Math 2210Q-008 - Applied Linear Algebra — Quiz 9

Name: Solns — Fall 2018

Show ALL work to receive full credit.

1. In part (a), if the statement is always true, circle True. If the statement is sometimes false, circle False. In both parts, write a careful and clear **justification** or **counterexample**.

   (a) In the standard basis $\mathcal{B}$ for $\mathbb{R}^2$, $[\,\mathbf{x}\,]_{\mathcal{B}} = \mathbf{x}$. **(True)** False

   $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ $\quad \vec{x} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   $\Rightarrow [x]_B = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{x}$

   (b) Provide a **counterexample** for the ***false*** statement:
   A basis is a linearly independent set that is as small as possible.

   If you remove an element from a basis, it stays linearly indep, but no longer spans.

2. Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Let $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$.

   Find $[\,x\,]_{\mathcal{B}}$.

   $\begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 2 & 1 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ cons

   $\Rightarrow \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

   $\Rightarrow [x]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

3. Assume that A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

basis for Col A is {pivot columns of A}

$= \{\vec{a}_1, \vec{a}_3, \vec{a}_5\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 8 \\ 9 \\ 9 \end{bmatrix} \right\}$

Nul $A = \{\vec{x} : A\vec{x} = \vec{0}\} = \{\vec{x} : B\vec{x} = \vec{0}\}$

$x_1 = -2x_2 - 2x_4$
x_2 free
$x_3 = 2x_4$
x_4 free $\quad x_5 = 0$

$\Rightarrow \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ basis

Show ALL work to receive full credit.

1. In part (a), if the statement is always true, circle True. If the statement is sometimes false, circle False. In both parts, write a careful and clear **justification** or **counterexample**.

   (a) In the standard basis $\mathcal{B}$ for $\mathbb{R}^2$, $\left[\,\mathbf{x}\,\right]_{\mathcal{B}} = \mathbf{x}$. True False

   (b) Provide a **counterexample** for the ***false*** statement:
   A basis is a linearly independent set that is as small as possible.

2. Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for Span$\{\mathbf{v}_1, \mathbf{v}_2\}$ where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Let $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$.

   Find $\left[\,x\,\right]_{\mathcal{B}}$.

3. Assume that A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Math 2210Q-008 - Applied Linear Algebra Fall 2018 - 9

This Survey is Anonymous

3-2-1 Survey

3. List (up to) 3 concepts that you learned this week.

2. List (up to) 2 concepts that were not clear.

1. What 1 thing would you like to see as a follow up to this week's lessons?

Math 2210Q-004 - Applied Linear Algebra Fall 2018 - 9

This Survey is Anonymous

3-2-1 Survey

3. List (up to) 3 concepts that you learned this week.

2. List (up to) 2 concepts that were not clear.

1. What 1 thing would you like to see as a follow up to this week's lessons?