Math 2210Q-008 - Applied Linear Algebra
Name: Soln

Quiz 7
Fall 2018

Show ALL work to receive full credit.

1. In part (a), if the statement is always true, circle True. If the statement is sometimes false, circle False. In both parts, write a careful and clear **justification** or **counterexample**.

(a) The determinant of $\begin{bmatrix} 4 & 3 & 2 & 1 \\ 7 & 6 & 5 & 0 \\ 9 & 8 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is -40. True False

$$\begin{vmatrix} 4 & 3 & 2 & 1 \\ 7 & 6 & 5 & 0 \\ 9 & 8 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -\left(-\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}\right) = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 40$$

swap

swap

(b) Justify the ***true*** statement: *If A is an $n \times n$ matrix with $\det(A) = 0$, then the transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ by $x \mapsto Ax$ is neither one-to-one nor onto.*

if $\det A = 0 \Rightarrow A$ not invertible by IMT $\Rightarrow$ not one-to-one or onto.

2. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$. Use Cramer's rule to find all solutions to $A\mathbf{x} = \mathbf{b}$.

$\det A = 24$

$$A_1(b) = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\det A_1(b) = 6 \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} = 48$$

$$A_2(b) = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\det A_2(b) = -12$$

$$A_3(b) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\det A_3(b) = 0$$

$$x_i = \frac{\det A_i(b)}{\det A}$$

$$\Rightarrow x_1 = \frac{48}{24} = 2$$

$$x_2 = \frac{-12}{24} = -\frac{1}{2}$$

$$x_3 = \frac{0}{24} = 0$$

Math 2210Q-004 - Applied Linear Algebra
Name:

Quiz 7
Fall 2018

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   (a) The determinant of $\begin{bmatrix} 4 & 3 & 2 & 1 \\ 7 & 6 & 5 & 0 \\ 9 & 8 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is -40. True False

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