



Introduction to Computer Graphics with WebGL

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Lighting and Shading II

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Objectives

- Continue discussion of shading
- Introduce modified Phong model
- Consider computation of required vectors



Ambient Light

- Ambient light is the result of multiple interactions between (large) light sources and the objects in the environment
- Amount and color depend on both the color of the light(s) and the material properties of the object
- Add $k_a I_a$ to diffuse and specular terms

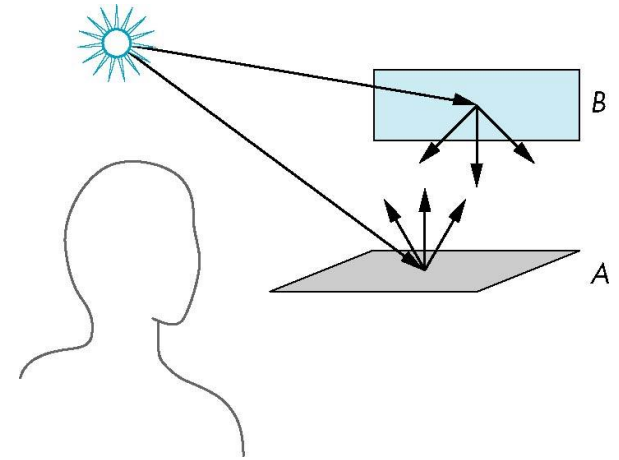
reflection coef

intensity of ambient light



Distance Terms

- The light from a point source that reaches a surface is inversely proportional to the square of the distance between them
- We can add a factor of the form $1/(a + bd + cd^2)$ to the diffuse and specular terms
- The constant and linear terms soften the effect of the point source





Light Sources

- In the Phong Model, we add the results from each light source
- Each light source has separate diffuse, specular, and ambient terms to allow for maximum flexibility even though this form does not have a physical justification
- Separate red, green and blue components
- Hence, 9 coefficients for each point source
 - $I_{dr}, I_{dg}, I_{db}, I_{sr}, I_{sg}, I_{sb}, I_{ar}, I_{ag}, I_{ab}$



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Material Properties

- Material properties match light source properties
 - Nine absorption coefficients
 - k_{dr} , k_{dg} , k_{db} , k_{sr} , k_{sg} , k_{sb} , k_{ar} , k_{ag} , k_{ab}
 - Shininess coefficient α

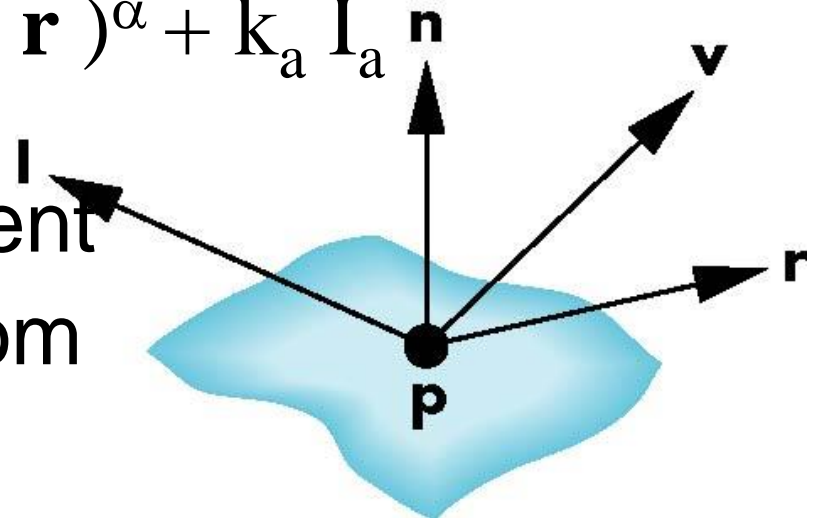


Adding up the Components

For each light source and each color component, the Phong model can be written (without the distance terms) as

$$I = k_d I_d \mathbf{l} \cdot \mathbf{n} + k_s I_s (\mathbf{v} \cdot \mathbf{r})^\alpha + k_a I_a$$

For each color component we add contributions from all sources





Modified Phong Model

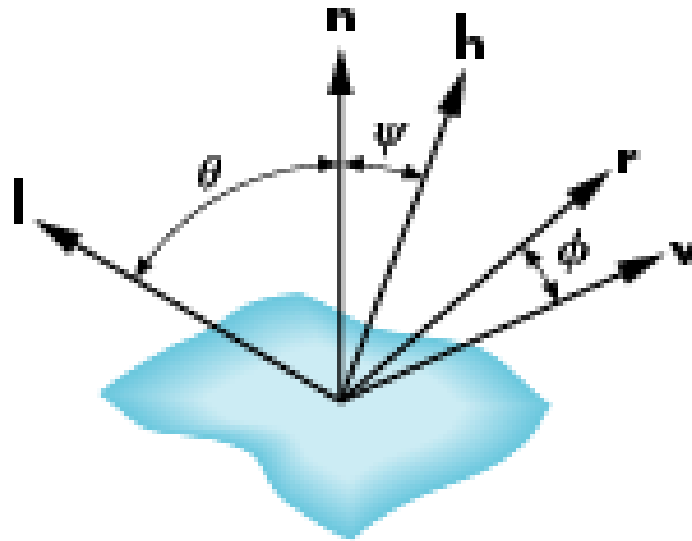
- The specular term in the Phong model is problematic because it requires the calculation of a new reflection vector and view vector for each vertex
- Blinn suggested an approximation using the halfway vector that is more efficient



The Halfway Vector

- **\mathbf{h}** is normalized vector halfway between **\mathbf{l}** and **\mathbf{v}**

$$\mathbf{h} = (\mathbf{l} + \mathbf{v}) / |\mathbf{l} + \mathbf{v}|$$





Using the halfway vector

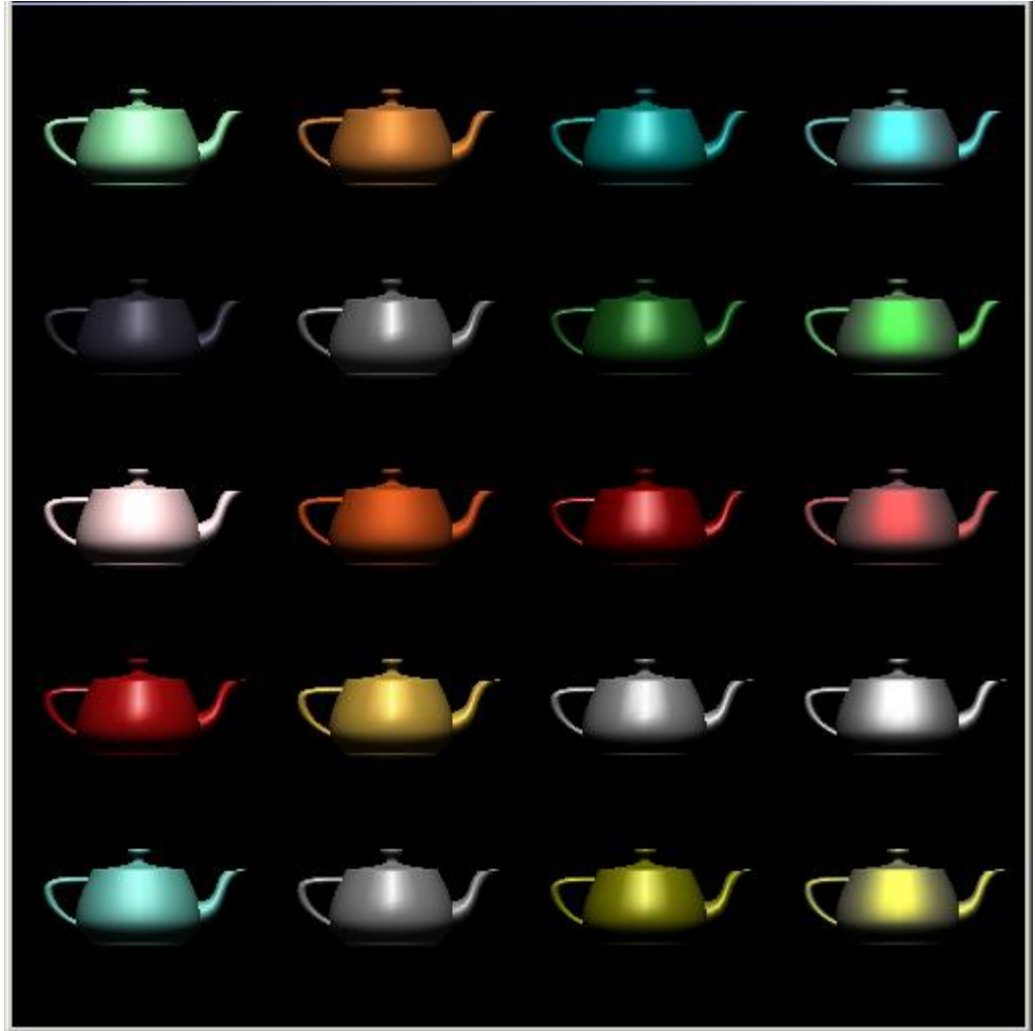
- Replace $(\mathbf{v} \cdot \mathbf{r})^\alpha$ by $(\mathbf{n} \cdot \mathbf{h})^\beta$
- β is chosen to match shininess
- Note that halfway angle is half of angle between \mathbf{r} and \mathbf{v} if vectors are coplanar
- Resulting model is known as the modified Phong or Phong-Blinn lighting model
 - Specified in OpenGL standard



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Example

Only differences in these teapots are the parameters in the modified Phong model





Computation of Vectors

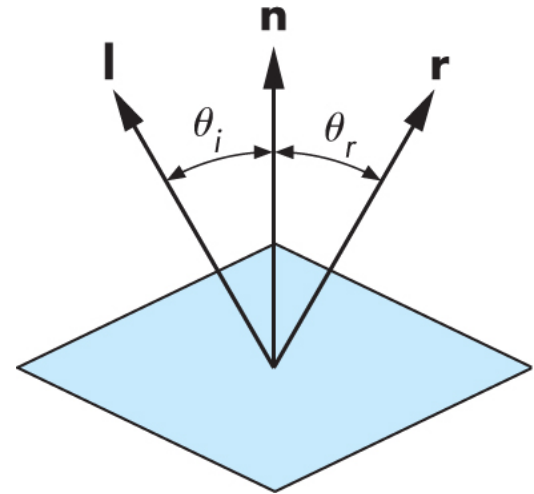
- \mathbf{l} and \mathbf{v} are specified by the application
- Can compute \mathbf{r} from \mathbf{l} and \mathbf{n}
- Problem is determining \mathbf{n}
- For simple surfaces \mathbf{n} can be determined but how we determine \mathbf{n} differs depending on underlying representation of surface
- OpenGL leaves determination of normal to application
 - Exception for GLU quadrics and Bezier surfaces was deprecated



Computing Reflection Direction

- Angle of incidence = angle of reflection
- Normal, light direction and reflection direction are coplaner
- Want all three to be unit length

$$r = 2(l \bullet n)n - l$$

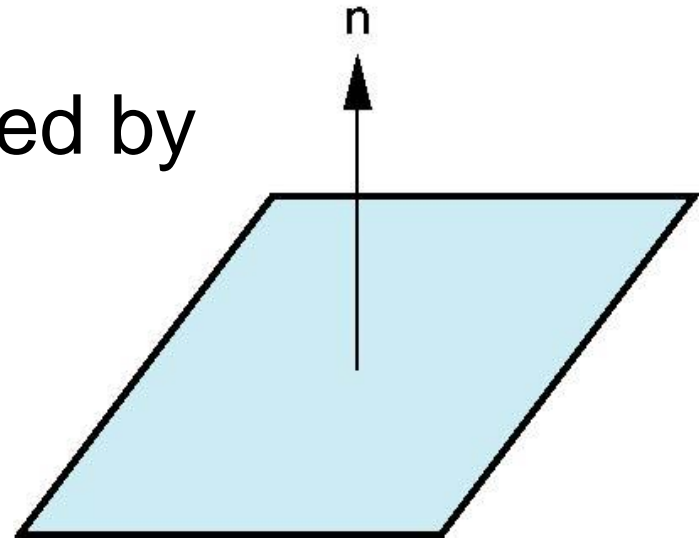




Plane Normals

- Equation of plane: $ax+by+cz+d = 0$
- From Chapter 4 we know that plane is determined by three points p_0, p_2, p_3 or normal \mathbf{n} and p_0
- Normal can be obtained by

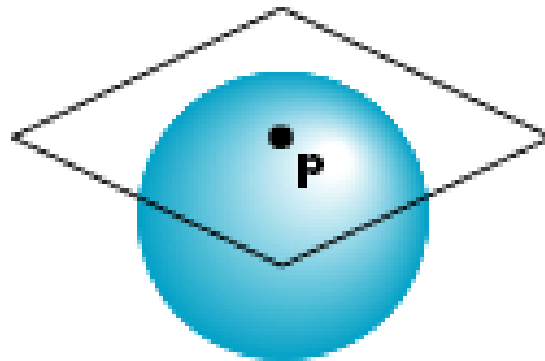
$$\mathbf{n} = (p_2 - p_0) \times (p_1 - p_0)$$





Normal to Sphere

- Implicit function $f(x,y,z)=0$
- Normal given by gradient
- Sphere $f(\mathbf{p})=\mathbf{p}\cdot\mathbf{p}-1$
- $\mathbf{n} = [\partial f/\partial x, \partial f/\partial y, \partial f/\partial z]^T = \mathbf{p}$





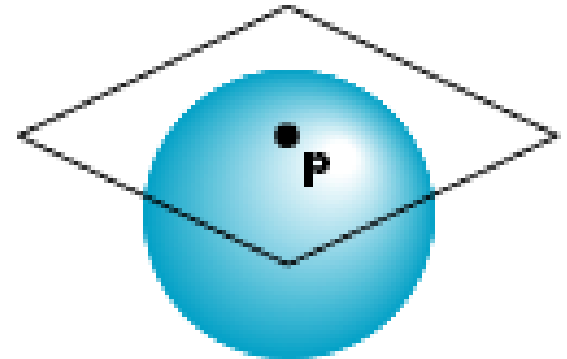
Parametric Form

- For sphere

$$x = x(u, v) = \cos u \sin v$$

$$y = y(u, v) = \cos u \cos v$$

$$z = z(u, v) = \sin u$$



- Tangent plane determined by vectors

$$\frac{\partial \mathbf{p}}{\partial u} = [\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}]^T$$

$$\frac{\partial \mathbf{p}}{\partial v} = [\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}]^T$$

- Normal given by cross product

$$\mathbf{n} = \frac{\partial \mathbf{p}}{\partial u} \times \frac{\partial \mathbf{p}}{\partial v}$$



General Case

- We can compute parametric normals for other simple cases
 - Quadrics
 - Parametric polynomial surfaces
 - Bezier surface patches (Chapter 11)



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Lighting and Shading in WebGL

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Objectives

- Introduce the WebGL shading methods
 - Light and material functions on MV.js
 - per vertex vs per fragment shading
 - Where to carry out



WebGL lighting

- Need
 - Normals
 - Material properties
 - Lights
- State-based shading functions have been deprecated (`glNormal`, `glMaterial`, `glLight`)
- Compute in application or in shaders



Normalization

- Cosine terms in lighting calculations can be computed using dot product
- Unit length vectors simplify calculation
- Usually we want to set the magnitudes to have unit length but
 - Length can be affected by transformations
 - Note that scaling does not preserved length
- GLSL has a normalization function



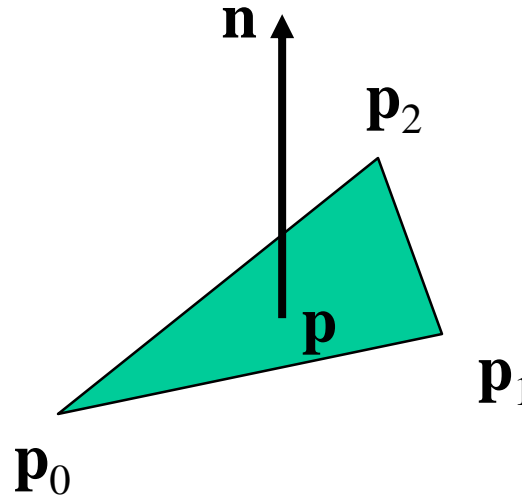
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Normal for Triangle

plane $\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0) = 0$

$$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_0) \times (\mathbf{p}_1 - \mathbf{p}_0)$$

normalize $\mathbf{n} \leftarrow \mathbf{n} / |\mathbf{n}|$



Note that right-hand rule determines outward face

Specifying a Point Light Source

- For each light source, we can set an RGBA for the diffuse, specular, and ambient components, and for the position

```
var diffuse0 = vec4(1.0, 0.0, 0.0, 1.0);  
var ambient0 = vec4(1.0, 0.0, 0.0, 1.0);  
var specular0 = vec4(1.0, 0.0, 0.0, 1.0);  
var light0_pos = vec4(1.0, 2.0, 3.0, 1.0);
```



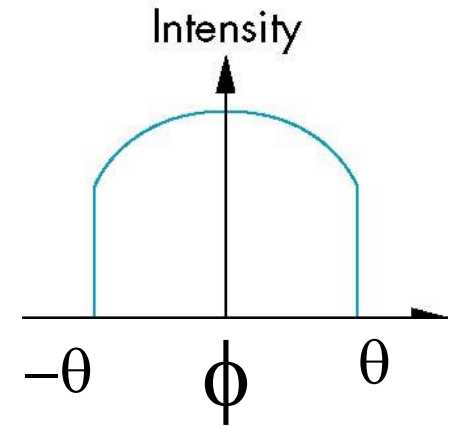
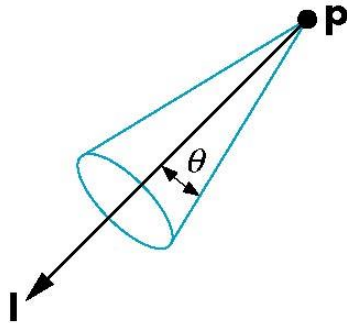
Distance and Direction

- The source colors are specified in RGBA
- The position is given in homogeneous coordinates
 - If $w = 1.0$, we are specifying a finite location
 - If $w = 0.0$, we are specifying a parallel source with the given direction vector
- The coefficients in distance terms are usually quadratic ($1/(a+b*d+c*d*d)$) where d is the distance from the point being rendered to the light source



Spotlights

- Derive from point source
 - Direction
 - Cutoff
 - Attenuation Proportional to $\cos^{\alpha}\phi$





Global Ambient Light

- Ambient light depends on color of light sources
 - A red light in a white room will cause a red ambient term that disappears when the light is turned off
- A global ambient term that is often helpful for testing



Moving Light Sources

- Light sources are geometric objects whose positions or directions are affected by the model-view matrix
- Depending on where we place the position (direction) setting function, we can
 - Move the light source(s) with the object(s)
 - Fix the object(s) and move the light source(s)
 - Fix the light source(s) and move the object(s)
 - Move the light source(s) and object(s) independently



Light Properties

```
var lightPosition = vec4(1.0, 1.0, 1.0, 0.0 );  
var lightAmbient = vec4(0.2, 0.2, 0.2, 1.0 );  
var lightDiffuse = vec4( 1.0, 1.0, 1.0, 1.0 );  
var lightSpecular = vec4( 1.0, 1.0, 1.0, 1.0 );
```



Material Properties

- Material properties should match the terms in the light model
- Reflectivities
- w component gives opacity

```
var materialAmbient = vec4( 1.0, 0.0, 1.0, 1.0 );  
var materialDiffuse = vec4( 1.0, 0.8, 0.0, 1.0);  
var materialSpecular = vec4( 1.0, 0.8, 0.0, 1.0 );  
var materialShininess = 100.0;
```



Using MV.js for Products

```
var ambientProduct = mult(lightAmbient, materialAmbient);
var diffuseProduct = mult(lightDiffuse, materialDiffuse);
var specularProduct = mult(lightSpecular, materialSpecular);
gl.uniform4fv(gl.getUniformLocation(program,
    "ambientProduct"),    flatten(ambientProduct));
gl.uniform4fv(gl.getUniformLocation(program,
    "diffuseProduct"),    flatten(diffuseProduct) );
gl.uniform4fv(gl.getUniformLocation(program,
    "specularProduct"),    flatten(specularProduct) );
gl.uniform4fv(gl.getUniformLocation(program,
    "lightPosition"),    flatten(lightPosition) );
gl.uniform1f(gl.getUniformLocation(program,
    "shininess"),materialShininess);
```




Adding Normals for Quads

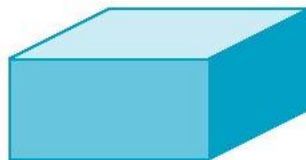
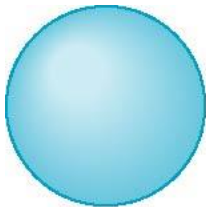
```
function quad(a, b, c, d) {  
    var t1 = subtract(vertices[b], vertices[a]);  
    var t2 = subtract(vertices[c], vertices[b]);  
    var normal = cross(t1, t2);  
    var normal = vec3(normal);  
    normal = normalize(normal);  
  
    pointsArray.push(vertices[a]);  
    normalsArray.push(normal);  
    .  
    .  
    .  
}
```



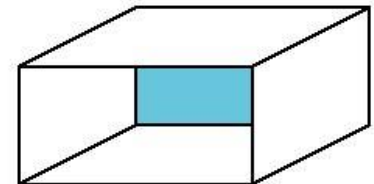
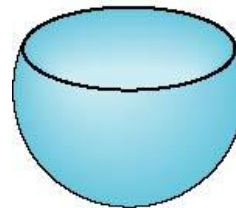
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Front and Back Faces

- Every face has a front and back
- For many objects, we never see the back face so we don't care how or if it's rendered
- If it matters, we can handle in shader



back faces not visible



back faces visible



Emissive Term

- We can simulate a light source in WebGL by giving a material an emissive component
- This component is unaffected by any sources or transformations



Transparency

- Material properties are specified as RGBA values
- The A value can be used to make the surface translucent
- The default is that all surfaces are opaque
- Later we will enable blending and use this feature
- However with the HTML5 canvas, $A < 1$ will mute colors



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Polygonal Shading

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Polygonal Shading

- In per vertex shading, shading calculations are done for each vertex
 - Vertex colors become vertex shades and can be sent to the vertex shader as a vertex attribute
 - Alternately, we can send the parameters to the vertex shader and have it compute the shade
- By default, vertex shades are interpolated across an object if passed to the fragment shader as a varying variable (smooth shading)
- We can also use uniform variables to shade with a single shade (flat shading)



Polygon Normals

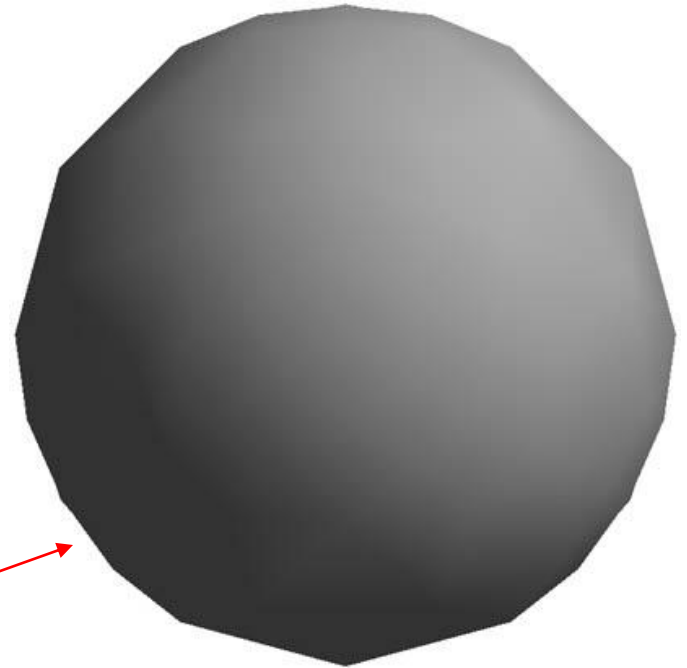
- Triangles have a single normal
 - Shades at the vertices as computed by the modified Phong model can be almost same
 - Identical for a distant viewer (default) or if there is no specular component
- Consider model of sphere
- Want different normals at each vertex even though this concept is not quite correct mathematically





Smooth Shading

- We can set a new normal at each vertex
- Easy for sphere model
 - If centered at origin $\mathbf{n} = \mathbf{p}$
- Now smooth shading works
- Note *silhouette edge*

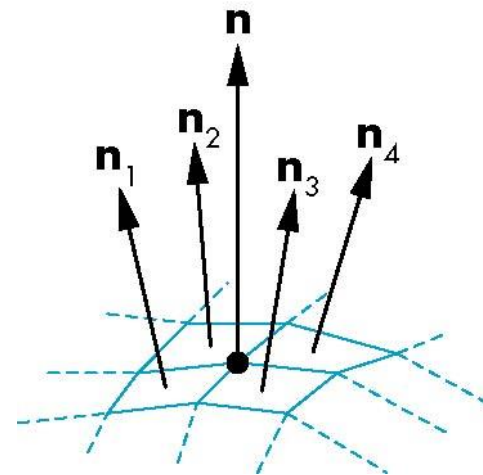




Mesh Shading

- The previous example is not general because we knew the normal at each vertex analytically
- For polygonal models, Gouraud proposed we use the average of the normals around a mesh vertex

$$\mathbf{n} = (\mathbf{n}_1 + \mathbf{n}_2 + \mathbf{n}_3 + \mathbf{n}_4) / |\mathbf{n}_1 + \mathbf{n}_2 + \mathbf{n}_3 + \mathbf{n}_4|$$





Gouraud and Phong Shading

- Gouraud Shading
 - Find average normal at each vertex (vertex normals)
 - Apply modified Phong model at each vertex
 - Interpolate vertex shades across each polygon
- Phong shading
 - Find vertex normals
 - Interpolate vertex normals across edges
 - Interpolate edge normals across polygon
 - Apply modified Phong model at each fragment



Comparison

- If the polygon mesh approximates surfaces with a high curvatures, Phong shading may look smooth while Gouraud shading may show edges
- Phong shading requires much more work than Gouraud shading
 - Until recently not available in real time systems
 - Now can be done using fragment shaders
- Both need data structures to represent meshes so we can obtain vertex normals



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Per Vertex and Per Fragment Shaders

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Vertex Lighting Shaders I

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```
// vertex shader
```

```
attribute vec4 vPosition;  
attribute vec4 vNormal;  
varying vec4 fColor;  
uniform vec4 ambientProduct, diffuseProduct, specularProduct;  
uniform mat4 modelViewMatrix;  
uniform mat4 projectionMatrix;  
uniform vec4 lightPosition;  
uniform float shininess;
```

```
void main()  
{
```



Vertex Lighting Shaders II

```
vec3 pos = -(modelViewMatrix * vPosition).xyz;  
vec3 light = lightPosition.xyz;  
vec3 L = normalize( light - pos );  
vec3 E = normalize( -pos );  
vec3 H = normalize( L + E );
```

```
// Transform vertex normal into eye coordinates
```

```
vec3 N = normalize( (modelViewMatrix*vNormal).xyz);
```

```
// Compute terms in the illumination equation
```




Vertex Lighting Shaders III

```
// Compute terms in the illumination equation
vec4 ambient = AmbientProduct;

float Kd = max( dot(L, N), 0.0 );
vec4 diffuse = Kd*DiffuseProduct;
float Ks = pow( max(dot(N, H), 0.0), Shininess );
vec4 specular = Ks * SpecularProduct;
if( dot(L, N) < 0.0 ) specular = vec4(0.0, 0.0, 0.0, 1.0);
gl_Position = Projection * ModelView * vPosition;

fColor = ambient + diffuse + specular;
fColor.a = 1.0;
}
```



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Vertex Lighting Shaders IV

```
// fragment shader
```

```
precision mediump float;
```

```
varying vec4 fColor;
```

```
void main()
```

```
{
```

```
    gl_FragColor = fColor;
```

```
}
```



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Fragment Lighting Shaders I

```
// vertex shader

attribute vec4 vPosition;
attribute vec4 vNormal;
varying vec3 N, L, E;
uniform mat4 modelViewMatrix;
uniform mat4 projectionMatrix;
uniform vec4 lightPosition;
```



Fragment Lighting Shaders II

```
void main()
{
    vec3 pos = -(modelViewMatrix * vPosition).xyz;
    vec3 light = lightPosition.xyz;
    L = normalize( light - pos );
    E = -pos;
    N = normalize( (modelViewMatrix*vNormal).xyz);
    gl_Position = projectionMatrix * modelViewMatrix * vPosition;
};
```



Fragment Lighting Shaders III

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```
// fragment shader
```

```
precision mediump float;
```

```
uniform vec4 ambientProduct;
```

```
uniform vec4 diffuseProduct;
```

```
uniform vec4 specularProduct;
```

```
uniform float shininess;
```

```
varying vec3 N, L, E;
```

```
void main()
```

```
{
```



Fragment Lighting Shaders IV

```
vec4 fColor;
vec3 H = normalize( L + E );
vec4 ambient = ambientProduct;
float Kd = max( dot(L, N), 0.0 );
vec4 diffuse = Kd*diffuseProduct;
float Ks = pow( max(dot(N, H), 0.0), shininess );
vec4 specular = Ks * specularProduct;
    if( dot(L, N) < 0.0 ) specular = vec4(0.0, 0.0, 0.0, 1.0);
fColor = ambient + diffuse +specular;
fColor.a = 1.0;
gl_FragColor = fColor;
}
```



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Teapot Examples





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Marching Squares

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Objectives

- Nontrivial two-dimensional application
- Important method for
 - Contour plots
 - Implicit function visualization
- Extends to important method for volume visualization
- This lecture is optional but should be interesting to most of you



Displaying Implicit Functions

- Consider the implicit function

$$g(x,y)=0$$

- Given an x , we cannot in general find a corresponding y
- Given an x and a y , we can test if they are on the curve



Height Fields and Contours

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- In many applications, we have the heights given by a function of the form $z=f(x,y)$
- To find all the points that have a given height t , we have to solve the implicit equation $g(x,y)=f(x,y)-t=0$
- Such a function determines the **isocurves** or **contours** of f for the **isovalue** t



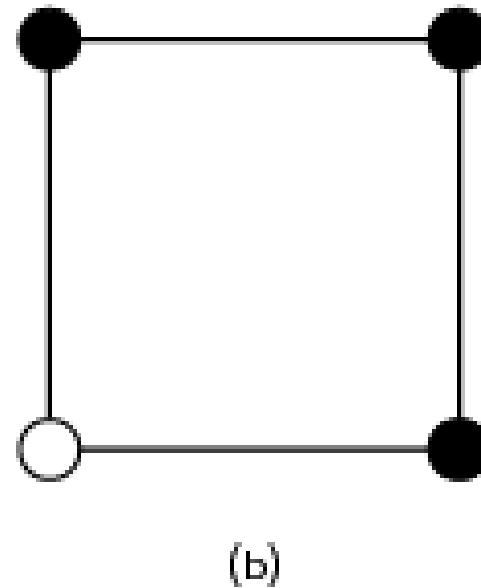
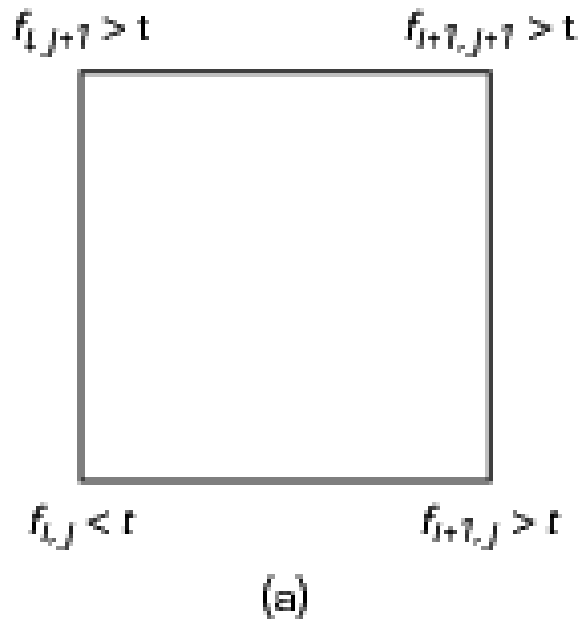
Marching Squares

-
- Displays isocurves or contours for functions $f(x,y) = t$
 - Sample $f(x,y)$ on a regular grid yielding samples $\{f_{ij}(x,y)\}$
 - These samples can be greater than, less than, or equal to t
 - Consider four samples $f_{ij}(x,y)$, $f_{i+1,j}(x,y)$, $f_{i+1,j+1}(x,y)$, $f_{i,j+1}(x,y)$
 - These samples correspond to the corners of a cell
 - Color the corners by whether they exceed or are less than the contour value t



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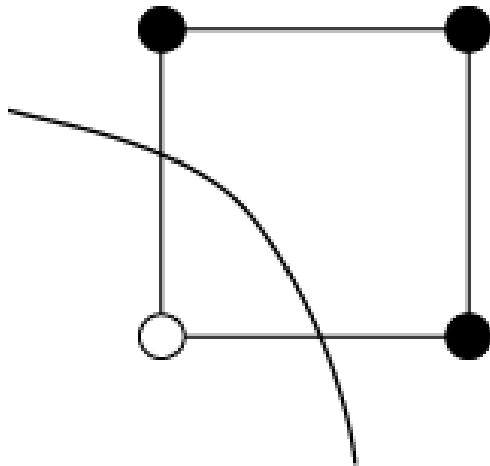
Cells and Coloring



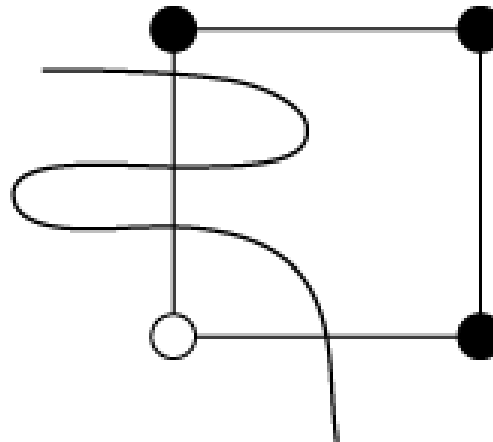


Occam's Razor

- Contour must intersect edge between a black and white vertex an odd number of times
- Pick simplest interpretation: one crossing



(a)

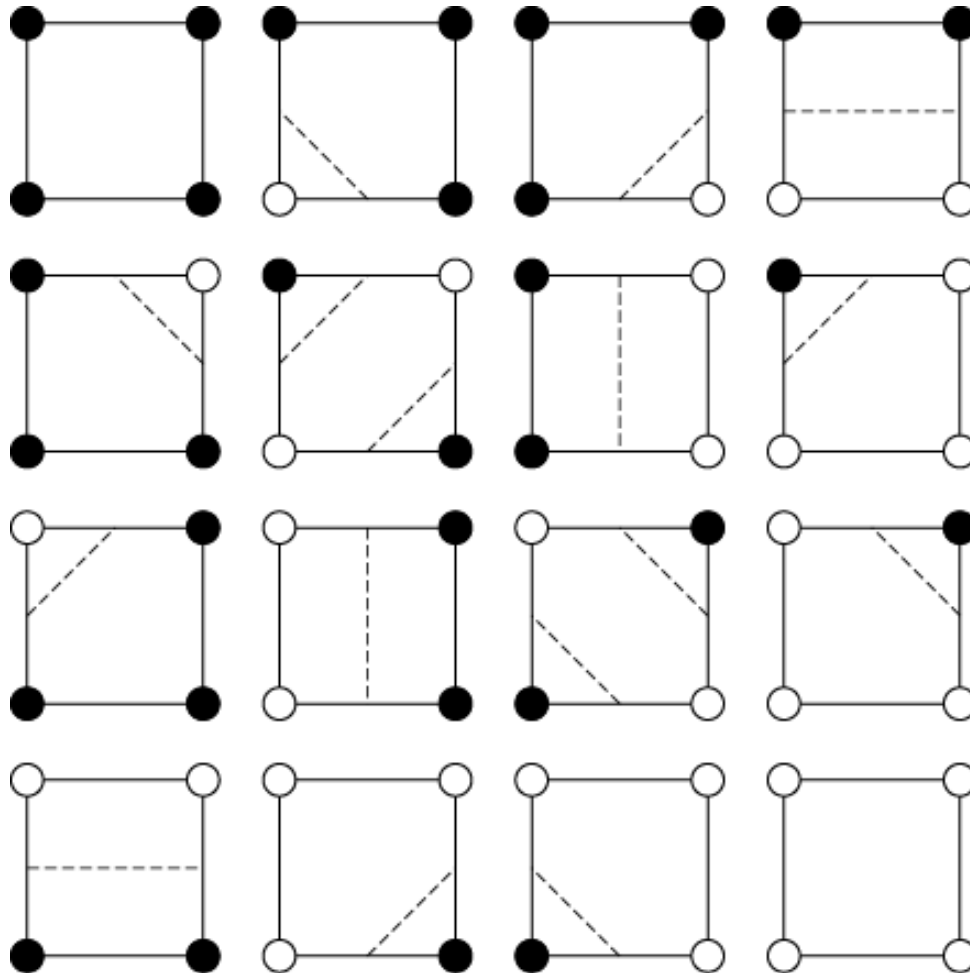


(b)



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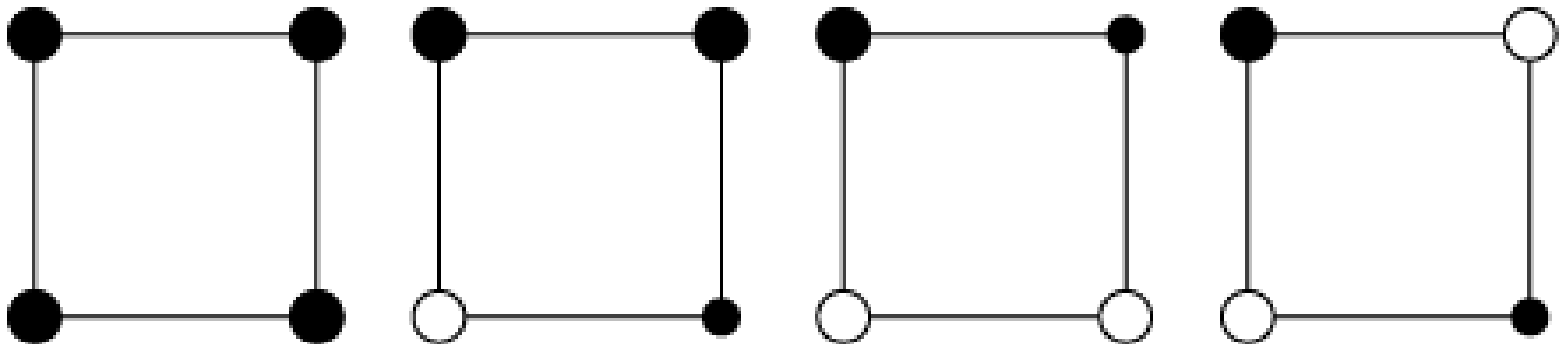
16 Cases





Unique Cases

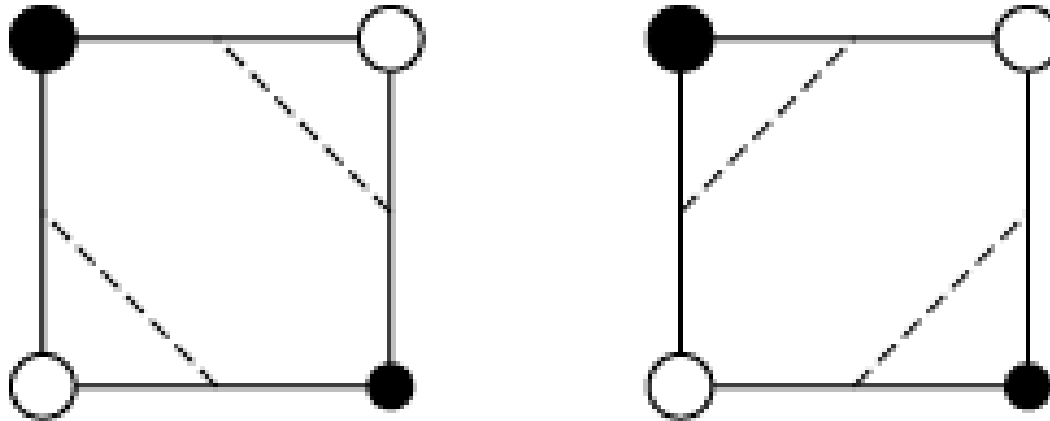
- Taking out rotational and color swapping symmetries leaves four unique cases
- First three have a simple interpretation





Ambiguity Problem

- Diagonally opposite cases have two equally simple possible interpretations

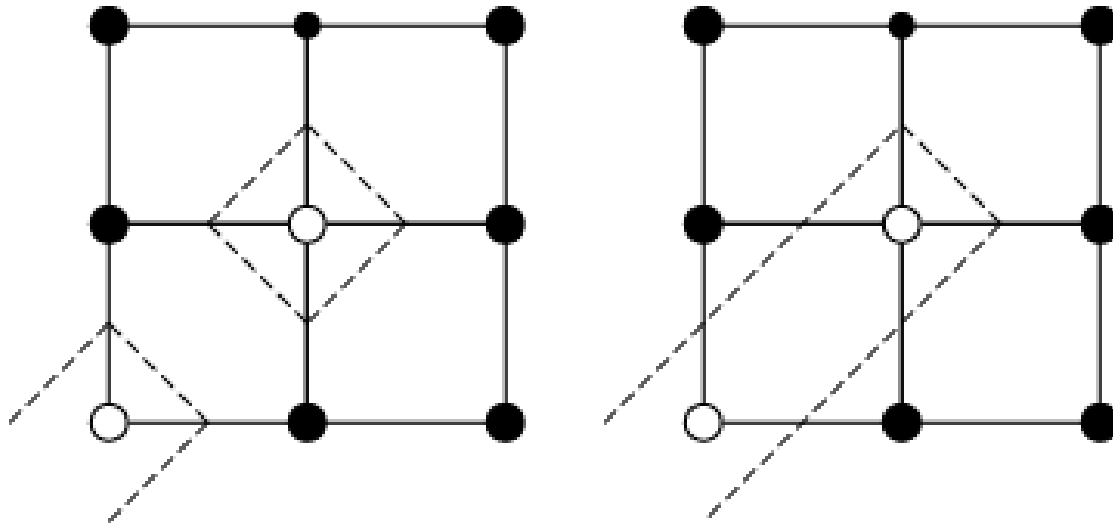




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Ambiguity Example

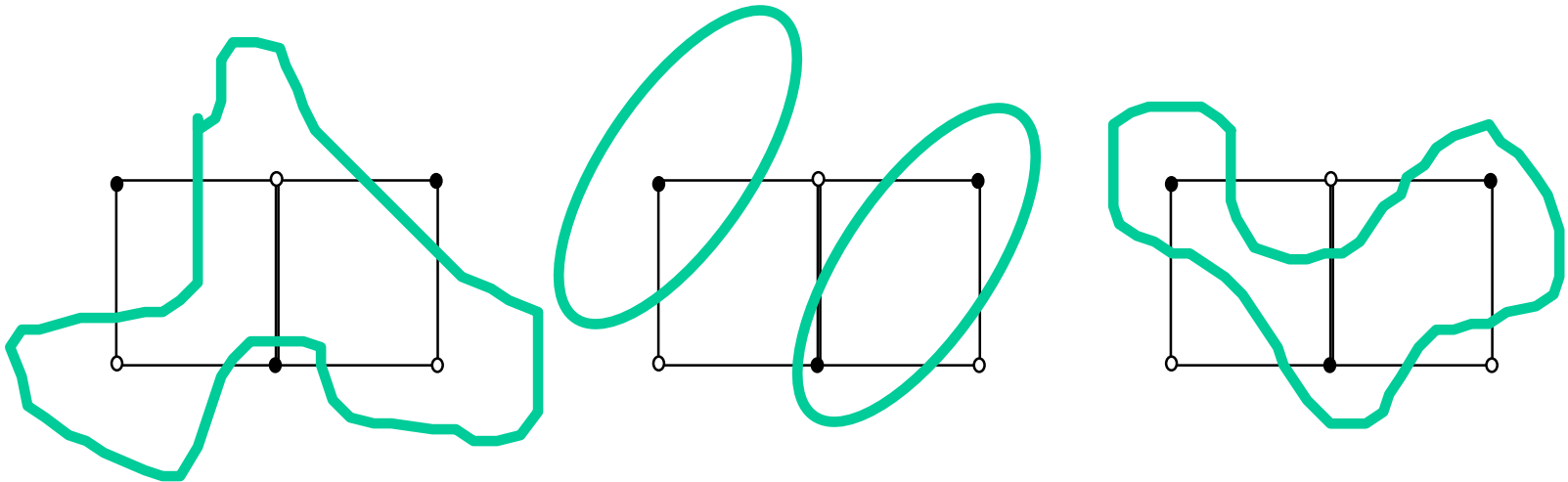
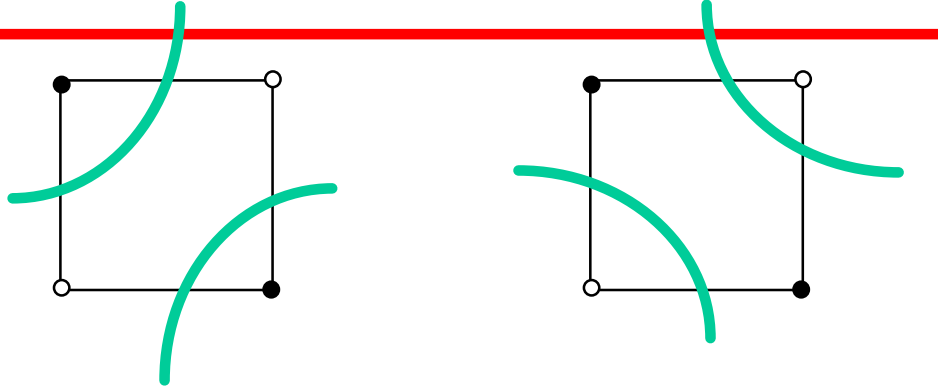
- Two different possibilities below
- More possibilities on next slide





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Ambiguity Problem





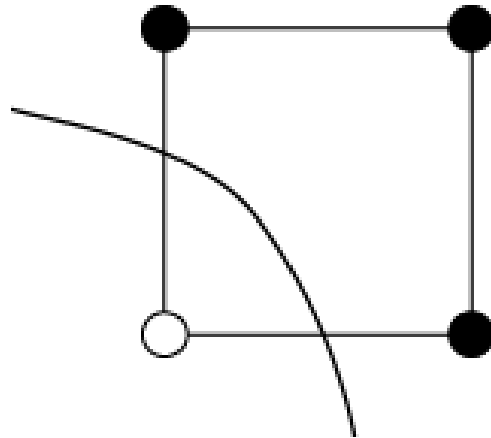
Is Problem Resolvable?

- Problem is a sampling problem
 - Not enough samples to know the local detail
 - No solution in a mathematical sense without extra information
- More of a problem with volume extension (marching cubes) where selecting “wrong” interpretation can leave a hole in a surface
- Multiple methods in literature to give better appearance
 - Supersampling
 - Look at larger area before deciding



Interpolating Edges

- We can compute where contour intersects edge in multiple ways
 - Halfway between vertices
 - Interpolated based on difference between contour value and value at vertices





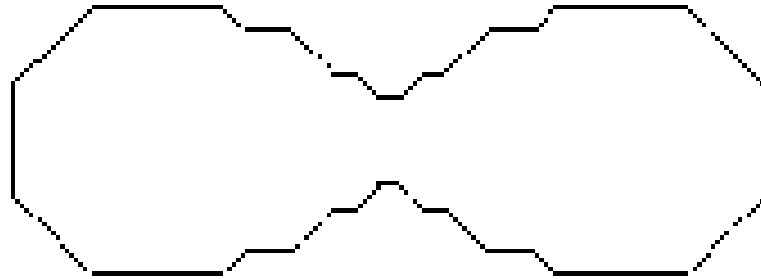
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Example: Oval of Cassini

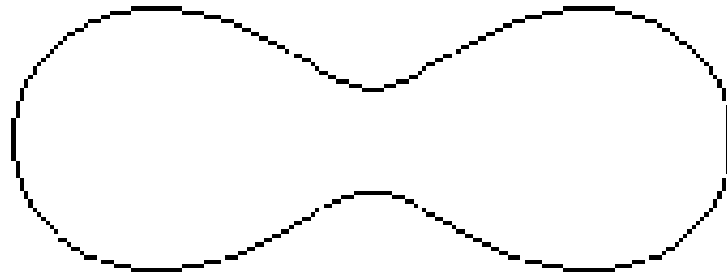
$$f(x,y)=(x^2+y^2+a^2)^2-4a^2x^2-b^4$$

Depending on a and b we can have 0, 1, or 2 curves

midpoint intersections



interpolating intersections

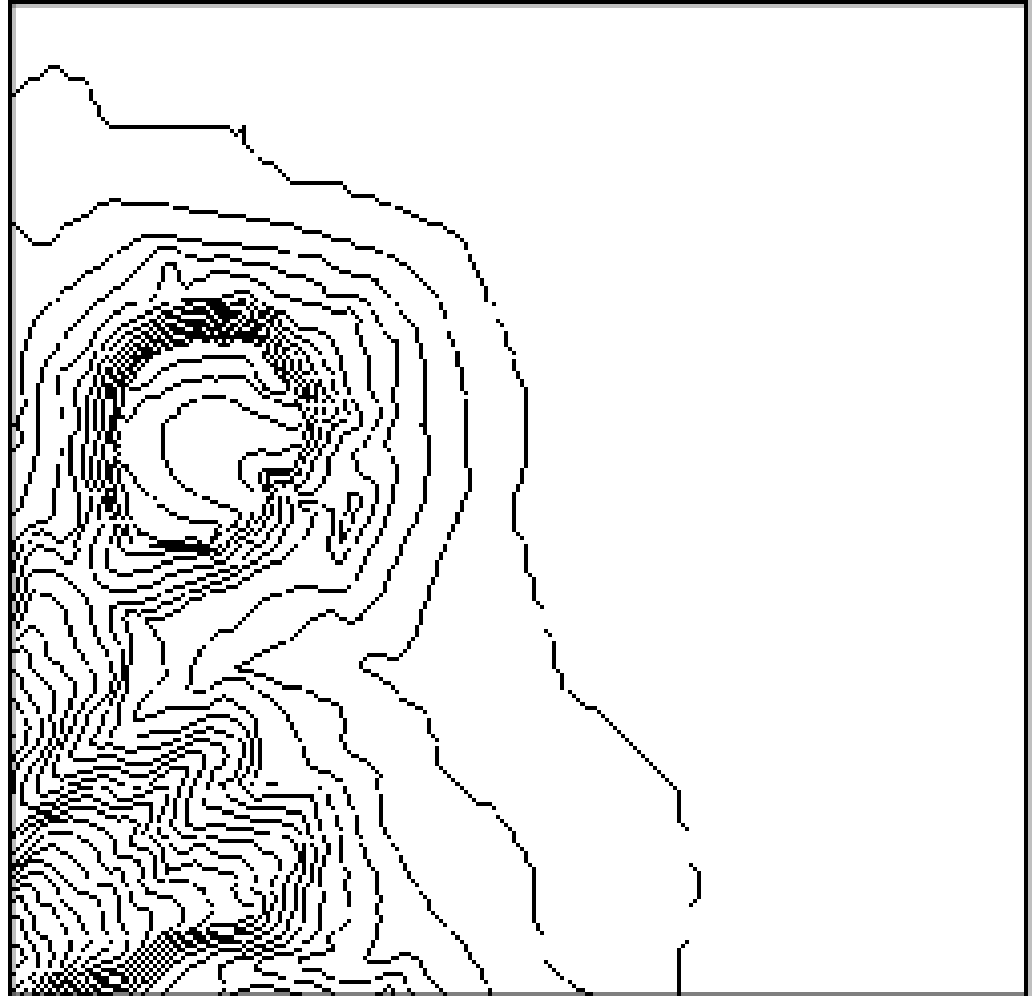




The University of New Mexico

Contour Map

- Diamond Head, Oahu Hawaii
- Shows contours for many contour values





Marching Cubes

- Isosurface: solution of $g(x,y,z)=c$
- Use same argument to derive method but with a cubic cell (8 vertices, 256 colorings)
- Standard method of volume visualization
- Suggested by Lorensen and Kline before marching squares
- Note inherent parallelism of both marching cubes and marching squares