Game Theory I
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Lecture Outline
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SPNE and Agenda Setting Models

1 Subgame Perfect Nash Equilibrium

- Backward Induction (informally)

  Each player deduces what the next player will do, given his own action and chooses his own action so as to maximize his payoff.

- Problems with BI:

  1. non-strict preferences
  2. games with infinite horizons
  3. simultaneous moves

  Because of the limitations of BI, we will need a way to find Nash Equilibria in extensive form games that avoids all the problems that we run into with BI. But for that we need to formally describe the concept of a strategy.

- What is a strategy in the context of extensive form game?

  Definition 1 *A strategy specifies the action for each player of each history where it is his time to move:* $\forall h$, *where* $P(h) = i$, $s_i(h) == a_i(h)$, *where* $a_i(h) \in A(h)$,

  where $A(h)$ *is the set of actions available to* i *at* h

  Example 2 *Tosca or La Boheme?*

- Strategy vs. "plans of action"

  Outcome, $O(s)$ is associated with the terminal history that profile, s, leads to. However, $O(s)$ is associated more with a plan of action than with a strategy profile

- Nash Equilibrium

  Definition 3 *Strategy profile s^* is a Nash Equilibrium in an extensive form game with perfect information iff* $\forall i \in N, u_i(O(s^*)) \geq (O(r_i, s^*_{-i})), \forall r_i \in S_i$

  Problem 4 *How do we find NE in extensive form games?*

  Claim 5 *The set of NE in any extensive form game with perfect information is the same as the set of NE of the same game presented in strategic form*

- Nash Equilibria of extensive form games in strategic form and the problem of incredible threats

  Problem 6 *In the entry game, how does the Challenger know that if he enters, the Incumbent will fight? In fact, the Incumbent is better of Acquiescing should the Challenger enter. The Incumbent can threaten the Challenger he will fight, but is this threat credible?*

  The entry game in extensive form represents the fact that the Incumbent cannot commit to fight in the event that the Challenger enters

  Exercise 7 *163.2 from Osborne*

- Subgame Perfect Equilibrium

  The Nash equilibrium in the context of extended form games does not reflect a robust steady state. The Subgame Perfect Equilibrium, however, does. In SPE strategies must be optimal given what each player knows not only at the start of the game, but after every possible history.

  Definition 8 *Let Γ be an extensive game with perfect information, with a player function, P. For any non-terminal history h, the subgame following h, $\Gamma(h)$ is the extensive game defined by*

  - *N: players in Γ that take actions in $\Gamma(h)$*

- *Player function is the player function from* Γ
- *terminal histories: all action sequences such that* (h, h') *is a terminal history of* Γ
- *preferences: each player prefers* h' *to* h" *iff she prefers* h, h' *to* h, h" *in* Γ

Exercise 9 *What is* $\Gamma(\varnothing)$?

Claim 10 *Every non-terminal history is associated with a subgame. So there are as many subgames as there are non-terminal histories*

A subgame perfect equilibrium is a strategy profile s^*, with the property that in no subgame can any player do better by choosing a strategy different from s_i^*, given that every other player j adheres to s_j^*

Notation 11 $O_h(s)$ *is the outcome in a terminal history generated by profile* s *in subgame* h

Definition 12 s^* *is a subgame perfect equilibrium in an extensive game with perfect information if*

$\forall i \in N, \forall h$ *s.t.* $P(h) = i, \forall r_i \in S_i$, *the terminal history* $O_h(s^*)$ *generated by* s^* *after history* h *is at least as good according to* $i's$ *preferences as the terminal history* $O_h(r_i, s^*_{-i})$ *generated by the strategy profile* (r_i, s^*_{-i}), *i.e.,:*

$\forall i \in N, u_i(O_h(s^*)) \geq u_i(O_h(r_i, s^*_{-i}))$, *for every* $r_i \in S_i$,

where u_i *is i's utility function*

$O_h(s)$ *is the terminal history consisting of* h, *followed by a sequence of actions generated by profile* s *after* h

Claim 13 *An SPE is a strategy profile that induces an NE in every subgame*

Claim 14 *Since* $O_\varnothing(s) = O(s)$, *by definition, every subgame perfect equilibrium is a Nash Equilibrium in the game itself*

2 The Classic Setter Model - analysis

We assume an issue space, which is a line R or $[0, \infty]$. The "Setter Model" is a sequential game with just two players: the [median] legislator L and the "Setter" or "Proposer" P. Assume both players have Euclidean preferences around their "bliss points" p and l respectively. Assume q - the staus quo is given. In period t_1 P makes a proposal x on the issue space. In period t_2, L accepts x, in which case x becomes the new bill, or rejects x, in which result the status quo q remains in force.

Definition 15 *The setter model is a sequential game* $G =< P, L; S_P S_L; u_P, u_L >$
P is the agenda setter (with an ideal point at p)
L is the legislator (with an ideal point at l);
$A_P = S_P = [0, \infty)$ *is P's action set equal to his strategy space;*
$A_L = \{yes, no\}$*is P's action set*
$S_L = \{f \mid f : [0, \infty) \longrightarrow \{no, yes\}\}$*is L's strategy space;*
S_L *is equivalent to* $\{Y^L | Y^L \subseteq [0, \infty)\}$

$$u_i(x, Y^L) = \begin{cases} -|x - b_i| & \textit{if } x \in Y^L \\ -|q - b_i| & \textit{if} x \notin Y^L \end{cases}$$

where $i = P, L$ and $b_i$defines i's "bliss point "
The solution concept is Subgame Perfect Equilbrium.

The given points p, l, q may be ordered in six different ways; adding to this the equality cases, would give a total of nine cases, but considering that the ordering $p > l > q$ will give symmetric results with $q > l > p$ we really have "only" six cases to consider:

- $p = l$
- $p = q$
- $p = l$
- $p < q < l$
- $q < p < l$
- $q < l < p$ (the most interesting and original formulation)

  Note: these cases are not always disjoint

  Let's assume $0 = q \leq l$.

Normally, to find a best reply we would assign to every proposal x of P an optimal action of L, but instead, we can just state the necessary condition that Y^L has to satisfy in order to be in SPE

$$Y^{L^*} = \arg\max_{Y^L} u_L(x, Y^L) = [q, 2l - q]$$

ad 1) $l = q$

$Y^L * = \arg\max_{Y^L} u_L(x, Y^L) = \{q\}$

Just notice, that

$$u_L = \begin{cases} -|x - l| & \text{if } x \in Y^L \\ 0 & \text{if } xY^L \end{cases}$$

ad 2) $p = q$

$$BR_P(Y^L *) = \arg\max_x u_P(x, Y^L) = q$$

ad 3) $p = l$

$$BR_P(Y^{L^*}) = \arg\max_x u_P(x, Y^L) = l$$

ad 4) $p < q < l$

$$BR_A(Y^{L^*}) = \arg\max_x u_P(x, Y^L) = q$$

ad 5) $q < p < l$

$$BR_P(Y^{L^*}) = \arg\max_x u_P(x, Y^L) = p$$

ad 6) $q < l < p$

There are really two cases:
a) $p < 2l - q$
b) $p \geq 2l - q$ or p does not exist (the Setter always prefers more than x)

ad a)

$$BR_P(Y^{L^*}) = \arg\max_x u_P(x, Y^L) = p$$

ad b)

$$BR_A(Y^{L^*}) = \arg\max_x u_P(x, Y^L) = 2l - q$$

Question 1:: If you are the setter in case 6, what is your favorite position of l given:

q..p..........................

Question 2: What would happen if NE were the solution concept?
Provide an example of NE that is not SPNE

3 Relaxing assumptions of the Classic Model

1. suppose that the location of the status quo is determined endogenously
   [sketch graph with $l < p, p < l$ of SPE outcome as function of sq]
2. Suppose an additional player gets to choose the status quo?
   Given $l < p$, where would you choose the status quo if:
   -your bliss point b=0
   -your bliss point b=1?
3. What if you are uncertain about the preferences of the median?
4. What if you are also uncertain about the preferences of the proposer?