



University of Connecticut
Department of Mathematics

MATH 2210 CHAPTER 5 AND 6 PRACTICE PROBLEMS SPRING 2019

NAME: _____

Instructor Name: _____ Section: _____

This practice exam *is not* an exhaustive list of problems you should expect to see on the actual exam. Look at the homework, quizzes, in class notes, or extra suggested problems from the book for more practice.

The following message appears on the actual exam:

Read This First!

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- No calculators permitted.

Grading - For Administrative Use Only

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1 Chapter Summaries

This list is incomplete. Here are some of the important topics covered in each chapter on the exam.

Chapter 5

5.1 Eigenvectors and Eigenvalues

- how to determine if a scalar is an eigenvalue
- how to determine if a vector is an eigenvector
- how to find eigenspace given an eigenvalue
- how to find eigenvalues of a triangular matrix
- row operations change eigenvalues
- what does it mean to have an eigenvalue of zero

5.2 The Characteristic Equation

- how to find eigenvalues and their multiplicities using the characteristic equation

5.3 Diagonalization

- know how to use eigenvalues and eigenvectors to diagonalize a matrix
- definition of eigenvector basis

5.4 Eigenvectors and Linear Transformations

- definition of matrix for T relative to \mathcal{B} and \mathcal{C}
- definition of \mathcal{B} -matrix for a transformation $T : V \rightarrow V$
- diagonalizable matrix transformations
- take home exam, video, and quiz 10

1 Chapter 6

6.1 Inner Product, Length, Orthogonality

- dot product
- norm
- distance
- orthogonality

6.2 Orthogonal Sets

- orthogonal set
- orthogonal basis
- matrices with orthonormal columns, and orthogonal matrices

6.3 Orthogonal Projections

- orthogonal decomposition theorem
- best approximation theorem
- orthonormal bases

6.4 The Gram-Schmidt Process

- how to find an orthogonal/orthonormal basis
- how to find a QR factorization

2 Practice Exam

1. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}$.

(a) Find an eigenvector basis for \mathbb{R}^3 corresponding to A .

(b) Diagonalize A .

2. Let $T : \mathbb{P}^2 \rightarrow \mathbb{P}^2$ be defined by

$$T(a_0 + a_1t + a_2t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2.$$

(a) Show that T is linear.

(b) Find the matrix representation for T relative to the standard basis for \mathbb{P}^n .

(c) Find the kernel and range of T using part (b).

3. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$.

(a) Show that \mathbf{u} and \mathbf{v} are orthogonal.

(b) Verify the Pythagorean theorem by checking $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \|\mathbf{u} + \mathbf{v}\|^2$.

4. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 8 \\ -4 \\ 3 \end{bmatrix}$.

(a) Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthonormal basis for \mathbb{R}^3 .

(b) Express \mathbf{x} as a linear combination of \mathbf{u}_1 , \mathbf{u}_2 and \mathbf{u}_3 .

5. Find the distance from \mathbf{y} to $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$ where

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

6. Find an orthonormal basis for the column space of

$$\begin{bmatrix} 3 & 8 \\ 0 & 5 \\ -1 & -6 \end{bmatrix}.$$

7. Find an orthogonal basis for the column space of

$$\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}.$$