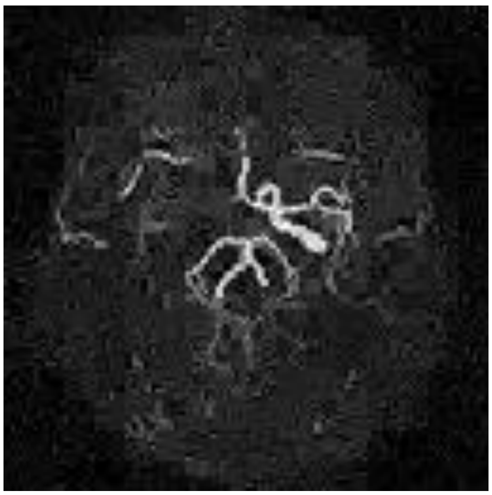
# of variables
$N = 128^2 = 16384$

128

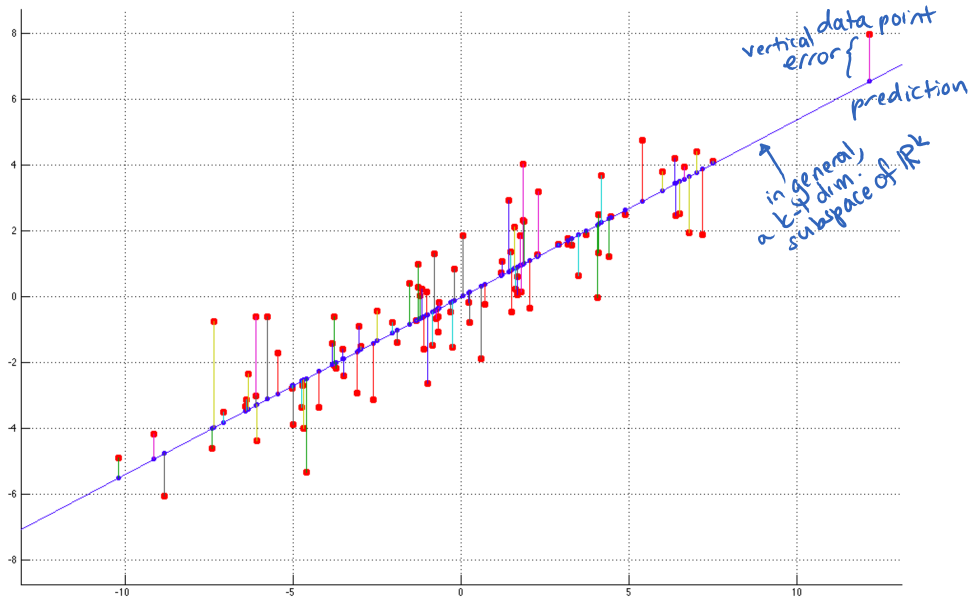


# of observations
$d = 4480 = 0.27N$

15 minutes in ℓ_1 magic

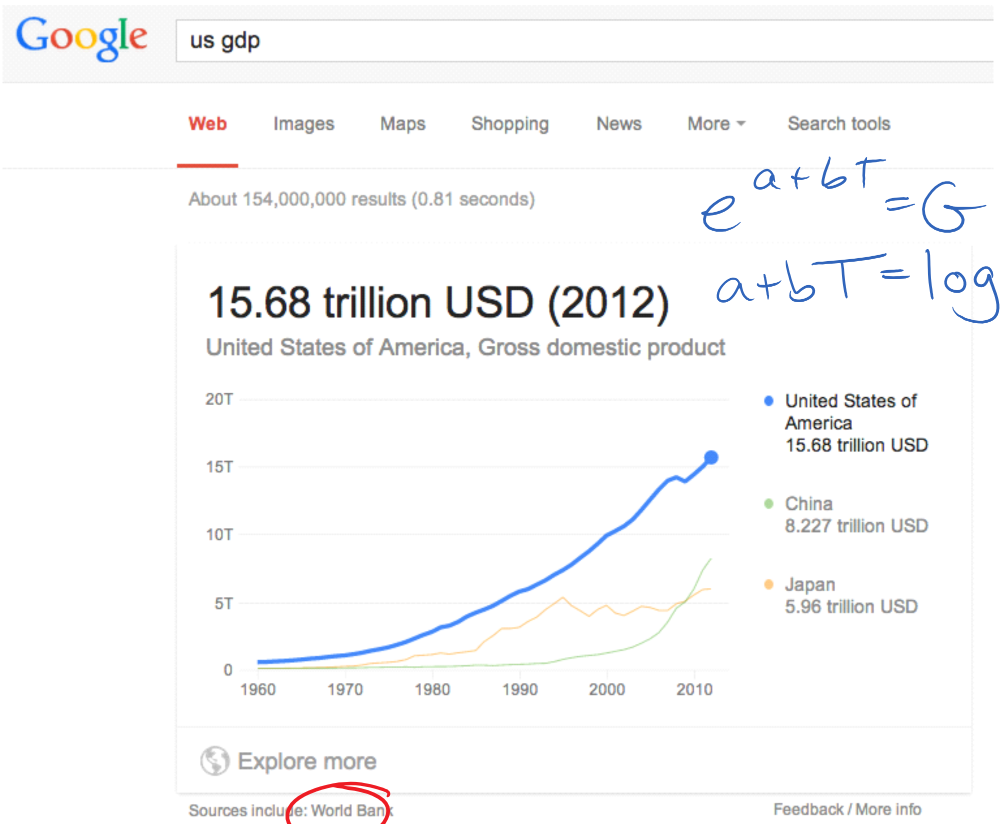


- Machine learning and statistics:
  dimension reduction, least-squares fitting



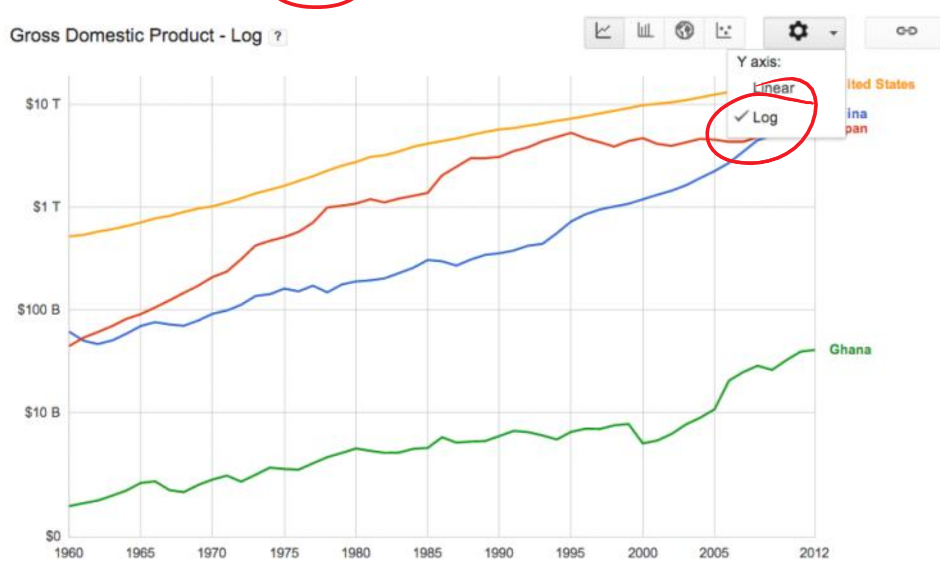
Example: Predict US gross domestic product (GDP) in 2050.

Answer:



$$e^{a+bT} = G$$

$$a + bT = \log G$$

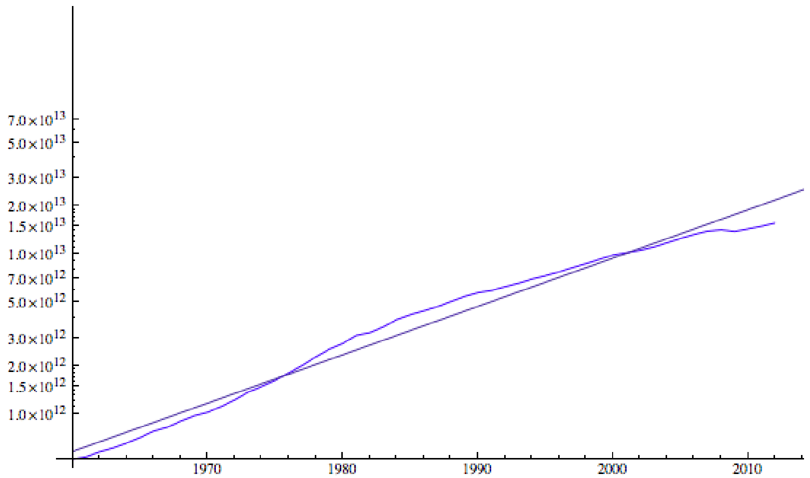


An exponential should fit the data better than a straight line.

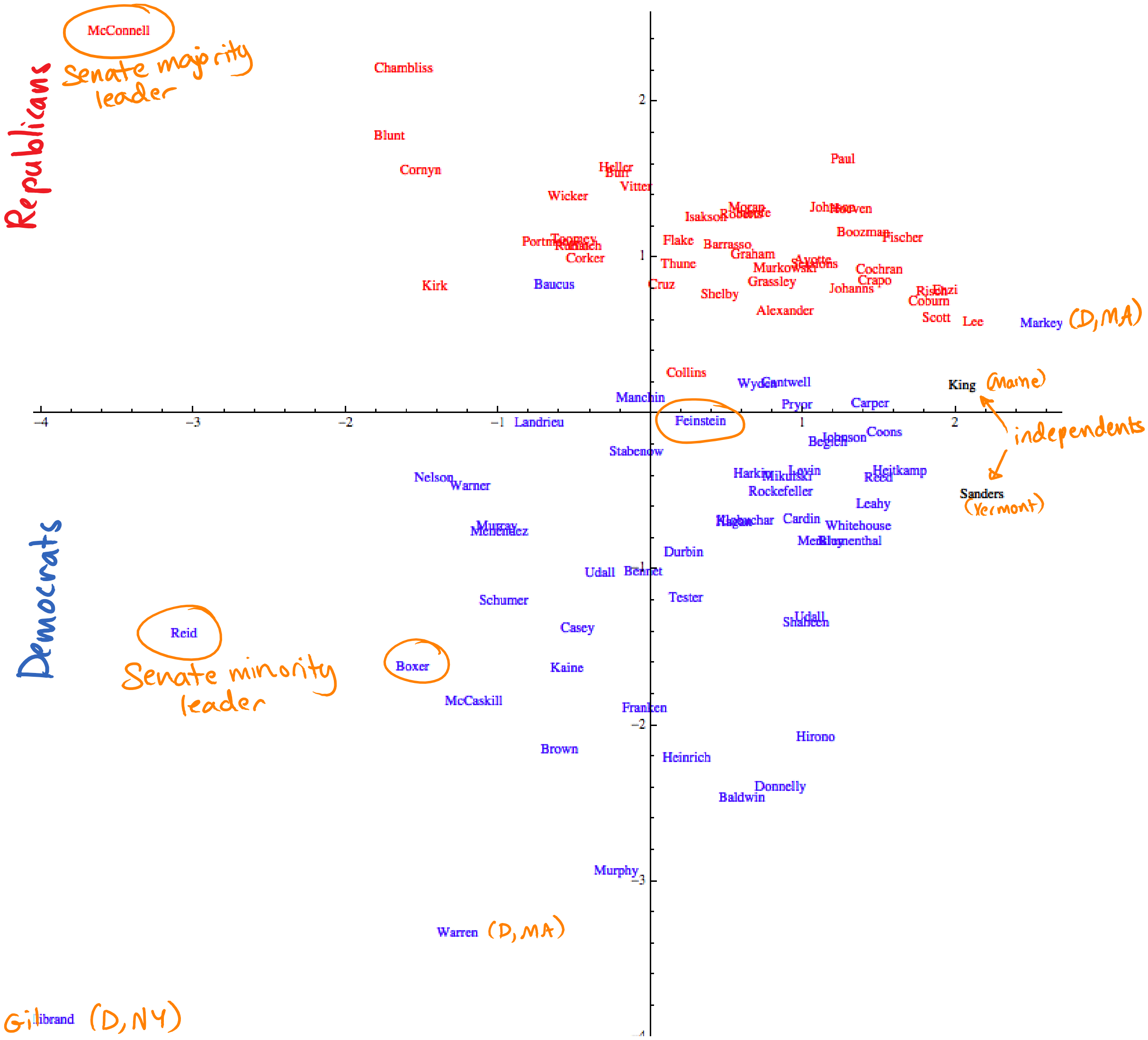
```
USGDPdata = [1960, 5.20531e11; 1961, 5.39051e11; 1962, 5.79748e11; 1963, 6.1167e11; 1964, 6.56912e11;
1965, 7.12082e11; 1966, 7.80761e11; 1967, 8.25056e11; 1968, 9.01456e11; 1969, 9.73385e11; 1970,
1.0248e12; 1971, 1.1131e12; 1972, 1.225e12; 1973, 1.3693e12; 1974, 1.4859e12; 1975, 1.6234e12; 1976,
1.8091e12; 1977, 2.0136e12; 1978, 2.276e12; 1979, 2.5435e12; 1980, 2.7675e12; 1981, 3.1038e12; 1982,
3.2277e12; 1983, 3.5069e12; 1984, 3.9004e12; 1985, 4.1848e12; 1986, 4.425e12; 1987, 4.6989e12; 1988,
5.0619e12; 1989, 5.4397e12; 1990, 5.7508e12; 1991, 5.9307e12; 1992, 6.2618e12; 1993, 6.5829e12; 1994,
6.9933e12; 1995, 7.3384e12; 1996, 7.7511e12; 1997, 8.2565e12; 1998, 8.741e12; 1999, 9.301e12; 2000,
9.8988e12; 2001, 1.02339e13; 2002, 1.05902e13; 2003, 1.10893e13; 2004, 1.17978e13; 2005, 1.25643e13;
2006, 1.33145e13; 2007, 1.39618e13; 2008, 1.42193e13; 2009, 1.38983e13; 2010, 1.44194e13; 2011,
1.49913e13; 2012, 1.56848e13];
```

```
>> years = USGDPdata(:,1);
A = [ones(length(years),1), years];
USFit = pinv(A) * log(USGDP)

USFit =

    -1.096282320582321e+02
     6.975400397258215e-02

>> exp(USFit' * [1; 2050])

ans =

     3.099636305012074e+14
```

= 30 trillion dollars

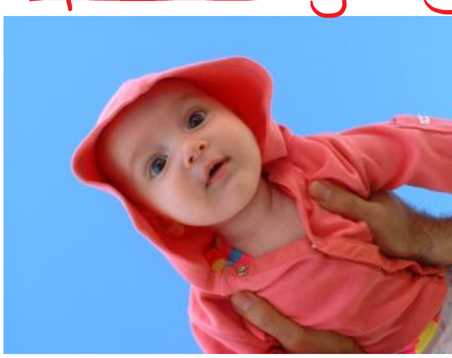


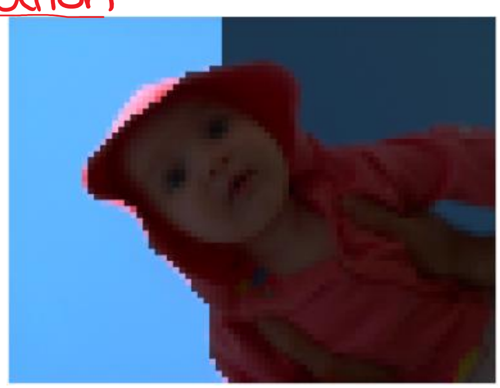
```
ListPlot[{#} & /@ dataprojectedmanually, AspectRatio → 1, PlotMarkers → data[[All, 1, 1]],
 PlotStyle → (data[[All, 1, 3]]) /. {"D" → Blue, "R" → Red, "I" → Black}]
{97, 17}
{17, 2}
```



- Even purely combinatorial applications

Spectral image segmentation

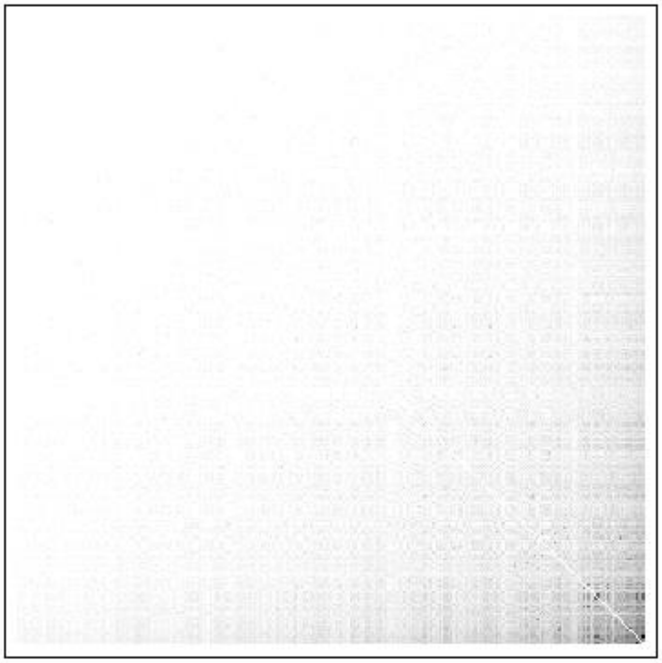




Spectral clustering

Spectral clustering

```
matrix = Import["/Users/breic/Desktop/adjacencymatrix.txt", "Table"];
matrix += Transpose[matrix];
matrix // ArrayPlot
```



```
laplacian = DiagonalMatrix[Plus @@ # & /@ matrix] - matrix // N;
di = DiagonalMatrix[(1 / Plus @@ #) & /@ matrix // N];
evs = Eigensystem[√di .laplacian.√di] // Transpose // Reverse;

coordinates = evs[[2 ;; 9, 2]] // Transpose;

numclusters = 16;

ClusteringComponents[coordinates, numclusters, 1, Method → "PAM"]
```

→ embedding into $\mathbb{R}^d$

	A Walk to Remember	Bend It Like Beckham	Con Air
Indiana Jones and the L	Coyote Ugly	Bridget Jones's Diary	Double Jeopardy
Lord of the Rings: The	Dirty Dancing	Frida	Gone in 60 Seconds
Lord of the Rings: The	How to Lose a Guy in 10	Life Is Beautiful	Independence Day
Lord of the Rings: The	Maid in Manhattan	Love Actually	Lethal Weapon 4
Raiders of the Lost Ark	Pretty Woman	Moulin Rouge	Men in Black II
Star Wars: Episode IV:	Sister Act	My Big Fat Greek Weddin	Pearl Harbor
Star Wars: Episode VI:	The Princess Diaries 2:	Pride and Prejudice	The Fast and the Furiou
Star Wars: Episode V: T	The Princess Diaries (W	Rabbit-Proof Fence	The Patriot
The Lord of the Rings:	The Wedding Planner	Shakespeare in Love	Tomb Raider
	What Women Want	Whale Rider	Twister

			2001: A Space Odyssey
	A Bug's Life	Amelie	All the President's Men
12 Angry Men	Breakfast at Tiffany's	American Beauty	Blade Runner
Airplane!	City of Angels	Being John Malkovich	Gandhi
American Pie	Ever After: A Cinderell	Crouching Tiger	Jaws
American Pie 2	Finding Nemo (Widescree	Election	L.A. Confidential
Austin Powers in Goldme	Grease	Eternal Sunshine of the	Lawrence of Arabia
Austin Powers: Internat	Harry Potter and the Ch	High Fidelity	Lord of the Rings: The
Austin Powers: The Spy	Harry Potter and the Pr	Lock	One Flew Over the Cucko
Interview with the Vamp	Harry Potter and the So	Lost in Translation	Seven Samurai
Liar Liar	Runaway Bride	Magnolia	The Aviator
Meet the Parents	The Lion King: Special	Run Lola Run	The Exorcist
Ransom	The NeverEnding Story	Rushmore	The Godfather
Spaceballs	The Princess Bride	Sideways	The Godfather
Spider-Man	The Sound of Music	The Royal Tenenbaums	The Graduate
Wayne's World	Willy Wonka & the Choco	Y Tu Mama Tambien	The Great Escape
			The Maltese Falcon

Cold Mountain
Collateral
Crash
Fahrenheit 9/11
Finding Neverland
Hotel Rwanda
Man on Fire
Master and Commander: T
Million Dollar Baby
Ocean's Twelve
Ray
Road to Perdition
Runaway Jury
Seabiscuit
The Manchurian Candidat
The Notebook
The Phantom of the Oper
The Pianist ,

A Fish Called Wanda
Alien: Collector's Edit
Back to the Future
Back to the Future Part
Batman
Die Hard 2: Die Harder
Die Hard With a Vengean
Goldfinger
Groundhog Day
Indiana Jones and the T
Men in Black
Mission: Impossible
Predator: Collector's E
Rocky
Speed
Star Trek II: The Wrath
Terminator 2: Extreme E
The Hunt for Red Octobe
The Terminator
True Lies ,

A Knight's Tale
Ice Age
Jurassic Park
Lara Croft: Tomb Raider
Minority Report
Pirates of the Caribbea
Rush Hour
Rush Hour 2
Sleeping Beauty: Specia
Spider-Man 2
Star Wars: Episode II:
Star Wars: Episode I: T
Terminator 3: Rise of t
The Fifth Element
The Incredibles
The Matrix
The Matrix: Reloaded
The Matrix: Revolutions
The Mummy
The Mummy Returns
X2: X-Men United
X-Men ,

Adaptation
A Few Good Men
Air Force One
Armageddon
Clear and Present Dange
Crimson Tide
Enemy of the State
Entrapment
High Crimes
In the Line of Fire
Lethal Weapon
Lethal Weapon 2
Lethal Weapon 3
Patriot Games
Rules of Engagement
Swordfish
The Bone Collector
The Client
The Fugitive
The Negotiator
The Pelican Brief
The Rock
The Sum of All Fears ,

12 Monkeys
Almost Famous
American History X
Anchorman: The Legend o
Donnie Darko
Garden State
GoodFellas: Special Edi
Grosse Pointe Blank
Heat: Special Edition
Kill Bill: Vol. 1
Kill Bill: Vol. 2
Memento
Napoleon Dynamite
Office Space
Pulp Fiction
Requiem for a Dream
Reservoir Dogs
Seven
Sin City ,

Ace Ventura: Pet Detect
A League of Their Own
A River Runs Through It
Basic Instinct
Cheaper by the Dozen
Daddy Day Care
Erin Brockovich
Face/Off
Father of the Bride
Kindergarten Cop
Legally Blonde
Mrs. Doubtfire
Notting Hill
Pay It Forward
Phenomenon
Serendipity
Shall We Dance?
Sleepless in Seattle
Steel Magnolias ,

50 First Dates
Anger Management
Bad Boys II
Behind Enemy Lines
Bruce Almighty
Dodgeball: A True Under
Harold and Kumar Go to
Hero
Hidalgo
Hitch
Hostage
I Robot
Meet the Fockers
National Treasure
Ocean's Eleven
Sahara
Shrek 2
The Bourne Identity
The Bourne Supremacy
The Count of Monte Cris ,

Apollo 13
As Good as It Gets
Black Hawk Down
Boys Don't Cry
Cast Away
Chocolat
Dances With Wolves: Spe
Dead Man Walking
Driving Miss Daisy
Enemy at the Gates
E.T. the Extra-Terrestr
Field of Dreams
Forrest Gump
Fried Green Tomatoes
Gladiator
Glory
Good Will Hunting
Jerry Maguire
Moonstruck
My Cousin Vinny
October Sky
Philadelphia
Primal Fear
Rain Man
Remember the Titans }

2. algorithms based on fast SDD solvers

* max-flow & multi-commodity flow problems
  - for decades, best algorithms were deterministic, combinatorial
    [Goldberg-Rao '98]: $\tilde{O}(m\sqrt{n}/\varepsilon)$ for $(1-\varepsilon)$-approx max flow   augmenting paths, blocking flows...
  - recent breakthroughs based on numerical linear algebra, spectral graph theory
    [S-H Teng et al. '11] (USC): $\tilde{O}(mn^{1/3}/\varepsilon^{11/3})$
    [Kelner et al., '13; Sherman '13]:
    $\tilde{O}(m/\varepsilon^2)$ or $\tilde{O}(km/\varepsilon^2)$ for k flows
* generating random spanning trees

[Kelner et al, [illegible]; Sherman [illegible]]

$\tilde{O}(m/\varepsilon^2)$ or $\tilde{O}(km/\varepsilon^2)$ for k flows

* generating random spanning trees
* graph sparsification
* sparsest cut
* distributed routing