

Show ALL work to receive full credit.

1. (3 points each) If the statement is always true, circle True. If the statement is sometimes false, circle False. In each case, write a careful and clear **justification** or **counterexample**.

(a)  $\mathbf{v} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$  is in  $\text{Nul} \left( \begin{bmatrix} 6 & -4 \\ 9 & -5 \end{bmatrix} \right)$ .

True

False

$$\begin{bmatrix} 6 & -4 \\ 9 & -5 \end{bmatrix} \begin{bmatrix} 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 36 - 36 \\ 54 - 45 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \end{bmatrix} \neq \vec{0}$$

(b)  $\mathbf{v} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$  is in  $\text{Col} \left( \begin{bmatrix} 6 & -4 \\ 9 & -5 \end{bmatrix} \right)$ .

True

False

2. (5 points) Show that  $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq y \right\}$  is closed under addition, but is not a subspace of  $\mathbb{R}^2$ .

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+x' \\ y+y' \end{bmatrix} \quad \begin{matrix} x \geq y \\ x' \geq y' \end{matrix}$$

$$\Rightarrow x+x' \geq y+y'$$

so this is still in  $H$

if  $x > y$  and  $c < 0$  then  $cx < cy$

ex.  $c = -1 \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad -1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$   
 $-2 \not\geq -1$

3. (5 points) Show  $H = \left\{ \begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix} : a \text{ is real} \right\}$  is a subspace of  $M_{2 \times 2}$ .

$a = 0$  gives  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix} + \begin{bmatrix} b & 2b \\ 3b & 4b \end{bmatrix} = \begin{bmatrix} a+b & 2(a+b) \\ 3(a+b) & 4(a+b) \end{bmatrix} \text{ in } H$$

$$c \begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix} = \begin{bmatrix} ac & 2(ac) \\ 3(ac) & 4(ac) \end{bmatrix}$$

Another way

$$H = \left\{ \begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix} : a \text{ real} \right\} = \left\{ a \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} : a \text{ real} \right\} = \text{Span} \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right\}$$

Since Span of a vector is a subspace  
this is a subspace.

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