Math 2210 Final

For questions 1-4, read the given information and decide if each statement is True or False. Circle your answer. You do not need to justify your answer. No partial credit will be given.

- 1. Consider the set $S = \left\{ \begin{bmatrix} 0\\0\\1 \end{bmatrix}, \begin{bmatrix} 4/\sqrt{5}\\1/\sqrt{5}\\0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{5}\\2/\sqrt{5}\\0 \end{bmatrix} \right\}.$
 - (a) S is a basis for \mathbb{R}^3 .

True False

True

(b) S is an orthogonal set.

True False

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[3]

- (c) The vectors in S are normal.
- 2. Consider the matrix $A = \begin{bmatrix} 1 & -3 \\ -3 & 9 \end{bmatrix}$. [4]
 - (a) The equation $A\vec{x} = \vec{b}$ is consistent for any $\vec{b} \in \mathbf{R}^2$.

True

(b) The equation $A\vec{x} = \vec{0}$ has exactly one solution, the trivial solution.

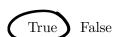
True

(c) A is invertible.

True

True

- (d) The equation $A\vec{x} = \vec{0}$ has infinitely many solutions, all of which are non-trivial. Wild word w. Technical
- [4]
- 3. Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\} = \{\begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \end{bmatrix}\}$ be a basis for \mathbf{R}^2 and let $\vec{x} = -1\vec{b}_1 + 2\vec{b}_2$. (a) $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -1\\2 \end{bmatrix}$.



(b) $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -7 \\ -11 \end{bmatrix}$.



(c) $P_{\mathcal{B}} = \begin{bmatrix} 5 & -1 \\ 3 & -4 \end{bmatrix}$.



(d) $P_{\mathcal{B}}$ sends \vec{x} to $[\vec{x}]_{\mathcal{B}}$.

True false

4. Let W be a vector space and let $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ be a basis for some subspace V of W.

[5]

(a) The set $\{\vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly independent.



(b) For any $\vec{x} \in V$, Span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{x}\} = V$.

True False

(c) For any $\vec{x} \in V$, the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{x}\}$ is linearly independent.

True False

(d) The set $\{\vec{v}_1, \vec{v}_2\}$ spans V.

(e) $\dim V \leq \dim W$.

True



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5. Answer the following using the matrix below.

$$A = \begin{bmatrix} 1 & -6 \\ -6 & 10 \end{bmatrix}$$

(a) Compute $\det A$.

[2]

(b) Compute
$$A^{-1}$$
.

$$\frac{1}{-26} \begin{pmatrix} 10 & 6 \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} -5/13 & -3/13 \\ -3/13 & -1/26 \end{pmatrix}$$
[2]

(c) Find the eigenvalues of A.

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6. Answer the following using the matrix below.

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & -2 & 2 \\ 3 & 0 & 6 \end{bmatrix}$$

(a) Compute $\det A$.

$$\det \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} &$$

[4]

(b) Compute
$$A^{-1}$$
.

$$\begin{bmatrix}
1 & 1 & 4 & 1 & 0 & 0 \\
2 & -2 & 2 & 0 & 0 & 0 \\
3 & 0 & 6 & 1 & 0 & 0
\end{bmatrix}$$

$$\sim
\begin{bmatrix}
0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}$$
[4]

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[7]

[7]

7. Let
$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
.

(a) Use the Gram-Schmidt process to construct and orthogonal basis for Col A.

(b) Find a basis for each eigenspace of A given that the eigenvalues of A are $\lambda = 1$ and $\lambda = 2$.

$$E_{1} = N_{U} III \left(\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \right) = N_{U} III \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)$$

$$= Span \left\{ \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix} \right\} = N_{U} II \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)$$

$$= Span \left\{ \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} = Span \left\{ \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

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[8]

8. Use Cramer's Rule to find a solution to $A\vec{x} = \vec{b}$ where A is the **triangular** matrix below and \vec{b} is the vector below. Show all work, no credit will be awarded for any other method.

$$A = \begin{bmatrix} -1 & 4 & 3 \\ 0 & -3 & 7 \\ 0 & 0 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{1}(b) = \begin{bmatrix} 4 & 4 & 3 \\ 0 & -3 & 7 \\ 0 & 0 & 1 \end{bmatrix} \quad det (A_{1}(b)) = -12$$

$$A_{2}(b) = \begin{bmatrix} -1 & 4 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \end{bmatrix} \quad det (A_{2}(b)) = 0$$

$$A_{3}(b) = \begin{bmatrix} -1 & 4 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \end{bmatrix} \quad det (A_{3}(b)) = 0$$

$$A = \begin{bmatrix} -1 & 4 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \end{bmatrix} \quad det (A) = 3$$

$$X_{1} = -4 \quad X_{2} = 0 \quad X_{3} = 0$$

9. Check your answer to the previous question. (If you were unable to answer the question, take a guess for the solution and show if your guess is correct or incorrect.)



[2]

incorrect

correct

[6]

[3]

[3]

- 10. Let $T: \mathbf{R}^3 \to \mathbf{R}^2$ be given by $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$.
 - (a) Show that $T(\vec{x})$ is a linear transformation.

(b) Is T one-to-one? Explain why or why not.

No, the columns are limarly dip

(c) Find a vector in the range of T. Explain how you know this vector is in the range of T.

[3] b/c Col(A) = Ronge (T)

11. Let $W = \text{Span}\{\vec{u}_1, \vec{u}_2\}$ and let A be the matrix whose columns are \vec{u}_1 and \vec{u}_2 .

$$\vec{u}_1 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(a) Find the vector in W closest to \vec{b} .

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[3]

[3]

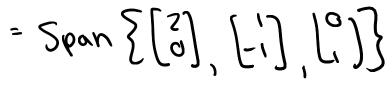
[3]

- 12. Construct a 3×4 matrix A that is in RREF (row reduced echelon form). Construct A such that it has linearly **dependent** columns and $A\vec{x} = \vec{b}$ is not consistent for every $\vec{b} \in \mathbf{R}^3$.
- lois of answers

13. (a) Let $W = \left\{ \begin{bmatrix} 2a+b\\c-b \end{bmatrix} : a,b,c \in \mathbf{R} \right\}$. Prove or disprove that W is a subspace of \mathbf{R}^2 .

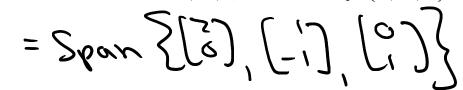


[2]



⇒ Subspace

(b) Write W as a span of vectors. i.e. Find vectors $\vec{v}_1, \ldots, \vec{v}_n$ such that $W = \operatorname{Span}\{\vec{v}_1, \ldots, \vec{v}_n\}$.



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14. Let $\mathcal{B} = \{1 + t + t^2, -t - t^2, t\}$ be a basis for \mathbf{P}_2 . Find the coordinate vector for the polynomial $p(t) = 5t^2 + 6t - 1$ with respect to \mathcal{B} .

[5]

$$\begin{cases} \frac{1}{2} - \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{$$

Bonus: Write as many equivalences of the invertible matrix theorem as you can. Let A be an $n \times n$ matrix. Then A is invertible if and only if: