

Lecture 2: Unpredictability

- Pick up a copy of the syllabus if you don't have one already
- Lab 1 posted on piazza.com,
due on gradescope.com on Monday 1/28 at 11pm

Bruce Schneier's 4 truths of computer security

- | | |
|---|--|
| 1. <i>Attackers have the advantage</i> | “Complexity is the worst enemy of security, [and] the Internet is the most complex machine man has ever built by a lot.” |
| 2. <i>Interconnections → New vulnerabilities</i> | “The more we connect things to each other, the more vulnerabilities in one thing affect other things.” |
| 3. <i>Attacks scale</i> | “The Internet is a massive tool for making things more efficient. That's also true for attacking.” |
| 4. <i>Defense requires smart people who know what to do</i> | “Our computers are secure [only because] the engineers at Google, Apple, Microsoft spent a lot of time on this.” |

kryptos = secret, hidden

Cryptology



Cryptography

the art of making codes

Cryptanalysis

the art of breaking codes

Schneier's law: Anyone, from the most clueless amateur to the best cryptographer, can create an algorithm that he himself can't break.

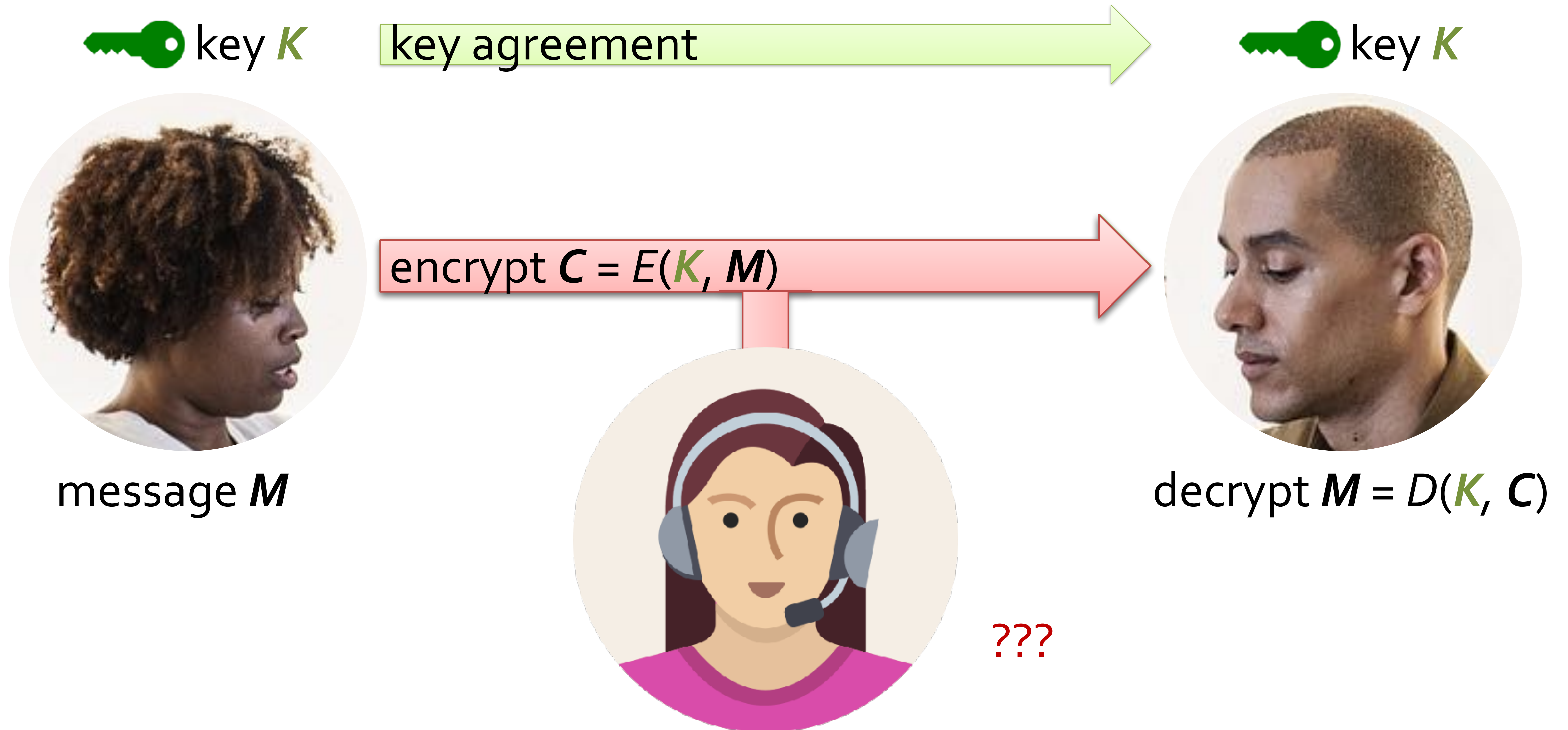
Course outline

1. Protecting data at rest
2. Attacking data at rest
3. Protecting data in transit
4. Crypto law and policy
5. Protecting data during use
6. Design + cryptanalysis of crypto building blocks

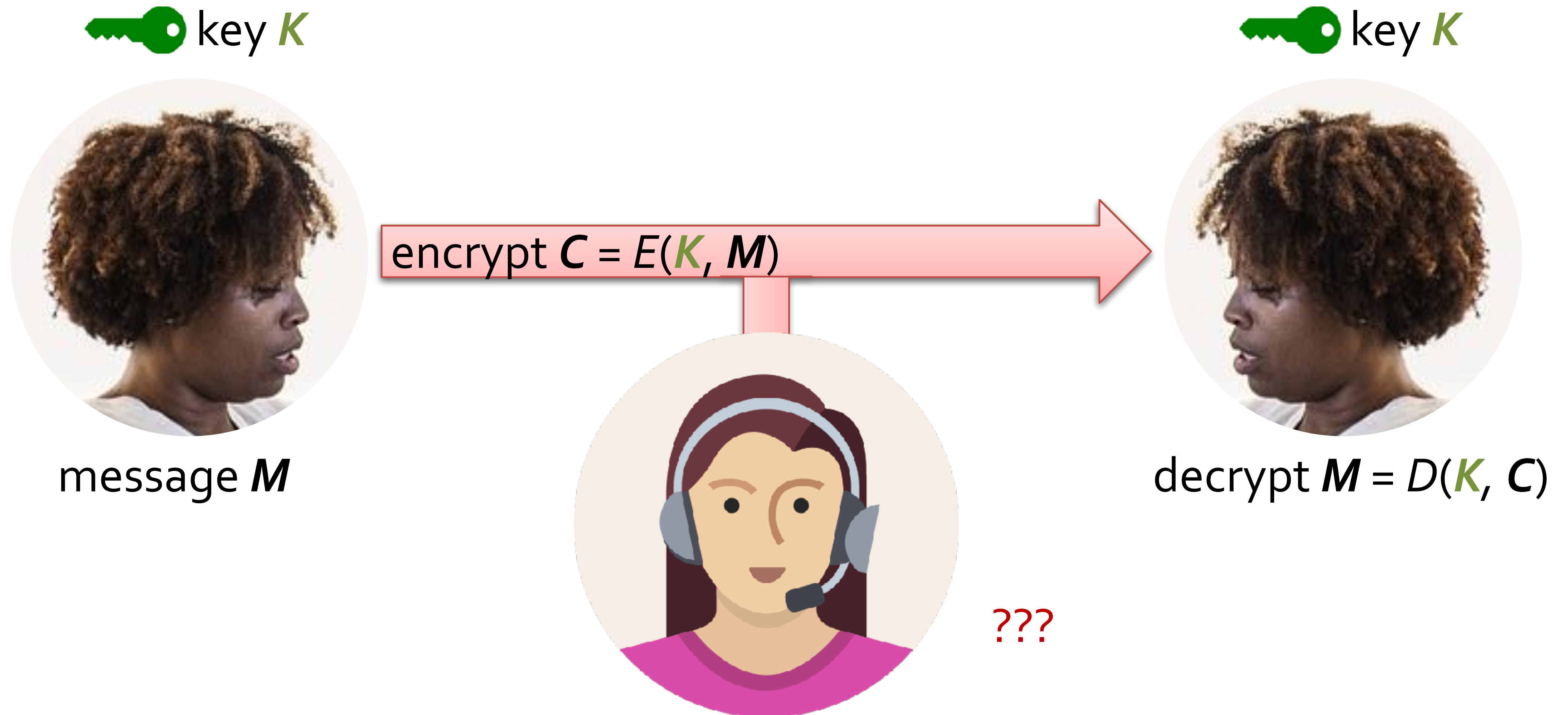
“Cryptography is about communication
in the presence of an adversary.”

–Ron Rivest

Protecting data in transit



Part 1: Protecting data at rest



How can Alice encode messages so Eve can't read them?

1. Substitute each character with another one
2. Write in a foreign language
3. Make it hard for Eve to determine where Alice wrote her message

Caesar cipher

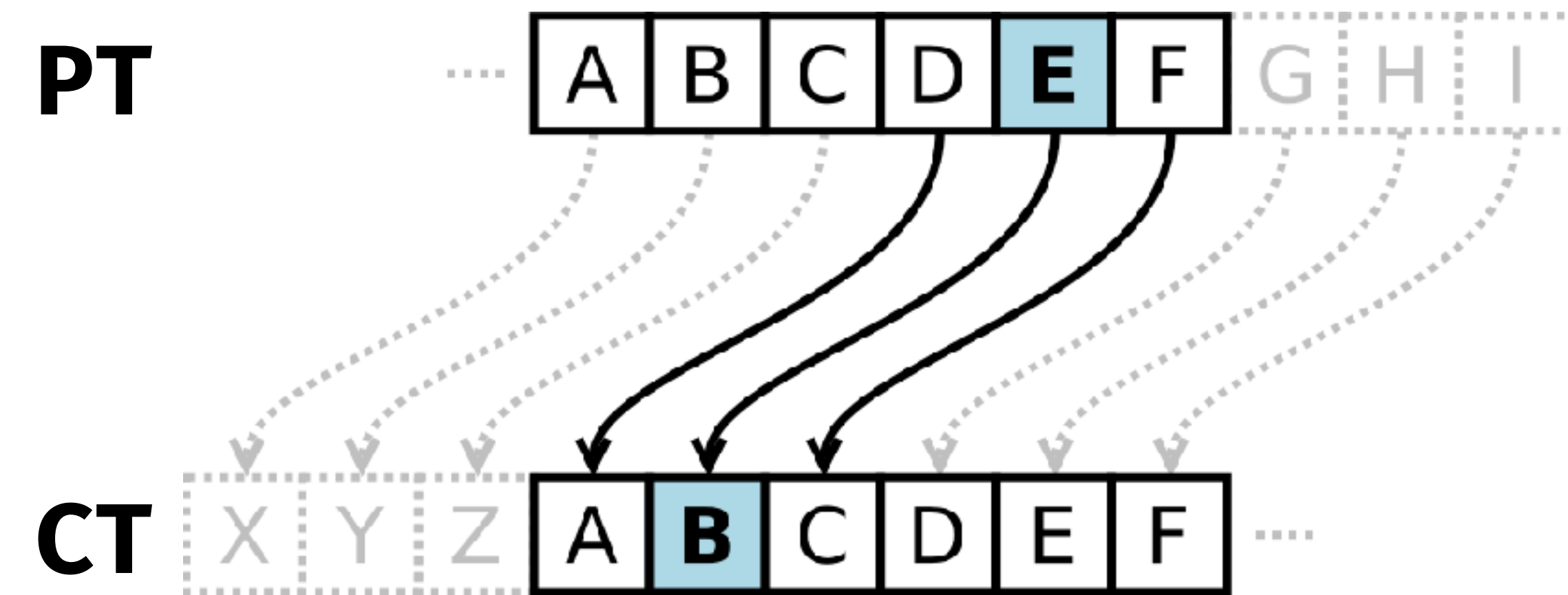


Image source: Wikipedia

- Encipher one character at a time
- Figure shows cipher with key $K = 3$
 - one \mapsto lkb
 - two \mapsto qtl
- Problem?
 - three \mapsto qeobb
- How to resolve?

Binary representation of data

Quantities

- bit $\in \{0,1\}$
- byte $\in \{0,1\}^8$

Formats

- Raw bits (some of which are ASCII printable)
- Hex characters

| Dec | Hx | Oct | Html | Chr |
|-----|----|-----|-------|-----|
| 64 | 40 | 100 | @ | @ |
| 65 | 41 | 101 | A | A |
| 66 | 42 | 102 | B | B |
| 67 | 43 | 103 | C | C |
| 68 | 44 | 104 | D | D |
| 69 | 45 | 105 | E | E |
| 70 | 46 | 106 | F | F |

Reminder: Keep track of the format of a string during the labs!

If you compute an output whose length is double what you expected, then you almost surely operated over a hex encoding rather than the raw string.

One time pad

| \oplus | 0 | 1 |
|----------|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

- XOR function measures whether 2 inputs are identical

- OTP “masks” the message by applying a Caesar cipher independently to each bit

```
message  0110 0110 1001
XOR key   0011 1100 1110
```

```
cipher   0101 1010 0111
```

- XOR is a “lossless” function, so it is invertible (XOR is its own inverse)

```
cipher   0101 1010 0111
XOR key   0011 1100 1110
```

- Drawbacks?

```
message  0110 0110 1001
```

- Key length == plaintext message length
- No integrity: easy to manipulate ciphertext

How can Alice encode messages so Eve can't read them?

1. Substitute each character with another one
2. Write in a foreign language
3. Make it hard for Eve to determine where Alice wrote her message

Slowenisch

Rumänisch



Russian

COLLINS

SPANISH

Französisch - Deutsch
Deutsch - Französisch

ROMANIAN-ENGLISH
ENGLISH-ROMANIAN


spanisch
deutsch
deutsch
spanisch

Slo

Praxis
Wörterbuch
Klett

Goal 1: Unintelligible to Eve

Goal 2: Simple for Alice

- Fast + easy to compute ~~✗~~ Slow
- Secret key  is small ~~✗~~ Big and easy to change
- Infinitely reusable ~~✗~~ Frequency analysis

Note that Alice chose her encoded words to be distinct. Why?

| Plain word | Encoded word |
|------------|--------------|
| aba | nrq |
| abs | mbk |
| ace | ybd |
| act | wxv |
| add | j en |
| ado | hhg |
| aft | uxv |
| age | zmx |
| ago | dgs |
| aha | ase |
| aid | ktf |
| : | : |
| zip | cyu |
| zoo | dux |



3-letter words

3 characters from random.org

Is Alice's custom codebook secure?

| X | Y |
|-----|-----|
| aba | nrq |
| abs | mbk |
| ace | ybd |
| act | wxv |
| add | jen |
| ado | hhg |
| aft | uxv |
| age | zmx |
| ago | dgs |
| aha | ase |
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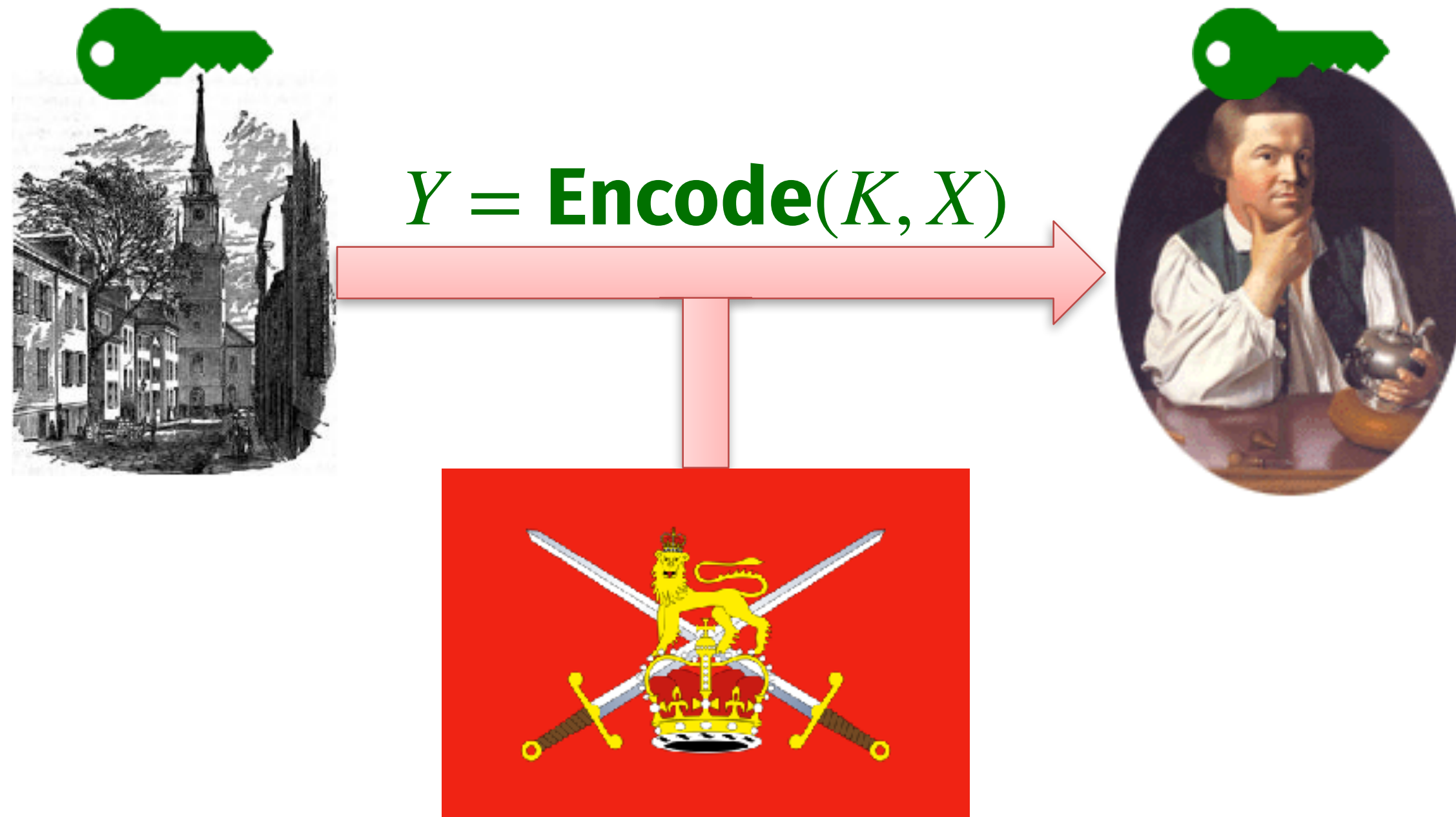
Alice has secret message X^* , sends $Y^* = \text{Encode}(X^*)$

Question: can Eve recover X^* when given:

1. Only Y^*
2. Above, plus many (X_i, Y_i) pairs chosen at random
3. Above, plus many (X_i, Y_i) pairs for X_i of Eve's choice
4. Above, plus Eve can choose the X_i one at a time, and adapt her choices based on the Y_i responses she receives
5. Above, plus Eve can also decipher Y_i of her choice

Upshot: security depends on the adversary's powers

When can we get away with a short key?



Two options:

-

| Plain word | Encoded word |
|------------|--------------|
| land | 1 |
| sea | 2 |

-

| Plain word | Encoded word |
|------------|--------------|
| land | 2 |
| sea | 1 |



Key == secret + unpredictable

Randomness \Rightarrow Unpredictability \Rightarrow Secrecy



Unpredictability

Suppose that Eve can adaptively make q queries into our codebook.

We call the codebook **unpredictable** if Eve has a very small chance to predict $\text{Enc}(X^*)$ for any unqueried X^* .

Note:

- An unpredictable codebook is *almost* secure against an Eve that conducts attack #4 (choose X adaptively, get Y).
- But, we have not addressed frequency analysis yet.
Unpredictability doesn't allow Eve to observe same X twice

| X | Y |
|-----|-----|
| aba | nrq |
| abs | mbk |
| ace | ybd |
| act | wxv |
| add | jen |
| ado | hhg |
| aft | uxv |
| age | zmx |
| ago | dgs |
| aha | ase |
| aid | ktf |
| : | : |
| zip | cyu |
| zoo | dux |

“If an adversary A has not **explicitly** queried a [perfect codebook] R on some point X , then the value of $R(X)$ is **completely random**... at least as far as A is concerned.”

–*Jon Katz and Yehuda Lindell, Introduction to Modern Cryptography*

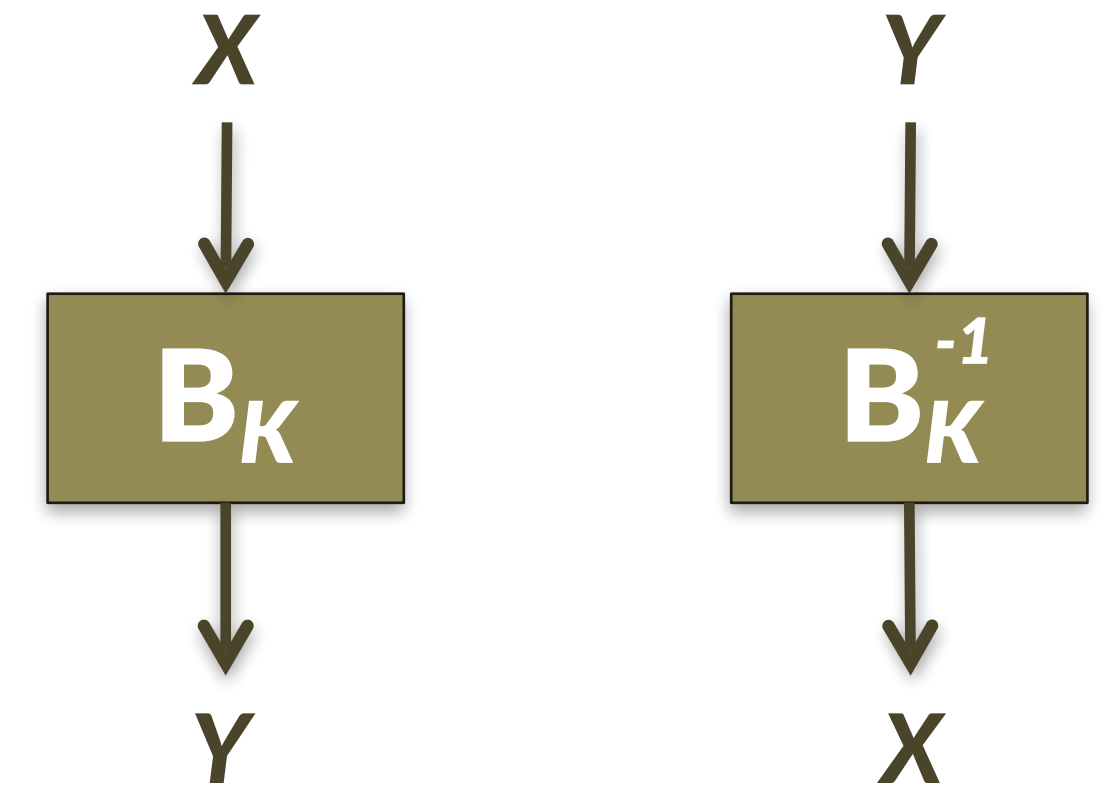
A crypto “Manhattan project”

- Imagine society spends an enormous effort to make a single codebook ***R*** and its inverse (so Alice can decipher her original message later)
- Can Alice use this codebook to protect her messages from Eve?
- Intuitively: no!
 - Eve can use the codebook too
 - Codebook is too large for Alice to carry around
 - Codebook’s input + output lengths may not suffice to encode Alice’s message




Block cipher

- Family of invertible permutations, indexed by a secret key
- Design goals
 1. **Simple** - built from native CPU operations like XOR, cyclic shifts, and table lookups
 2. **Makes no sense** - unpredictable
 3. **Simple to see that it makes no sense** - we have simple, convincing arguments that the cipher is unpredictable (remember Schneier's law!)



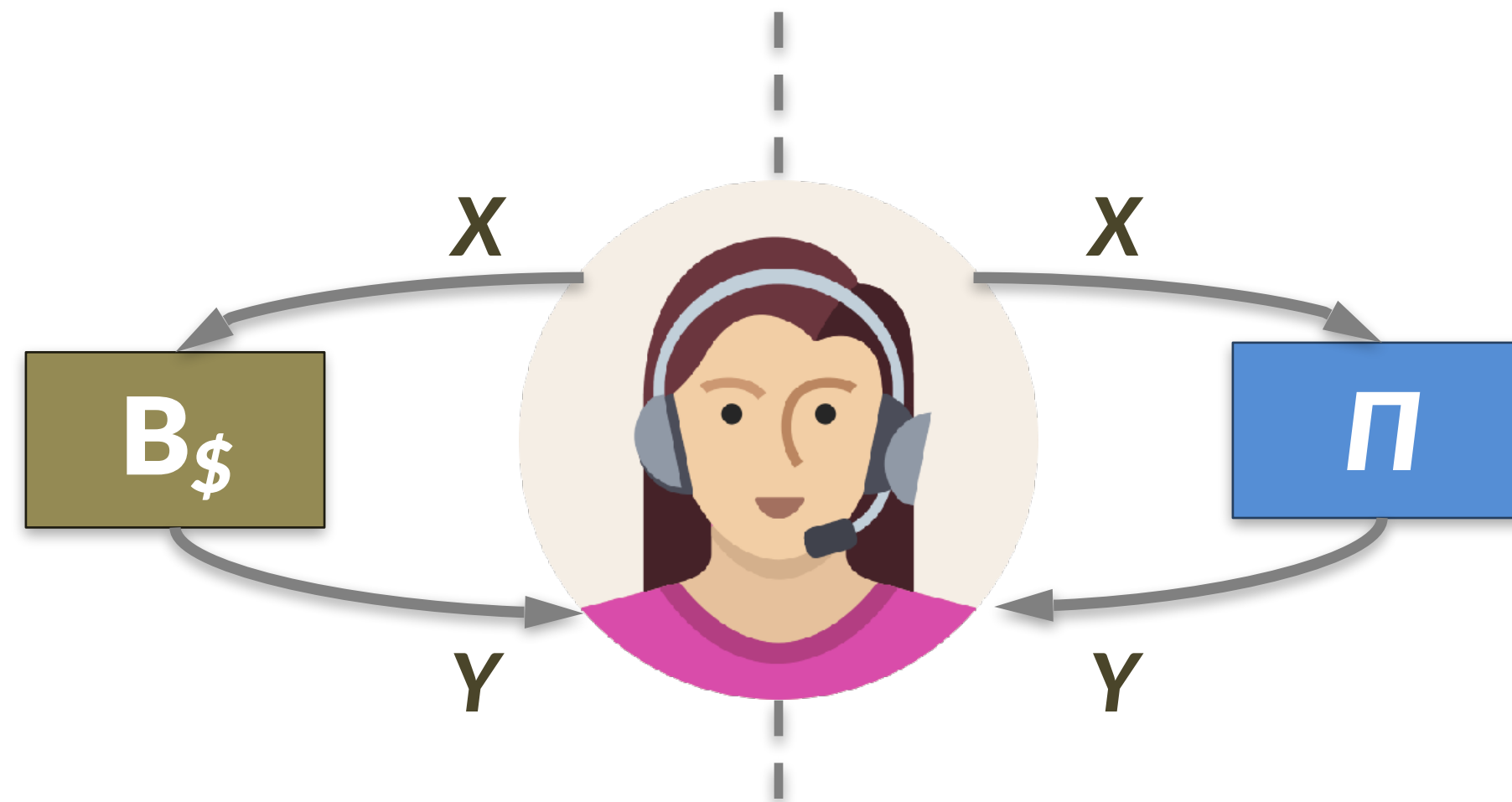
Cryptanalysis via brute force

- There is a large (but not infinite!) universe of possible keys
 - Alice's key  identifies the cipher that she decided to use
- *Brute force attack:*
Eve can try all possible keys and test against an observed (X, Y) pair
- Alice's objectives
 - Make brute forcing infeasibly difficult
 - Ensure that cipher cannot be broken faster than brute force search

| Game | Search size | Solved? |
|----------------------|----------------|---------|
| Connect 4 | 10^{13} | ✓ |
| Limit hold 'em | 10^{14} | ✓ |
| Checkers | 10^{20} | ✓ |
| Chess | 10^{50} | |
| <i>Modern crypto</i> | $\sim 10^{70}$ | |
| No limit hold 'em | 10^{140} | |
| Go (19 × 19) | 10^{171} | |

Security guarantee: pseudorandomness

- Let Π = truly random permutation made by a *secret* Manhattan project
- Goal: Eve cannot tell apart B_K and Π , so they are effectively the same



Security guarantee: **strong** pseudorandomness

- Let Π = truly random permutation made by a secret Manhattan project
- Goal: Eve cannot tell apart B_K and Π , so they are effectively the same



- ...even if Eve gets access to both enciphering and deciphering
- Question: How do we build something that looks truly chaotic but isn't?

A crypto “Manhattan project”

- Imagine society spends an enormous effort to make a single codebook R and its inverse (so Alice can decipher her original message later)
- Can Alice use this codebook to protect her messages from Eve?
- ~~Intuitively: no!~~ **Actually: Yes! (We can fix all of the problems below)**
 - Eve can use the codebook too
 - Codebook is too large for Alice to carry around
 - Codebook's input + output lengths may not suffice to encode Alice's message