#### Lecture 15: Public key infrastructure

- Lab 7 due Monday 4/1 at 11pm
- Office hours this week moved to Wednesday at 11am-1pm
- Guest lectures on cryptography + law starting on Thursday
- Read The Moral Character of Cryptographic Work by Friday

#### Part 3: Generate, exchange, evolve, and delete keys





#### Last time: Needham–Schroeder protocol

Objective: with the help of a trusted server, Alice + Bob agree on a shared key



#### **Kerberos = Key Management for Access Control**

![](_page_3_Picture_1.jpeg)

"This bar is pretty good, but you have to go stand in line for a ticket before they serve you."

Source: twitter.com/sweis/status/982272891948421120

![](_page_3_Picture_4.jpeg)

### **Today: Public key infrastructure**

Lower trust in the server, at the expense of using more expensive cryptography

![](_page_4_Picture_2.jpeg)

![](_page_5_Figure_0.jpeg)

![](_page_5_Picture_1.jpeg)

![](_page_5_Picture_2.jpeg)

![](_page_6_Figure_0.jpeg)

![](_page_6_Picture_1.jpeg)

![](_page_6_Picture_2.jpeg)

### Diffie-Hellman key agreement (last Friday's discussion)

**Protocol** (for a publicly known g)

Choose *a* randomly Compute A = g<sup>a</sup> Choose *b* randomly Compute B = g<sup>b</sup>

![](_page_7_Picture_4.jpeg)

Output Ba

Output A<sup>b</sup>

#### Shared secret = g<sup>ab</sup>

![](_page_7_Picture_8.jpeg)

#### Analysis

- Correctness: commutativity  $A^{b} = (g^{a})^{b} = g^{ab} = (g^{b})^{a} = B^{a}$
- Security: to learn the key, a passive Eve must solve following problem
  - Knows g, g<sup>a</sup>, g<sup>b</sup>
  - Wants to find gab
- Forward secrecy: Choices of a, b are *ephemeral*, can delete when done
- Active attacker can cause problems!

#### How to perform key exchange securely?

#### **Modular arithmetic**

![](_page_8_Picture_2.jpeg)

• Raise a constant to any power, e.g.  $x \mapsto 3^x \pmod{7}$ 

X	1	2	3	4	5	6
<b>3</b> ×	3	2	6	4	5	1

Permutation, but hard\* to invert

\* = really need to take the group of quadratic residues (i.e., the even half of the truth table)

#### **Elliptic curves**

![](_page_8_Figure_8.jpeg)

- Elliptic curve: a cubic equation  $y^2 = x^3 + ax + b \pmod{p}$
- Consider set of points on this curve
- We can "multiply" points using the rule  $P \cdot Q \cdot R = 1$

### Diffie-Hellman key agreement

**Protocol** (for a publicly known g)

Choose *a* randomly Compute A = g<sup>a</sup> Choose *b* randomly Compute B = g<sup>b</sup>

![](_page_9_Picture_4.jpeg)

Output Ba

Output A<sup>b</sup>

Shared secret = g<sup>ab</sup>

How do Alice and Bob know that they're talking with each other?

Solution: Use a MAC?

![](_page_9_Picture_10.jpeg)

![](_page_9_Picture_11.jpeg)

#### Let's build a public method to authenticate message origin

![](_page_10_Picture_1.jpeg)

- In the symmetric case, Alice & Bob have a key that nobody else has
  - As a result, Bob knows Alice sent A *and* that the message was intended for him
  - Tag is also deniable because either Alice or Bob could have made it
- In the public case, Alice has a secret SK and everyone knows corresponding PK
  - So, anyone in the world can verify that Alice wrote that message (to somebody...)
  - Also, asymmetry leads to non-deniability: Bob can't make **o** anymore

![](_page_10_Picture_8.jpeg)

# Security for public-key signatures

![](_page_11_Picture_1.jpeg)

• choose SK, give Mallory PK

![](_page_11_Picture_4.jpeg)

![](_page_11_Picture_5.jpeg)

EU-CMA security similar to before: Alice baits Mallory into producing a forgery

Mallory

![](_page_11_Picture_8.jpeg)

Mallory wins if 2. It's new

![](_page_11_Picture_10.jpeg)

# How to make digital signatures?

#### Modular arithmetic

 Similar math as with key exchange

![](_page_12_Picture_3.jpeg)

- Two common methods
  - (EC)DSA NIST standard
  - Schnorr signatures simpler but patented, will see on Friday

#### RSA (Rivest, Shamir, Adleman)

- Relies (more or less) on the hardness of factoring N = p q
- Less commonly used nowadays
- Will explore in this week's lab

# **Combining symmetric encryption + public signatures**

- In the symmetric case, we learned that Enc-then-MAC is the best option
  - Intuition: Never expose the decryption key to an invalid message
- Does this technique work as well with public key signatures?

# PublicSign<sub>A</sub>(SymEnc<sub>AB</sub>(P))

![](_page_13_Picture_5.jpeg)

![](_page_13_Picture_6.jpeg)

# **Combining symmetric encryption + public signatures**

- In the symmetric case, we learned that Enc-then-MAC is the best option
  - Intuition: Never expose the decryption key to an invalid message
- Does this technique work as well with public key signatures?
- Answer: No!
  - Issue: Mallory can receive ciphertexts from Alice, claim them as her own!
- Can lead to an oracle attack, as occurs with Apple's iMessage

#### Pretend to be Alice, send PublicSign<sub>A</sub>(C)

![](_page_14_Picture_8.jpeg)

Let C = SymEnc<sub>AM</sub>(P) Send PublicSign<sub>A</sub>(C)

Will decrypt C using symmetric key K<sub>AB</sub>!

![](_page_14_Picture_11.jpeg)

#### Non-repudiable crypto (xkcd.com/538)

![](_page_15_Figure_1.jpeg)

![](_page_15_Picture_2.jpeg)

#### Better combination of public signatures + symmetric crypto

![](_page_16_Figure_1.jpeg)

Remaining question: how do Alice and Bob learn the other's public key?

- y 1. Alice + Bob sign their D-H key exchange messages
  - 7. Alice + Bob verify signatures on each others' messages
    - 3. Use agreed-upon key for (deniable) symmetric authenticated encryption

Google.com in Firefox:

Technical Details

Connection Encrypted (TLS\_ECDHE\_RSA\_WITH\_AES\_128\_GCM\_SHA256, 128 bit keys, TLS 1.2)

BU login page in Firefox (2017):

Technical Details

Connection Encrypted (TLS\_RSA\_WITH\_AES\_256\_CBC\_SHA, 256 bit keys, TLS 1.2)

![](_page_16_Picture_12.jpeg)

![](_page_16_Picture_13.jpeg)

#### Public key infrastructure

- There is a certificate authority that knows everybody's keys
  - (Think of it like a telephone book)
- Anyone can query the authority to learn someone else's key
- CA signs responses so that everybody knows they are legit
- Alice knows the CA's public key because it is included in her OS

![](_page_17_Figure_6.jpeg)

![](_page_17_Picture_7.jpeg)

![](_page_17_Picture_8.jpeg)

# Slight improvement

- Alice wants to talk to Bob, not CA
- Bob can forward CA's attestation that signing key belongs to him
- (Shown: simplified TLS handshake)

![](_page_18_Picture_4.jpeg)

![](_page_18_Figure_5.jpeg)

#### What happens if Bob's secret key SK is compromised?

![](_page_19_Picture_1.jpeg)

![](_page_19_Figure_2.jpeg)

![](_page_19_Picture_5.jpeg)

#### **Backward security technique #1: Cert expiration**

- Alice wants to talk to Bob, not CA
- Bob can forward CA's attestation that signing key belongs to him
- (Shown: simplified TLS handshake)

#### "Hi, who are you?" + nonce

![](_page_20_Picture_5.jpeg)

![](_page_20_Figure_6.jpeg)

#### **Backward security technique #2: Key revocation**

- The PKI binds a public key to your identity
- If you lose control of your public key, you should tell the CA to break this binding
- Every CA maintains a certificate revocation list that anyone can query

![](_page_21_Figure_4.jpeg)

#### **Backward security technique #2: Key revocation**

- The PKI binds a public key to your identity
- If you lose control of your public key, you should tell the CA to break this binding
- Every CA maintains a certificate revocation list that anyone can query

![](_page_22_Figure_4.jpeg)

#### Key management = initial exchange + subsequent evolution

Server trust?	Crypto used	Method
Full	Symmetric	Needham
Partial	Asymmetric	(Authenti
None	Symmetric	Key evolu

# Next time: Key evolution

- n-Schroeder  $\Rightarrow$  Kerberos system
- icated) Diffie-Hellman key exchange
- ution, starting from an initial shared symmetric key

![](_page_23_Figure_6.jpeg)