3.1 # 4, 8, 11, 13, 20, 21, 31, 32, 37, 39

4.) Compute the determinant using a cofactor expansion across the first row. Also compute it by a cofactor expansion down the second column.

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$$

8.) Compute the determinant using a cofactor expansion across the first row.

$$\begin{vmatrix} 8 & 1 & 6 \\ 4 & 0 & 3 \\ 3 & -2 & 5 \end{vmatrix}$$

11.) Compute the determinant by cofactor expansion. At each step choose a row or column that involves the least computation.

$$\begin{vmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

3.1

13.) $\begin{vmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{vmatrix}$

20.) State the row operation and describe how it affects the determinant.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a & b \\ kc & kd \end{bmatrix}$

21.) $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 5+3k & 6+4k \end{bmatrix}$

3.1 continued

31.) What is the determinant of an elementary row replacement matrix?

32.) What is the determinant of an elementary scaling matrix with k on the diagonal?

37.) Let $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$. Write $5A$. Is $\det 5A = 5 \det A$?

39.) True/False (A is $n \times n$ matrix)

a.) An $n \times n$ ~~determinant~~ determinant is defined by determinants of $(n-1)\times(n-1)$ submatrices.

b.) The (i,j)-cofactor of a matrix A is the matrix A_{ij} obtained by deleting from A its ith row and jth column.

3.2 # 2, 3, 8, 10, 16, 17, 20, 26, 27, 32, 34, 40

2.) The equation illustrates a property of determinants. State the property.

$$\begin{vmatrix} 2 & -6 & 4 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 2 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix}$$

3.) $$\begin{vmatrix} 1 & 3 & -4 \\ 2 & 0 & -3 \\ 5 & -4 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -4 \\ 0 & -6 & 5 \\ 5 & -4 & 7 \end{vmatrix}$$

8.) Find the determinant by row reduction to echelon form.

$$\begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix}$$

10.) $$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix}$$

16.) Find the determinant where $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$.

$$\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix}$$

3.2

17.) $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$

20.) $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix}$

26.) Use determinants to decide if the set of vectors is linearly independent.

$\begin{bmatrix} 3 \\ 5 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \end{bmatrix}$

27.) True/False (A, B are $n \times n$ matrices)

a.) A row replacement operation does not affect the determinant of a matrix.

b.) The determinant of A is the product of pivots in any echelon form U of A, multiplied by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U.

c.) If the columns of A are linearly dependent, then $\det A = 0$.

d.) $\det(A+B) = \det A + \det B$.

3.2 Continued

32.) Find a formula for $\det(rA)$ when A is an $n \times n$ matrix.

34.) Let A and P be square matrices, with P invertible. Show that $\det(PAP^{-1}) = \det A$.

40.) Let A and B be 4×4 matrices, with $\det A = -1$ and $\det B = 2$. Compute:

a.) $\det AB$ b.) $\det B^5$ c.) $\det 2A$ d.) $\det A^T A$ e.) $\det B^{-1}AB$

3.3 # 4, 5, 6, 22, 23, 26, 29, 30

4.) Use cramer's rule to compute the solution.

$$-5x_1 + 3x_2 = 9$$
$$3x_1 - x_2 = -5$$

5.)
$$2x_1 + x_2 = 7$$
$$-3x_1 + x_3 = -8$$
$$x_2 + 2x_3 = -3$$

6.)
$$2x_1 + x_2 + x_3 = 4$$
$$-x_1 + \quad +2x_3 = 2$$
$$3x_1 + x_2 + 3x_3 = -2$$

3.3

22.) Find the area of the parallelogram whose vertices are $(0,-2), (6,-1), (-3,1), (3,2)$

23.) Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1,0,-2), (1,2,4)$ and $(7,1,0)$.

26.) Let $T: \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation, and let $\vec{p}$ be a vector and S a set in $\mathbb{R}^m$. Show that the image of $\vec{p}+S$ under T is the translated set $T(\vec{p})+T(S)$ in $\mathbb{R}^n$.

3.3 continued

29.) Find a formula for the area of the triangle whose vertices are $\vec{0}$, $\vec{v}_1$, and $\vec{v}_2$ in $\mathbb{R}^2$.

30.) Let R be the triangle w/ vertices at (x_1, y_1), (x_2, y_2) and (x_3, y_3). Show that $\{\text{area of triangle}\} = \frac{1}{2} \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$

4.1 # 1, 3, 8, 12, 13, 15, 17, 22, 23, 31, 32

1.) Let V be the first quadrant in the xy-plane; that is, let
$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0,\ y \geq 0 \right\}$

a.) If $\vec{u}$ and $\vec{v}$ are in V, is $\vec{u} + \vec{v}$ in V? Why

b.) Find a specific vector $\vec{u}$ in V and a specific scalar c such that $c\vec{u}$ is not in V. (This is enough to show V is not a vector space)

3.) Let H be the set of points inside and on the unit circle in the xy-plane. That is, let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$. Find a specifice example - two vectors or a vector & a scalar - to show that H is not a subspace of $\mathbb{R}^2$.

8.) Determine if the set of all polynomials in $\mathbb{P}_n$ such that $\vec{p}(0) = 0$ is a subspace of $\mathbb{P}_n$.

4.1

12.) Let W be the set of all vectors of the form $\begin{bmatrix} 2s+4t \\ 2s \\ 2s-3t \\ 5t \end{bmatrix}$.
Show that W is a subspace of $\mathbb{R}^4$.

13.) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

a.) Is $\vec{w}$ in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$? How many vectors are in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?
b.) How many vectors are in Span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?
c.) Is $\vec{w}$ in the subspace Spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$? Why?

15.) Let W be the set of all vectors of the form $\begin{bmatrix} 2a+3b \\ -1 \\ 2a-5b \end{bmatrix}$, where a, b are arbitrary real numbers. Find a set S of vectors that spans W or give an example to show W is not a vector space.

4.1 continued

17.) (Same directions as #15)

$$\begin{bmatrix} 2a-b \\ 3b-c \\ 3c-a \\ 3b \end{bmatrix}$$

22.) Let F be a fixed 3×2 matrix, and let H be the set of all matrices A in $M_{2\times 4}$ with the property that $FA=0$. Determine if H is a subspace of $M_{2\times 4}$.

23.) True/False

a.) If $\vec{f}$ is a function in the vector space V of all real-valued functions on $\mathbb{R}$ and if $\vec{f}(t)=0$ for some t, then $\vec{f}$ is the zero vector in V.

b.) A vector is an arrow in three-dimensional space.

c.) A subset H of a vector space V is a subspace of V if the zero vector is in H.

d.) A subspace is also a vector space.

31.) Let $\vec{u}$ and $\vec{v}$ be vectors in a vector space V, and let H be any subspace of V that contains both $\vec{u}$ & $\vec{v}$. Explain why H also contains Span$\{\vec{u}, \vec{v}\}$. This shows that Span$\{\vec{u}, \vec{v}\}$ is the smallest subspace of V that contains both $\vec{u}$ & $\vec{v}$.

32.) Let H and K be subspaces of a vector space V. The intersection of H and K, written $H \cap K$, is the set of $\vec{v} \in V$ that belong to both H and K. Show that $H \cap K$ is a subspace of V. Then give an example in $\mathbb{R}^2$ to show that the union of two subspaces is not, in general, a subspace.

4.2 # 3, 6, 11, 14, 17, 19, 21, 24, 25, 33, 34

3.) $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix}$ Find an explicit description of Nul A by listing vectors that span the null space.

6.) $A = \begin{bmatrix} 1 & 3 & -4 & -3 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

11.) Either use an appropriate theorem to show that the given set W is a vector space, or find a specific example to the contrary.

$$\left\{ \begin{bmatrix} s-2t \\ 3+3s \\ 3s+t \\ 2s \end{bmatrix} : s, t \text{ real} \right\}$$

14.) (same directions as # 11)

$$\left\{ \begin{bmatrix} -s+3t \\ s-2t \\ 5s-t \end{bmatrix} : s,t \text{ real} \right\}$$

17.) a.) Find k such that Nul A is a subspace of $\mathbb{R}^k$

b.) Find k such that Col A is a subspace of $\mathbb{R}^k$

$$A = \begin{bmatrix} 6 & -4 \\ -3 & 2 \\ -9 & 6 \\ 9 & -6 \end{bmatrix}$$

19.) $A = \begin{bmatrix} 4 & 5 & -2 & 6 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

21.) With A as in #17, find a non-zero vector in Nul A and a nonzero vector in Col A.

4.2 Continued

24.) $A = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$ Determine if $\vec{w}$ is in Col A. Is $\vec{w}$ in Nul A?

25.) True/False A is an $m \times n$ matrix.

a.) The null space of A is the soln set of $A\vec{x} = \vec{0}$.

b.) The null space of an $m \times n$ matrix is in $\mathbb{R}^m$.

c.) The column space of A is the range of the mapping $\vec{x} \mapsto A\vec{x}$.

d.) If the equation $A\vec{x} = \vec{b}$ is consistent, then Col A is $\mathbb{R}^m$.

e.) The Kernel of a linear transformation is a vector space.

f.) Col A is the set of all vectors that can be written as $A\vec{x}$ for some $\vec{x}$.

4.2

33.) Let M_{2x2} be the vector space of all 2x2 matrices and define $T: M_{2x2} \rightarrow M_{2x2}$ by $T(A) = A + A^T$, where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

a.) Show that T is a linear transformation.

b.) Let B be any element of M_{2x2} such that $B^T = B$. Find an A in M_{2x2} such that $T(A) = B$.

c.) Show that the range of T is the set of B in M_{2x2} with the property that $B^T = B$.

d.) Describe the Kernel of T.

4.2 continued

34) Define $T: C[0,1] \to C[0,1]$ as follows: For $\vec{f} \in C[0,1]$, let $T(\vec{f})$ be the antiderivative $\vec{F}$ of $\vec{f}$ such that $\vec{F}(0) = 0$. Show that T is a linear transformation and describe the Kernel of T.

Let $\vec{f}, \vec{g}$ be elements in $C[0,1]$.

$T(\vec{f}+\vec{g})$ is the antiderivative of $\vec{f}+\vec{g}$, from calculus we know this is the antiderivative of $\vec{f}$ plus the antiderivative of $\vec{g}$.

So $T(\vec{f}+\vec{g}) = \vec{F} + \vec{G}$ such that $(\vec{F}+\vec{G})(0) = 0$.

Then $T(\vec{f}+\vec{g}) = T(\vec{f}) + T(\vec{g})$. Similarly,

$T(c\vec{f}) = cT(\vec{f})$.

The Kernel of T is the set of all functions $\vec{f}$ whose antiderivative is zero and $\vec{F}(0) = 0$. Therefore $\vec{f} = \vec{0}$. The Kernel of T is $\{\vec{0}\}$.

4.3 # 3, 4, 8, 10, 14, 15, 21, 23, 24, 29, 30, 31

3.) Determine whether the sets are bases for $\mathbb{R}^3$. Of the sets that are not bases, determine which ones are linearly independent and which ones span $\mathbb{R}^3$.

$$\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

4.) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 4 \end{bmatrix}$

8.) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

10.) Find a basis for the null space of the matrix.

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -8 & 0 & 16 \end{bmatrix}$$

14.) Assume A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 0 & 3 & -6 & 3 \\ 0 & 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.3

15.) Find a basis for the space spanned by the given vectors.

$$\begin{bmatrix}1\\0\\-2\\3\end{bmatrix}, \begin{bmatrix}0\\1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\-2\\-8\\0\end{bmatrix}, \begin{bmatrix}2\\-1\\10\\3\end{bmatrix}, \begin{bmatrix}3\\-1\\-6\\9\end{bmatrix}$$

21.) True / False

a.) A single vector by itself is linearly dependent.

b.) If $H = \text{Span}\{\vec{b}_1, \ldots \vec{b}_p\}$, then $\{\vec{b}_1, \ldots \vec{b}_p\}$ is a basis for H.

c.) The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

d.) A basis is a spanning set that is as large as possible.

e.) In some cases, the linear dependence relations among the columns of a matrix can be affected by certain elementary row operations on the matrix.

23.) Suppose $\mathbb{R}^4 = \text{Span}\{\vec{v}_1, \ldots \vec{v}_4\}$. Explain why $\{\vec{v}_1, \ldots \vec{v}_4\}$ is a basis for $\mathbb{R}^4$.

24.) Let $\mathcal{B} = \{\vec{v}_1, \ldots \vec{v}_n\}$ be a linearly independent set in $\mathbb{R}^n$. Explain why $\mathcal{B}$ must be a basis for $\mathbb{R}^n$.

4.3 continued

29.) Let $S = \{\vec{v}_1, \ldots \vec{v}_K\}$ be a set of K vectors in $\mathbb{R}^n$, with $K < n$. Use a theorem from Section 1.4 to explain why S cannot be a basis for $\mathbb{R}^n$.

30.) Let $S = \{\vec{v}_1, \ldots \vec{v}_K\}$ be a set of K vectors in $\mathbb{R}^n$, with $K > n$. Use a theorem from chapter 1 to explain why S cannot be a basis for $\mathbb{R}^n$.

31.) Let V, W be vector spaces, $T: V \to W$ be a linear transformation and $\{\vec{v}_1, \ldots \vec{v}_p\}$ is a subset of V. Show that if $\{\vec{v}_1, \ldots \vec{v}_p\}$ is linearly dependent in V, then the set of images, $\{T(\vec{v}_1), \ldots T(\vec{v}_p)\}$ is linearly dependent in W.

(This also shows if $\{T(\vec{v}_1), \ldots T(\vec{v}_p)\}$ is linearly independent, then the original set $\{\vec{v}_1, \ldots \vec{v}_p\}$ is linearly independent.

4.4 # 2, 3, 5, 7, 10, 11, 13, 15, 17, 21, 23, 32

2.) Find the vector $\vec{x}$ determined by the given coordinate vector $[\vec{x}]_{\mathcal{B}}$ and the given basis $\mathcal{B}$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\}, \quad [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

3.) $$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} \right\}, \quad [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

5.) Find the coordinate vector $[\vec{x}]_{\mathcal{B}}$ of $\vec{x}$ relative to the given basis $\mathcal{B} = \{\vec{b}_1, \dots \vec{b}_n\}$

$$\vec{b}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

7.) $$\vec{b}_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \quad \vec{b}_2 = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, \quad \vec{b}_3 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$$

10.) Find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^n$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$$

11.) Use an inverse matrix to find $[\vec{x}]_{\mathcal{B}}$ for $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$.

4.4

13.) The set $\mathcal{B} = \{1+t^2, t+t^2, 1+2t+t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\vec{p}(t) = 1+4t+7t^2$ relative to $\mathcal{B}$.

15.) True/False. $\mathcal{B}$ is a basis for a vector space V.

a.) If $\vec{x}$ is in V and if $\mathcal{B}$ contains n vectors, then the $\mathcal{B}$ coordinate vector of $\vec{x}$ is in $\mathbb{R}^n$.

b.) If $P_{\mathcal{B}}$ is the change-of-coordinates matrix, then $[\vec{x}]_{\mathcal{B}} = P_{\mathcal{B}}\vec{x}$ for $\vec{x}$ in V.

c.) The vector spaces $\mathbb{P}_3$ and $\mathbb{R}^3$ are isomorphic.

17.) The vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ span $\mathbb{R}^2$ but do not form a basis. Find two different ways to express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

4.4 continued

21.) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \end{bmatrix} \right\}$. Since the coordinate mapping determined by $\mathcal{B}$ is a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$, this mapping must be implemented by some 2×2 matrix A. Find it. (Hint: Multiplication by A should transform a vector $\vec{x}$ into $[\vec{x}]_{\mathcal{B}}$.)

23.) V is a vector space, $\mathcal{B} = \{\vec{b}_1, \ldots \vec{b}_n\}$ is a basis and $\vec{x} \mapsto [\vec{x}]_{\mathcal{B}}$ is the coordinate mapping. Show that the coordinate mapping is one-to-one. (Hint: Suppose $[\vec{u}]_{\mathcal{B}} = [\vec{w}]_{\mathcal{B}}$ for some $\vec{u}, \vec{w} \in V$ and show that $\vec{u} = \vec{w}$.)

32.) Let $\vec{p}_1(t) = 1 + t^2$, $\vec{p}_2(t) = t - 3t^2$, $\vec{p}_3(t) = 1 + t - 3t^2$

a.) Use coordinate vectors to show that these polynomials form a basis for $\mathbb{P}^2$.

b.) Consider the basis $\mathcal{B} = \{\vec{p}_1, \vec{p}_2, \vec{p}_3\}$ for $\mathbb{P}^2$. Find $\vec{q} \in \mathbb{P}^2$ s.t. $[\vec{q}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.