5.1 # 2, 6, 7, 11, 13, 15, 19, 21, 23, 24, 25, 27, 31

2.) Is $\lambda = -3$ an eigenvalue of $\begin{bmatrix} -1 & 4 \\ 6 & 9 \end{bmatrix}$? Why or why not?

6.) Is $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ an eigen vector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix}$? If so, find the eigenvalue.

7.) Is $\lambda = 4$ an eigen value of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If so, find one corresponding eigenvector.

11.) Find a basis for the eigenspace corresponding to the eigenvalues

$A = \begin{bmatrix} 1 & -3 \\ -4 & 5 \end{bmatrix}$, $\lambda = -1, 7$

11.) continued

13.) Same directions as in #11.

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad \lambda = 1, 2, 3$$

5.1 Continued

15.) $A = \begin{bmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{bmatrix}$, $\lambda = -5$

19.) For $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$, find one eigenvalue, with no calculation. Justify youranswer.

21.) True/False. Let A be an nxn matrix.

a.) If $A\vec{x} = \lambda\vec{x}$ for some vector $\vec{x}$, then λ is an eigen value of A.

b.) A matrix A is not invertible iff zero is an eigenvalue of A.

c.) A number c is an eigenvalue of A iff the equation $(A - cI)\vec{x} = \vec{0}$ has a nontrivial soln.

d.) Finding an eigen vector of A may be dificult, but checking whether a given vector is an eigenvector is easy.

e.) To find the eigenvalues of A, reduce A to echelon form.

23.) Explain why a 2x2 matrix can have at most two distinct eigen values. Explain why an nxn matrix can have at most n distinct eigenvalues.

24.) Construct an example of a 2×2 matrix with only one distinct eigenvalue.

25.) Let λ be an eigenvalue of an invertible matrix A. Show λ^{-1} is an eigenvalue of A^{-1}.

27.) Show that λ is an eigenvalue of A if and only if λ is an eigenvalue of A^T.

31.) A is the standard matrix of T. T is the transformation on $\mathbb{R}^2$ that reflects points across some line through the origin. Without writing A, find an eigenvalue of A and describe the eigenspace.

5.2 # 2, 5, 9, 12, 15, 19, 20, 21

2.) Find the characteristic polynomial and the real eigenvalues of the matrix.

$$\begin{bmatrix} -4 & -1 \\ 6 & 1 \end{bmatrix}$$

5.) $\begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$

9.) Find the characteristic polynomial of $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}$.

12.) $\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

15.) List the real eigenvalues repeated according to their multiplicities.

$$\begin{bmatrix} 5 & 5 & 0 & 2 \\ 0 & 2 & -3 & 6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

19.) Let A be an $n \times n$ matrix, and suppose A has n real eigenvalues, $\lambda_1, \lambda_2, \ldots, \lambda_n$, repeated according to multiplicities, so that $\det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\ldots(\lambda_n - \lambda)$. Explain why $\det(A)$ is the product of the n eigenvalues of A.

20.) Use a property of determinants to show that A and A^T have the same characteristic Polynomial.

21.) True/False. A and B are $n \times n$ matrices.

a.) The determinant of A is the product of the diagonal entries in A.

b.) An elementary row operation on A <u>doesn't</u> change the determinant.

c.) $(\det A)(\det B) = \det(AB)$

d.) If $\lambda + 5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of A.

5.3 # 1, 4, 5, 9, 11, 15, 17, 21, 24, 26

1.) Let $A = PDP^{-1}$ and compute A^4. $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

4.) $A = PDP^{-1}$ is given. Compute A^k where k is an arbitrary positive integer.

5.) $A = PDP^{-1}$ is given. Use the diagonalization thm to find the eigenvalues of A and a basis for each eigenspace.

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & 0 \end{bmatrix}$$

9.) Diagonalize the matrix, if possible.

$$\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$$

11.) $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}$

$\lambda = -1, 5$

15.) $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

$\lambda = 0, 1$

17.) $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$

5.3 continued

21.) True/False. A, B, P, D are all $n \times n$ matrices.

a.) A is Diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P.

b.) If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable.

c.) A is diagonalizable iff A has n eigenvalues, counting multiplicities.

d.) If A is diagonalizable, then A is invertible.

24.) A is a 3×3 matrix with two eigenvalues. Each eigenspace is one-dimensional. Is A diagonalizable? Why?

26.) A is a 7×7 matrix with three eigenvalues. One eigenspace is two dimensional, and one of the other eigenspaces is three (3) dimensional. is it possible that A is not diagonalizable? Justify.

5.4 #1, 3, 6, 7, 10, 15, 16, 23, 25

1.) Let $\mathcal{B}=\{\vec{b}_1,\vec{b}_2,\vec{b}_3\}$ and $\mathcal{D}=\{\vec{d}_1,\vec{d}_2\}$ be bases for V and W respectively. Let $T: V\to W$ be a linear transformation s.t $T(\vec{b}_1)=3\vec{d}_1-5\vec{d}_2$, $T(\vec{b}_2)=-\vec{d}_1+6\vec{d}_2$ and $T(\vec{b}_3)=4\vec{d}_2$. Find the matrix for T relative to $\mathcal{B}$ and $\mathcal{D}$.

3.) Let $\mathcal{E}=\{\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ be the standard basis for $\mathbb{R}^3$, let $\mathcal{B}=\{\vec{b}_1,\vec{b}_2,\vec{b}_3\}$ be a basis for a vector space V, and let $T:\mathbb{R}^3\to V$ be a linear transformation s.t $T(x_1,x_2,x_3)=(2x_3-x_2)\vec{b}_1-(2x_2)\vec{b}_2+(x_1+3x_3)\vec{b}_3$

a.) Compute $T(\vec{e}_1), T(\vec{e}_2), T(\vec{e}_3)$

b.) Compute $[T(\vec{e}_1)]_{\mathcal{B}}, [T(\vec{e}_2)]_{\mathcal{B}}, [T(\vec{e}_3)]_{\mathcal{B}}$

c.) Compute the matrix for T relative to $\mathcal{E}$ an $\mathcal{B}$.

6.) Let $T:\mathbb{P}_2\to\mathbb{P}_4$ be the transformation that maps a polynomial $\vec{p}(t)$ into the polynomial $\vec{p}(t)+2t^2\vec{p}(t)$.

a.) Find the image of $\vec{p}(t)=3-2t+t^2$.

b.) Show that T is a linear transformation.

7.) Assume the mapping $T: \mathbb{P}_2 \to \mathbb{P}_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$ is linear. Find the matrix representation of T relative to the basis $\mathcal{B} = \{1, t, t^2\}$.

10.) Define $T: \mathbb{P}_3 \to \mathbb{R}^4$ by $T(\vec{p}) = \begin{bmatrix} \vec{p}(-2) \\ \vec{p}(3) \\ \vec{p}(1) \\ \vec{p}(0) \end{bmatrix}$.

a.) Show that T is a lin. trans.

b.) Find the matrix of T relative to the basis $\{1, t, t^2, t^3\}$ for $\mathbb{P}_3$ and the standard basis for $\mathbb{R}^4$.

5.4 Continued

15.) Define $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\vec{x}) = A\vec{x}$. Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ with the property that $[T]_{\mathcal{B}}$ is diagonal.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}.$$

16.) $A = \begin{bmatrix} 4 & -2 \\ -1 & 5 \end{bmatrix}$

23.) If $B = P^{-1}AP$ and $\vec{x}$ is an eigenvector of A corresponding to an eigenvalue λ, then $P^{-1}\vec{x}$ is an eigenvector of B corresponding to λ. Verify.

25.) The trace of a square matrix A is the sum is the sum of the diagonal entries of A and is denoted by $\operatorname{tr} A$.

It can be verified that $\operatorname{tr}(FG) = \operatorname{tr}(GF)$ for any two $n \times n$ matrices F and G. Show that if A, and B are similar then $\operatorname{tr} A = \operatorname{tr} B$.

6.1 # 3, 5, 10, 16, 18, 19, 23, 25, 27, 29

3.) Let $\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ Compute:

$$\frac{1}{\vec{w} \cdot \vec{w}} \vec{w}$$

5.) $\left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$

10.) Find a unit vector in the direction of $\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$.

16.) Determine if the vectors are orthogonal.

$\vec{u} = \begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$

18.) $\vec{y} = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}$, $\vec{z} = \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$

19.) True/False. All vectors are in $\mathbb{R}^n$.

a.) $\vec{v}\cdot\vec{v} = \|\vec{v}\|^2$

b.) For any scalar c, $\vec{u}\cdot(c\vec{v}) = c(\vec{u}\cdot\vec{v})$.

c.) If the distance from $\vec{u}$ to $\vec{v}$ equals the distance from $\vec{u}$ to $-\vec{v}$, then $\vec{u}$ and $\vec{v}$ are orthogonal

d.) For a square matrix A, vectors in Col A are orthogonal to vectors in Nul A.

e.) If vectors $\vec{v}_1, \dots \vec{v}_p$ span a subspace W and if $\vec{x}$ is orthogonal to each $\vec{v}_j$ for $j = 1, \dots, p$, then $\vec{x}$ is in $W^\perp$.

23.) Let $\vec{u} = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$. Compute and compare $\vec{u}\cdot\vec{v}$, $\|\vec{u}\|^2$, $\|\vec{v}\|^2$ and $\|\vec{u}+\vec{v}\|^2$. Do not use the pythagorean thm.

25.) Let $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$. Describe the set H of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ that are orthogonal to $\vec{v}$.

Hint: Consider $\vec{v} = \vec{0}$ and $\vec{v} \neq 0$.

6.1 continued

27.) Suppose a vector $\vec{y}$ is orthogonal to vectors $\vec{u}$ and $\vec{v}$. Show that $\vec{y}$ is orthogonal to $\vec{u}+\vec{v}$.

29.) Let $W = \text{Span}\{\vec{v}_1, \ldots, \vec{v}_p\}$. Show that if $\vec{x}$ is orthogonal to each $\vec{v}_j$, for $1 \leq j \leq p$, then $\vec{x}$ is orthogonal to every vector in W.

6.2 # 3, 6, 8, 9, 11, 14, 20, 21, 23, 26, 27, 28, 29

3.) Determine if the set of vectors is orthogonal.

$$\left\{ \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}$$

6.) $$\left\{ \begin{bmatrix} 5 \\ -4 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -3 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 5 \\ -1 \end{bmatrix} \right\}$$

8.) Show that $\{u_1, u_2\}$ or $\{u_1, u_2, u_3\}$ is an orthogonal basis for $\mathbb{R}^2$ or $\mathbb{R}^3$, respectively. Then express $\vec{x}$ as a linear combination of the $\vec{u}$'s.

$\vec{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} -6 \\ 3 \end{bmatrix}$

9.) Same directions as #8.

$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 8 \\ -4 \\ 3 \end{bmatrix}$

11.) Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and the origin.

14.) Let $\vec{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Write $\vec{y}$ as the sum of a vector in Span$\{\vec{u}\}$ and a vector orthogonal to $\vec{u}$.

6.2 Continued

20.) Determine if the set is orthonormal. If a set is only orthogonal, normalize the vectors to produce an orthonormal set.

$$\left\{ \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix} \right\}$$

21.) $$\left\{ \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{20} \\ 3/\sqrt{20} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{20} \\ -1/\sqrt{20} \end{bmatrix}, \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \right\}$$

23.) True/False. All vectors are in $\mathbb{R}^n$.

a) Not every linearly independent set in $\mathbb{R}^n$ is an orthogonal set.

b.) If $\vec{y}$ is a linear combination of nonzero vectors from an orthogonal set, then the weights in the linear combination can be computed without row operations on a matrix.

c.) If the vectors in an orthogonal set of nonzero vectors are normalized, then some of the new vectors may not be orthogonal.

d.) A matrix with orthonormal columns is an orthogonal matrix.

e.) If L is a line through $\vec{0}$ and if $\hat{y}$ is the orthogonal projection of $\vec{y}$ onto L, then $\|\hat{y}\|$ gives the distance from $\vec{y}$ to L.

26.) Suppose W is a subspace of $\mathbb{R}^n$ spanned by n nonzero orthogonal vectors. Explain why $W = \mathbb{R}^n$

27.) Let U be a square matrix with orthonormal columns. Explain why U is invertible.

28.) Let U be an $n \times n$ orthogonal matrix. Show that the rows of U form an orthonormal basis of $\mathbb{R}^n$.

29.) Let U and V be $n \times n$ orthogonal matrices. Explain why UV is an orthogonal matrix.

6.3 # 1, 6, 7, 9, 11, 13, 17, 21, 23, 24

1.) Assume $\{\vec{u}_1, \ldots, \vec{u}_4\}$ is an orthogonal basis for $\mathbb{R}^4$.

$\vec{u}_1 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}$, $\vec{u}_4 = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix}$ Write $\vec{x}$ as a sum of two vectors, one in Span$\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$, the other in Span$\{\vec{u}_4\}$.

6.) Verify that $\{\vec{u}_1, \vec{u}_2\}$ is an orthogonal set, and then find the orthogonal projection of $\vec{y}$ onto Span$\{\vec{u}_1, \vec{u}_2\}$.

$\vec{y} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$, $\vec{u}_1 = \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

7.) Let W be the subspace spanned by the $\vec{u}$'s and write $\vec{y}$ as a sum of a vector in W and a vector orthogonal to W.

$\vec{y} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, $\vec{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$

9.) $\vec{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}$, $\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

11.) Find the closest point to $\vec{y}$ in the subspace W spanned by $\vec{v}_1, \vec{v}_2$.

$\vec{y} = \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

6.3 continued

13.) Find the best approximation to $\vec{z}$ by vectors of the form $c_1\vec{v}_1 + c_2\vec{v}_2$.

$\vec{z} = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

17.) Let $\vec{y} = \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}$, $\vec{u}_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$ and $W = \text{Span}\{\vec{u}_1, \vec{u}_2\}$.

a.) Let $U = [\vec{u}_1 \ \vec{u}_2]$. Compute $U^T U$ and UU^T.

b.) Compute $\text{proj}_W \vec{y}$ and $(UU^T)\vec{y}$.

21.) True / False. All vectors and subspaces are in $\mathbb{R}^n$.

a.) If $\vec{z}$ is orthogonal to $\vec{u}_1$ and to $\vec{u}_2$ and if $W = \text{Span}\{\vec{u}_1, \vec{u}_2\}$ then $\vec{z}$ must be in $W^\perp$.

b.) For each $\vec{y}$ and each subspace W, the vector $\vec{y} - \text{proj}_W \vec{y}$ is orthogonal to W.

c.) The orthogonal projection $\hat{y}$ of $\vec{y}$ onto a subspace W can sometimes depend on the orthogonal basis for W used to compute $\hat{y}$.

d.) If $\vec{y}$ is in a subspace W, then the orthogonal projection of $\vec{y}$ onto W is $\vec{y}$ itself.

e.) If the columns of an $n \times p$ matrix U are orthonormal, then $UU^T\vec{y}$ is the orthogonal projection of $\vec{y}$ onto the column space U.

23.) Let A be an $m \times n$ matrix. Prove that every vector $\vec{x}$ in $\mathbb{R}^n$ can be written in the form $\vec{x} = \vec{p} + \vec{u}$, where $\vec{p}$ is in $\text{Row}\,A$ and $\vec{u}$ is in $\text{Nul}\,A$. Also, show that if the equation $A\vec{x} = \vec{b}$ is consistent, then there is a unique $\vec{p}$ in $\text{Row}\,A$ s.t. $A\vec{p} = \vec{b}$.

6.3 Continued

24.) Let W be a subspace of $\mathbb{R}^n$ with an orthogonal basis $\{\vec{w}_1, \dots \vec{w}_p\}$ and let $\{\vec{v}_1, \dots \vec{v}_q\}$ be an orthogonal basis for $W^\perp$.

a.) Explain why $\{\vec{w}_1, \dots \vec{w}_p, \vec{v}_1, \dots \vec{v}_q\}$ is an orthogonal set.

b.) Explain why the set in part (a) spans $\mathbb{R}^n$.

c.) Show that $\dim W + \dim W^\perp = n$.