Understanding Bell’s Inequality

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Inspired from: https://youtu.be/sAXxSKifgtU?list=PL193BC0532FE7B02C

Experiment

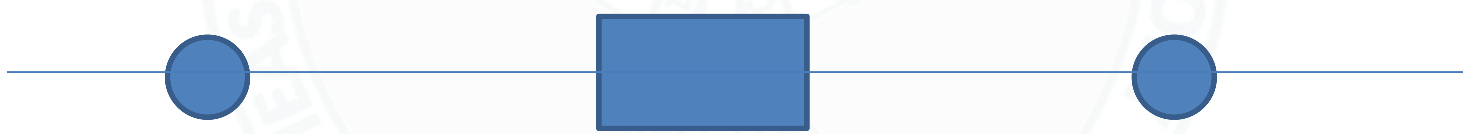
- Two decks of cards
  - Specially shuffled
  - Give one to each student
    - Each sees what they have
      - The outcomes are random
      - But the same for both
      - Synchronized randomness
      - This is classical entanglement
        - » In QM Entanglement, the action on one of them forces the other to have a property
        - » If someone tries to observe the state of the photon, that photon is absorbed/destroyed
  - But this is useful
    - Transmission of data
      - Amina and Bilal each have a synchronized deck
      - Amina wants to send a message to Bilal which has encoded in a bitstring
      - If the bit is 0 and the card that they have is in A,2,3,4,5,6,7 then she tells bilal "Same" and if it is not one of those then she tells him different
      - If the bit is 1 and the card they have is in A,2,3,4,5,6,7 then she tells Bilal "different" and otherwise "same"
      - The stream "same", "different" is meaningless to Everyone else

Implications of QM

- QM implies that
  - An object’s behavior can be described in terms of its wave function
  - However, the wave function only gives probabilistic estimates
  - Measurement affects/disturbs the system being observed
  - Heisenberg Uncertainty Principle
    - It is impossible to accurately measure both the momentum and position of a particle

Einstein's Argument

- Einstein, Podolsky, and Rosen (1935)
  - "Can Quantum Mechanical Description of Physical Reality be considered complete?"
    - "A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system."



Consider two entangled particles: we can measure position of one (without affecting the other) and get position of the other as well. We can also measure the momentum of the other particle (without affecting the first) and get information on both. Alternatively, it should be possible to measure the polarization of an entangled pair of photons along two different axes of polarization.
According to Quantum Entanglement Concepts: An action on one instantly affects the other and this would be impossible.

EPR

- Reality of particle 1 is dependent upon measurements of particle 2
  - No reasonable definition of reality could be expected to permit this
    - QM/Wavefunction based reality is incomplete
    - Rethink Reality!

Locality

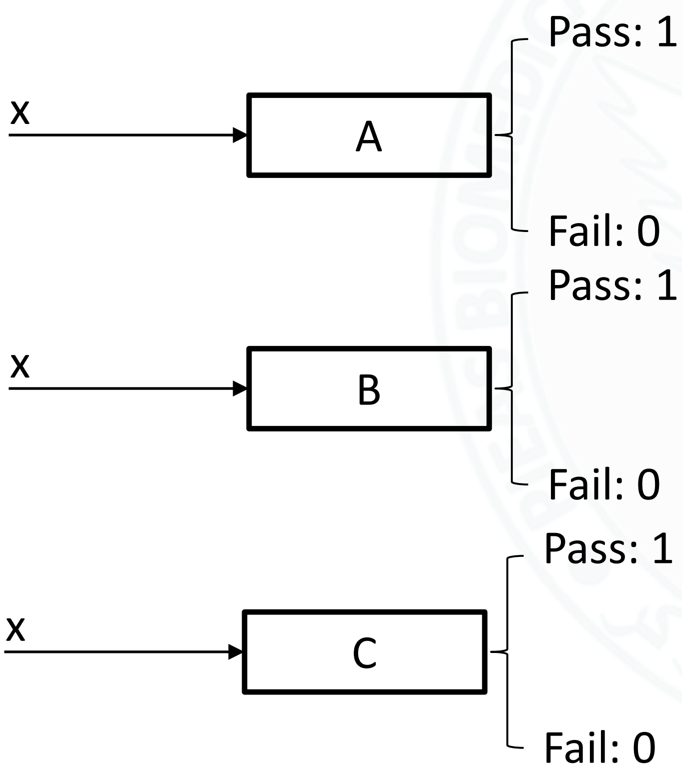
- A pizza cannot reach you before D/c
- You cannot get pizza instantaneously

Locality

- This pen being dropped in this class doesn't/shouldn't be affected by the motion of Pluto
- According to QM: Motion of one object instantaneously affects the other
- Assume a particle whose "state" can be described by x
  - Any and all measurable properties of the state are captured by x
  - The outcome of any experiment on the particle is dependent upon its state and the experiment
  - An event, occurring a distance D away, can affect the outcome of the experiment but only after D/c time

Implications of Locality

- Assume three pass/fail tests (A,B,C) on state of x

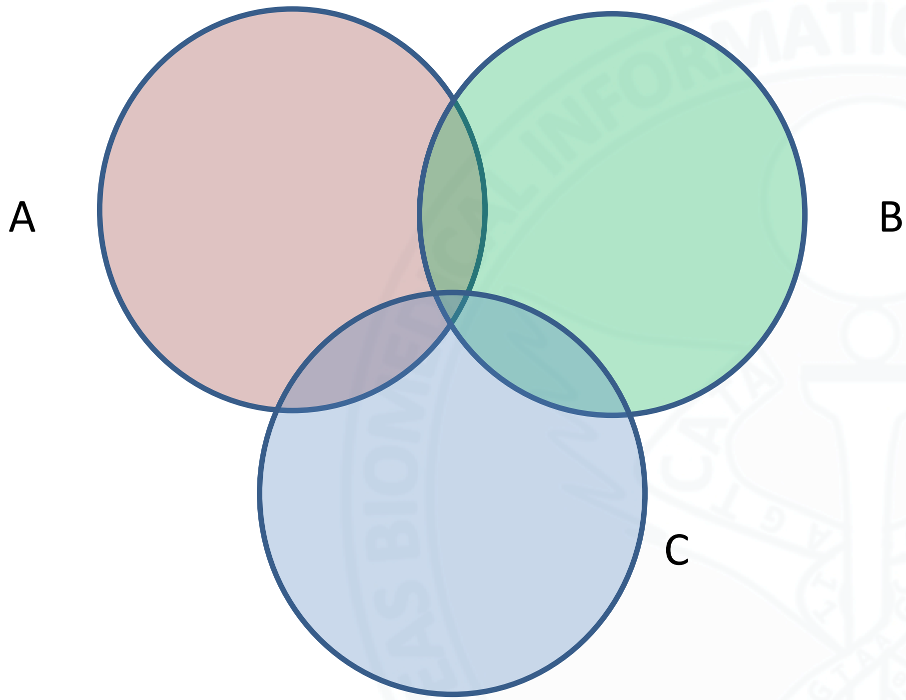


A	B	C	$A\bar{B}$	$B\bar{C}$	$A\bar{C}$
0	0	0			
0	0	1			
0	1	0		+	
0	1	1			
1	0	0	+		+
1	0	1	+		
1	1	0		+	+
1	1	1			

If locality holds, i.e., the measurement of/action on one object does not affect the state of the other, then each state "x" corresponds to only one row and in that case we must have:

$$N(A\bar{B}) + N(B\bar{C}) \geq N(A\bar{C})$$

Alternate View



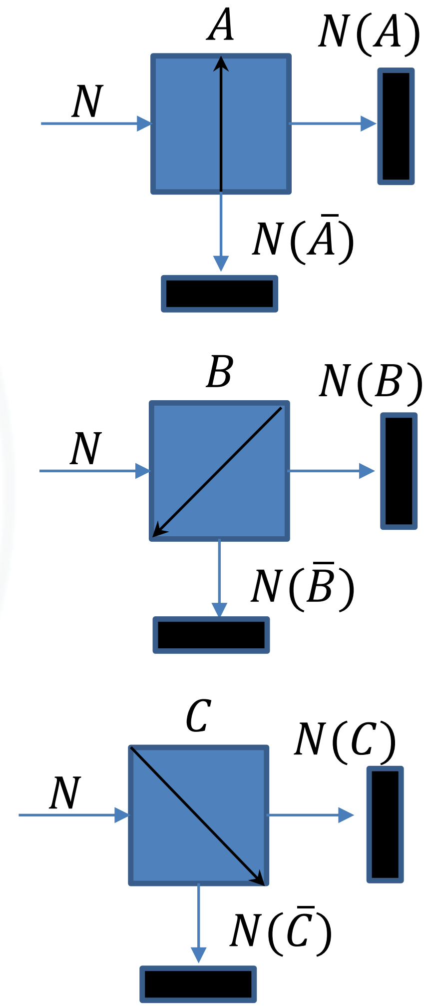
$$P(A\bar{B}) + P(B\bar{C}) \geq P(A\bar{C})$$

- If two entangled objects are in the same state then the same action on each of them should produce the same result.
- However, different actions on one of them should not affect the other at a distance.
- Thus, both of them should correspond to the same row in the table (but we do not know which one) and in that case the Bell's inequality must be satisfied

- What if Bell's inequality is satisfied?
  - The assumption that the outcome of the test depends only on "x" and the test is correct
  - Nothing weird is happening!
- What if Bell's Inequality is violated?
  - The assumption that the outcome of the experiment only depends on "x" and the setup is wrong
  - Local realism needs reconsideration?

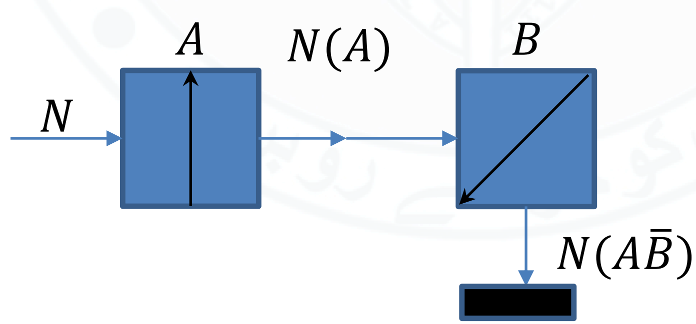
Testing Bell's Inequality

- Take 3 polarizers at angles $0, \theta, 2\theta$ angles from the vertical (A,B,C)
- And shine randomly polarized through light through a randomly picked pair of the polarizers
- Use a detector that measures the number of photons passing through (or not passing through) each polarizer
- Better to use polarizing beam-splitters
- Sensitive Photodetectors
  - Glorified webcams for intensity measurement



Single Photon Bell Inequality Testing

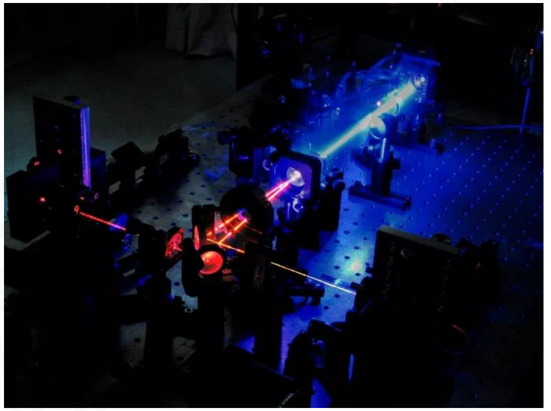
- Algorithm form Bell Inequality Testing
- Pick two different polarizers at random, for example, A and B
  - Count the number of times this polarizer pair has been picked
  - Count the number of times a photon passes through the first but not the second
  - Calculate the probability
- Repeat until bored
- Check if $\boldsymbol{P}(\boldsymbol{A}\overline{\boldsymbol{B}}) + \boldsymbol{P}(\boldsymbol{B}\overline{\boldsymbol{C}}) \geq \boldsymbol{P}(\boldsymbol{A}\overline{\boldsymbol{C}})$



Something weird happens!

- The Bell's Inequality is violated
  - But we know this!
    - Because light that manages to pass through the first polarizer is now polarized in the direction of the polarizer (i.e., x is not longer x – rather it changes to a different y)
      - Measurement of the polarization state of the photon changes it

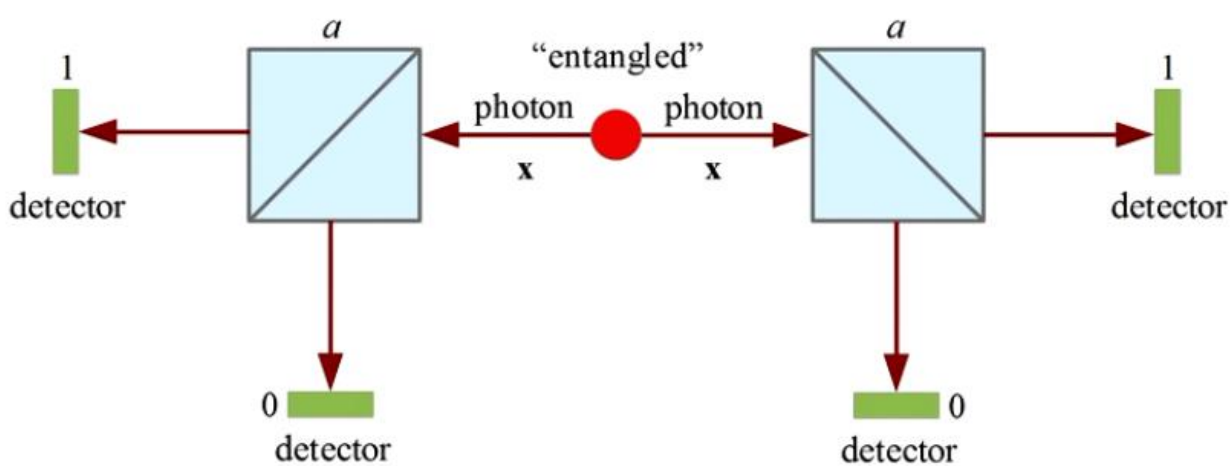
Entangled Photons



- How to generate entangled photons
  - Take an electron and a positron, when they annihilate, two photons of equal (but random) polarization are emitted
  - Take a high frequency laser and pass it through a BBO crystal which generates two lower energy photons of equal (but again random) polarization
  - One can also generate entangled electrons of opposite spins
- How to ensure if two particles are entangled?
  - Passing them through the same type of polarizer would always result in the same output
    - Either both will pass or both will fail

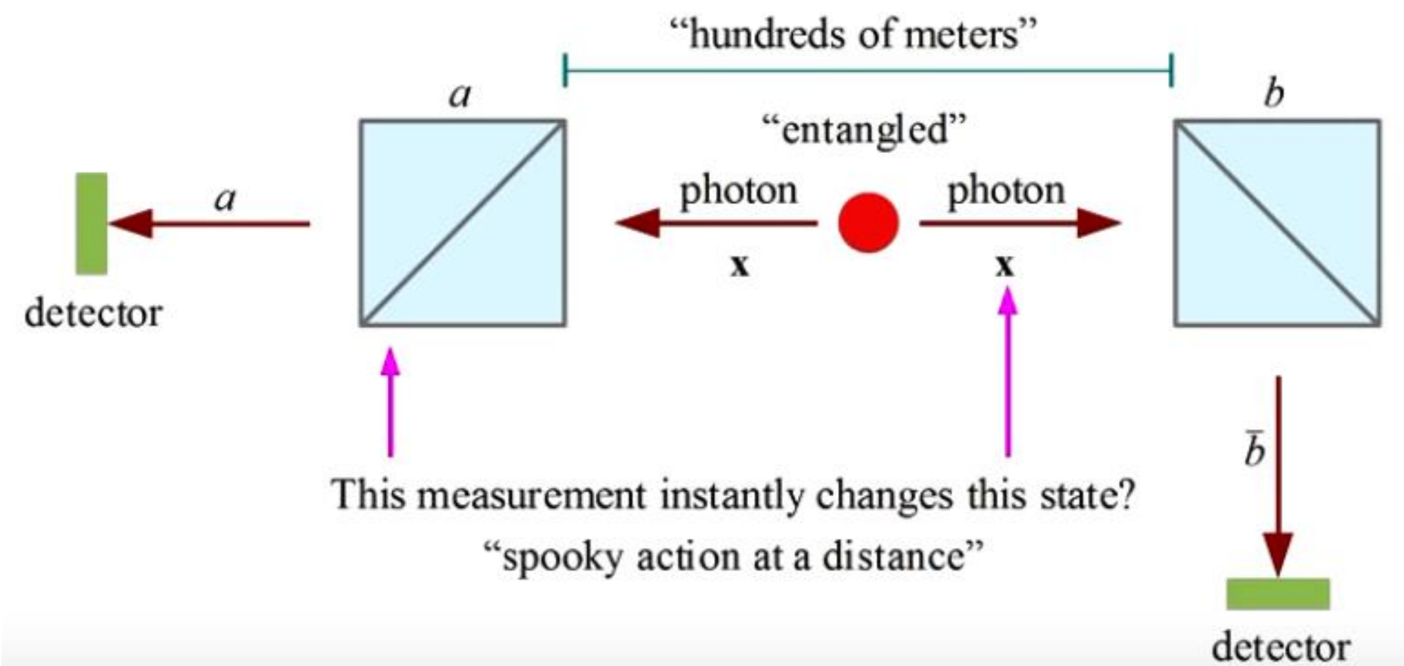
Testing for entangled photons

- Passing two entangled photons through the same polarizer will result in the same detection pattern (random but same)
  - So reliable that this is used for generating shared keys for quantum cryptography (along with other applications)



Testing Bell's Inequality

- Generate Entangled Photons
- Generate pairs of (different) polarizers
- Calculate probabilities to test $P(A\bar{B}) + P(B\bar{C}) \geq P(A\bar{C})$



Observations

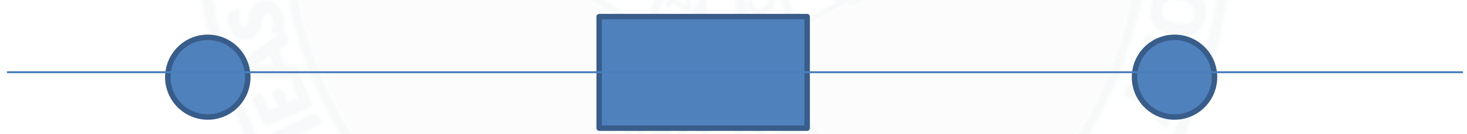
- The experimental probabilities violate the Bell's Inequality
  - Thus:
    - It is impossible to measure the polarization of an entangled pair of photons along two different axes of polarization
      - EPR Hypothesized that measurement of polarization along one axis and the other's along the other axis is possible
    - Measurement of polarization of one photon, instantly affects the polarization of the other
    - This violates locality
    - Spooky action at a distance is real!

So What was the flaw with EPR?

- The system is not the single photon
- But the pair of photons will be regarded as a single system
  - Described by a single wave function
  - Affects are non-local in this case
- However, if we consider that the system is non-local and the two entangled pairs are described by the same wave function, this easily explains how a measurement on one particle instantly affects the other!

Alternatively

- It is not possible to measure the position of one particle without affecting the momentum of both the particles and vice-versa
  - Nonlocality is real!



- The experimental violation of Bell's inequalities confirms that a pair of entangled photons separated by hundreds of meters must be considered a single non-separable object
  - It is impossible to assign local physical reality to each photon
    - Alain Aspect "Bell's inequality test: more ideal than ever", Nature (398, 1999)

"Challenging local realism with human choices" (2018)

- A Bell test is a randomized trial that compares experimental observations against the philosophical worldview of local realism[1], in which the properties of the physical world are independent of our observation of them and no signal travels faster than light. A Bell test requires spatially distributed entanglement, fast and high-efficiency detection and unpredictable measurement settings[2,3]. Although technology can satisfy the first two of these requirements[4,5,6,7], the use of physical devices to choose settings in a Bell test involves making assumptions about the physics that one aims to test. Bell himself noted this weakness in using physical setting choices and argued that human 'free will' could be used rigorously to ensure unpredictability in Bell tests[8]. Here we report a set of local-realism tests using human choices, which avoids assumptions about predictability in physics. We recruited about 100,000 human participants to play an online video game that incentivizes fast, sustained input of unpredictable selections and illustrates Bell-test methodology[9]. The participants generated 97,347,490 binary choices, which were directed via a scalable web platform to 12 laboratories on five continents, where 13 experiments tested local realism using photons[5,6], single atoms[7], atomic ensembles[10] and superconducting devices[11]. Over a 12-hour period on 30 November 2016, participants worldwide provided a sustained data flow of over 1,000 bits per second to the experiments, which used different human-generated data to choose each measurement setting. The observed correlations strongly contradict local realism and other realistic positions in bipartite and tripartite[12] scenarios. Project outcomes include closing the 'freedom-of-choice loophole' (the possibility that the setting choices are influenced by 'hidden variables' to correlate with the particle properties[13]), the utilization of video-game methods[14] for rapid collection of human-generated randomness, and the use of networking techniques for global participation in experimental science.

https://www.nature.com/articles/s41586-018-0085-3

Other Explanations

- https://www.wired.com/2014/01/bells-theorem/

End of Lecture-1

We want to make a machine that will be proud of us.

- Danny Hillis