University of Connecticut
Department of Mathematics

Math 2210 — **Chapter 4 Worksheet** — Spring 2019

Due: Thursday, April 11

Directions: This worksheet covers Chapter 4 from Lay's Linear Algebra book. Turn in all numbered problems below (problems 2 through 11). Use only one side of the paper, remove all fringes, and staple this cover sheet to your submission.

Any of these problems is fair game for the exam.
Ask questions if you're unsure about anything.

Group Member(s) (up to four)

Name: ____________________ Name: ____________________
Name: ____________________ Name: ____________________

Name **and** Section (of the student I will hand this back to):________________

Checklist for formatting:

- ☐ write every group member's name
- ☐ remove all fringes
- ☐ use only one side of the paper
- ☐ label each problem clearly
- ☐ give a clear justification for your answer
- ☐ circle your answer
- ☐ staple this coversheet to your submission

b. If $\{\mathbf{v}_1, \dots, \mathbf{v}_{p-1}\}$ spans V, then S spans V.

c. If $\{\mathbf{v}_1, \dots, \mathbf{v}_{p-1}\}$ is linearly independent, then so is S.

d. If S is linearly independent, then S is a basis for V.

e. If Span $S = V$, then some subset of S is a basis for V.

f. If $\dim V = p$ and Span $S = V$, then S cannot be linearly dependent.

g. A plane in $\mathbb{R}^3$ is a two-dimensional subspace.

h. The nonpivot columns of a matrix are always linearly dependent.

i. Row operations on a matrix A can change the linear dependence relations among the rows of A.

j. Row operations on a matrix can change the null space.

k. The rank of a matrix equals the number of nonzero rows.

l. If an $m \times n$ matrix A is row equivalent to an echelon matrix U and if U has k nonzero rows, then the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ is $m - k$.

m. If B is obtained from a matrix A by several elementary row operations, then rank B = rank A.

n. The nonzero rows of a matrix A form a basis for Row A.

o. If matrices A and B have the same reduced echelon form, then Row A = Row B.

p. If H is a subspace of $\mathbb{R}^3$, then there is a 3×3 matrix A such that H = Col A.

q. If A is $m \times n$ and rank $A = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

r. If A is $m \times n$ and the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then rank $A = m$.

s. A change-of-coordinates matrix is always invertible.

t. If $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ and $\mathcal{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_n\}$ are bases for a vector space V, then the jth column of the change-of-coordinates matrix $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$ is the coordinate vector $[\mathbf{c}_j]_{\mathcal{B}}$.

2. Find a basis for the set of all vectors of the form

$$\begin{bmatrix} a - 2b + 5c \\ 2a + 5b - 8c \\ -a - 4b + 7c \\ 3a + b + c \end{bmatrix}. \quad \text{(Be careful.)}$$

3. Let $\mathbf{u}_1 = \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, and $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$. Find an *implicit* description of W; that is, find a set of one or more homogeneous equations that characterize the points of W. [*Hint:* When is $\mathbf{b}$ in W?]

4. Explain what is wrong with the following discussion: Let $\mathbf{f}(t) = 3 + t$ and $\mathbf{g}(t) = 3t + t^2$, and note that $\mathbf{g}(t) = t\mathbf{f}(t)$. Then $\{\mathbf{f}, \mathbf{g}\}$ is linearly dependent because $\mathbf{g}$ is a multiple of $\mathbf{f}$.

5. Consider the polynomials $\mathbf{p}_1(t) = 1 + t$, $\mathbf{p}_2(t) = 1 - t$, $\mathbf{p}_3(t) = 4$, $\mathbf{p}_4(t) = t + t^2$, and $\mathbf{p}_5(t) = 1 + 2t + t^2$, and let H be the subspace of $\mathbb{P}_5$ spanned by the set $S = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5\}$. Use the method described in the proof of the Spanning Set Theorem (Section 4.3) to produce a basis for H. (Explain how to select appropriate members of S.)

6. Suppose $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$, and $\mathbf{p}_4$ are specific polynomials that span a two-dimensional subspace H of $\mathbb{P}_5$. Describe how one can find a basis for H by examining the four polynomials and making almost no computations.

7. What would you have to know about the solution set of a homogeneous system of 18 linear equations in 20 variables in order to know that every associated nonhomogeneous equation has a solution? Discuss.

8. Let H be an n-dimensional subspace of an n-dimensional vector space V. Explain why $H = V$.

9. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.

a. What is the dimension of the range of T if T is a one-to-one mapping? Explain.

b. What is the dimension of the kernel of T (see Section 4.2) if T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$? Explain.

10. Let S be a maximal linearly independent subset of a vector space V. That is, S has the property that if a vector not in S is adjoined to S, then the new set will no longer be linearly independent. Prove that S must be a basis for V. [*Hint:* What if S were linearly independent but not a basis of V?]

11. Let S be a finite minimal spanning set of a vector space V. That is, S has the property that if a vector is removed from S, then the new set will no longer span V. Prove that S must be a basis for V.