Secure Computation & Yao's Protocol

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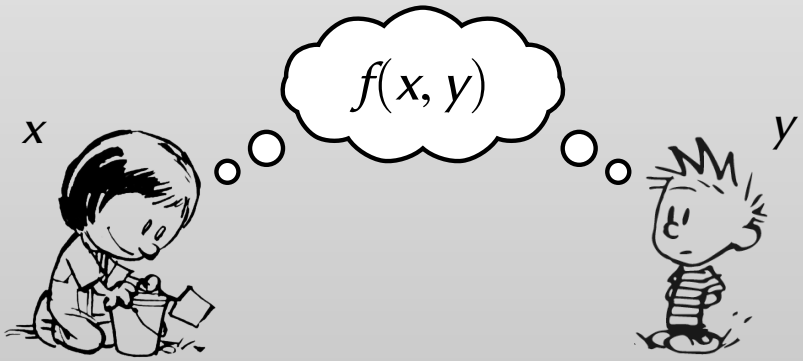
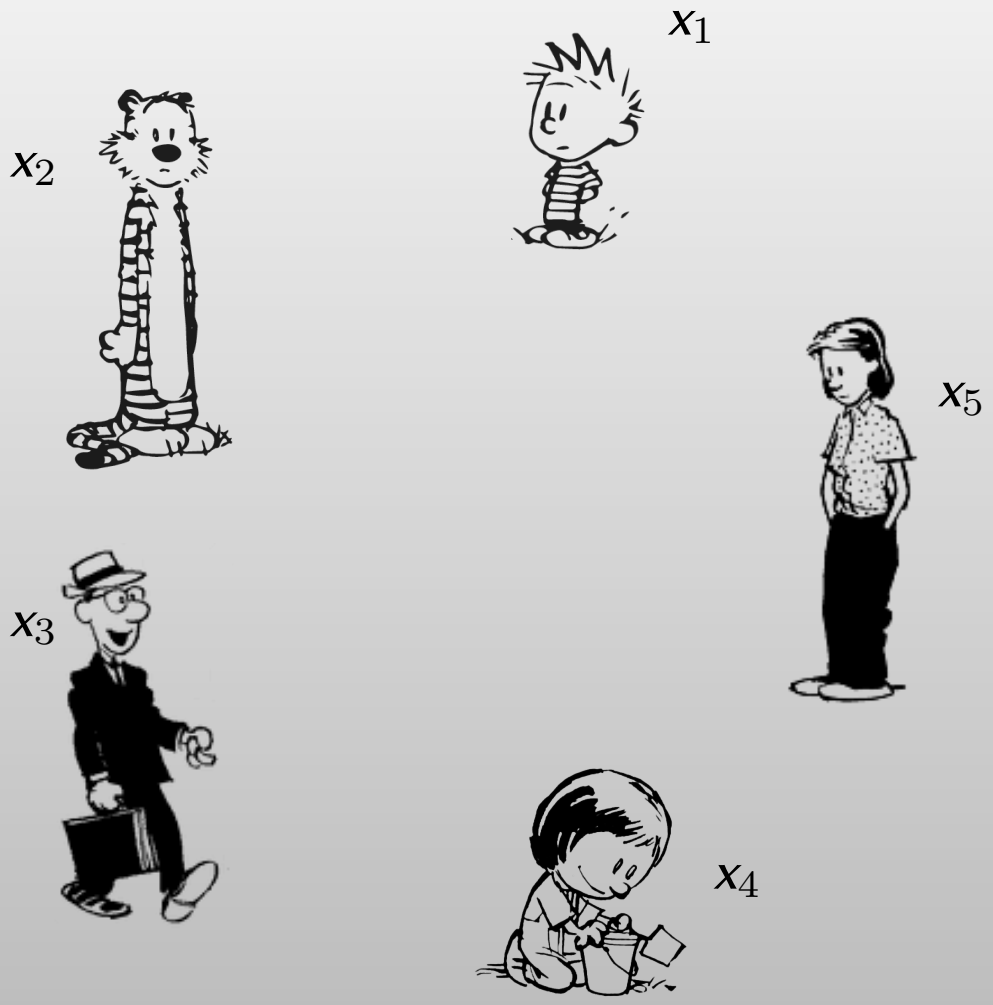
crypt@b-it 2018



Roadmap

1. **Secure computation:** Concepts & definitions
2. **Yao's protocol:** semi-honest secure computation for boolean circuits

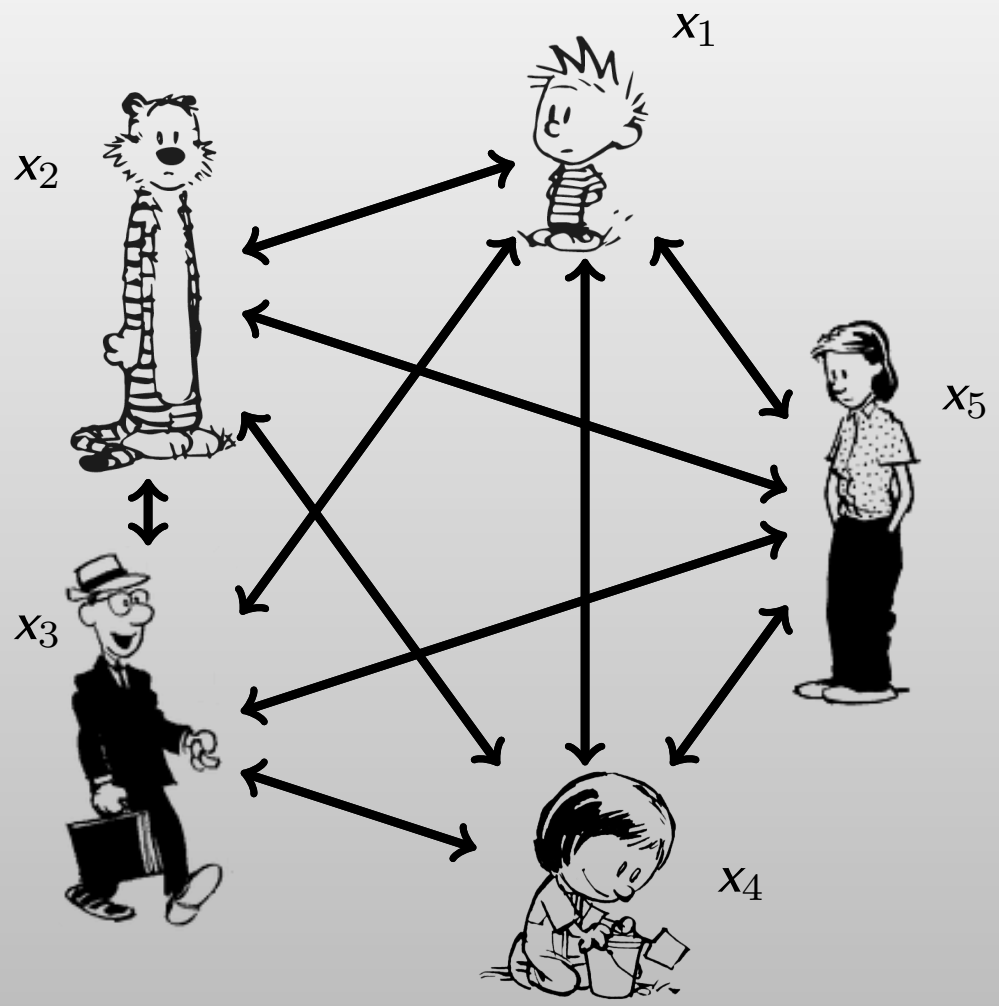
Secure computation



Premise:

- Mutually distrusting parties, each with a private input

Secure computation

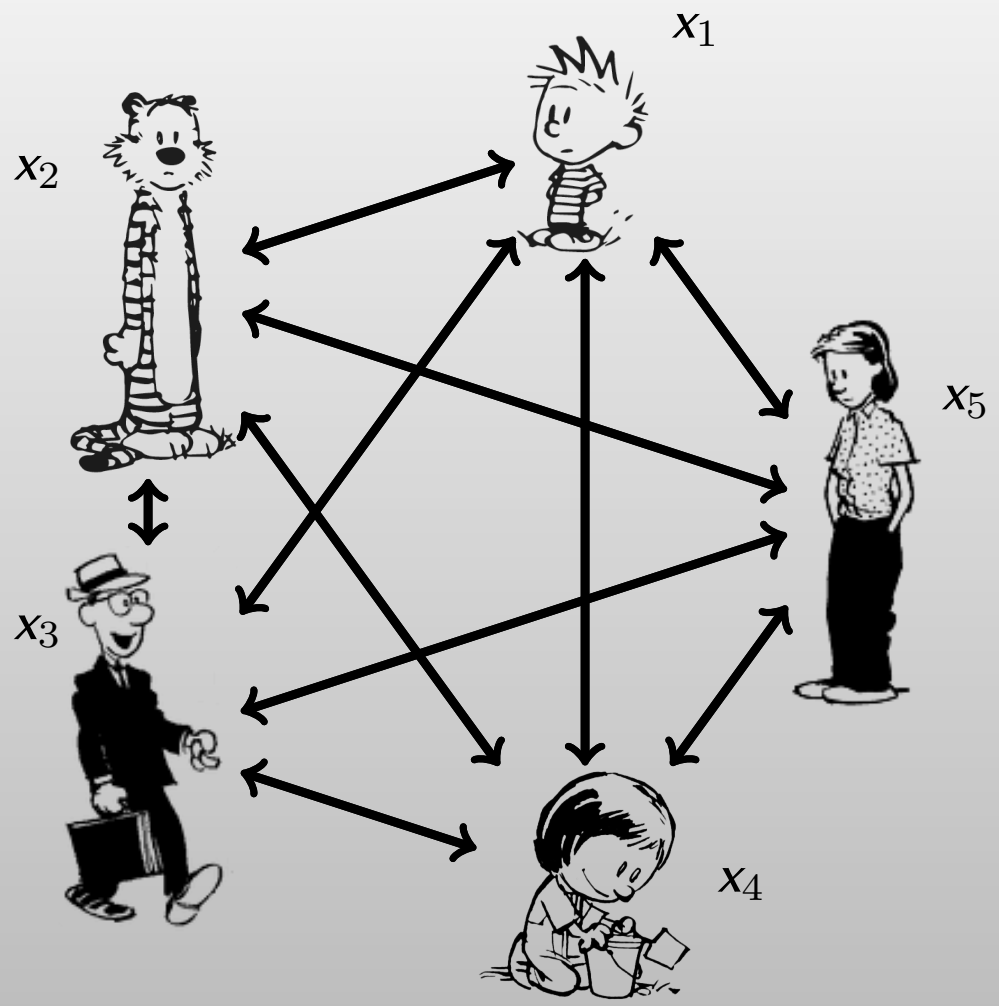


Premise:

- Mutually distrusting parties, each with a private input
- Learn the result of agreed-upon computation
- *Ex:* election, auction, etc.

$$\therefore f(x_1, x_2, x_3, x_4, x_5)$$

Secure computation



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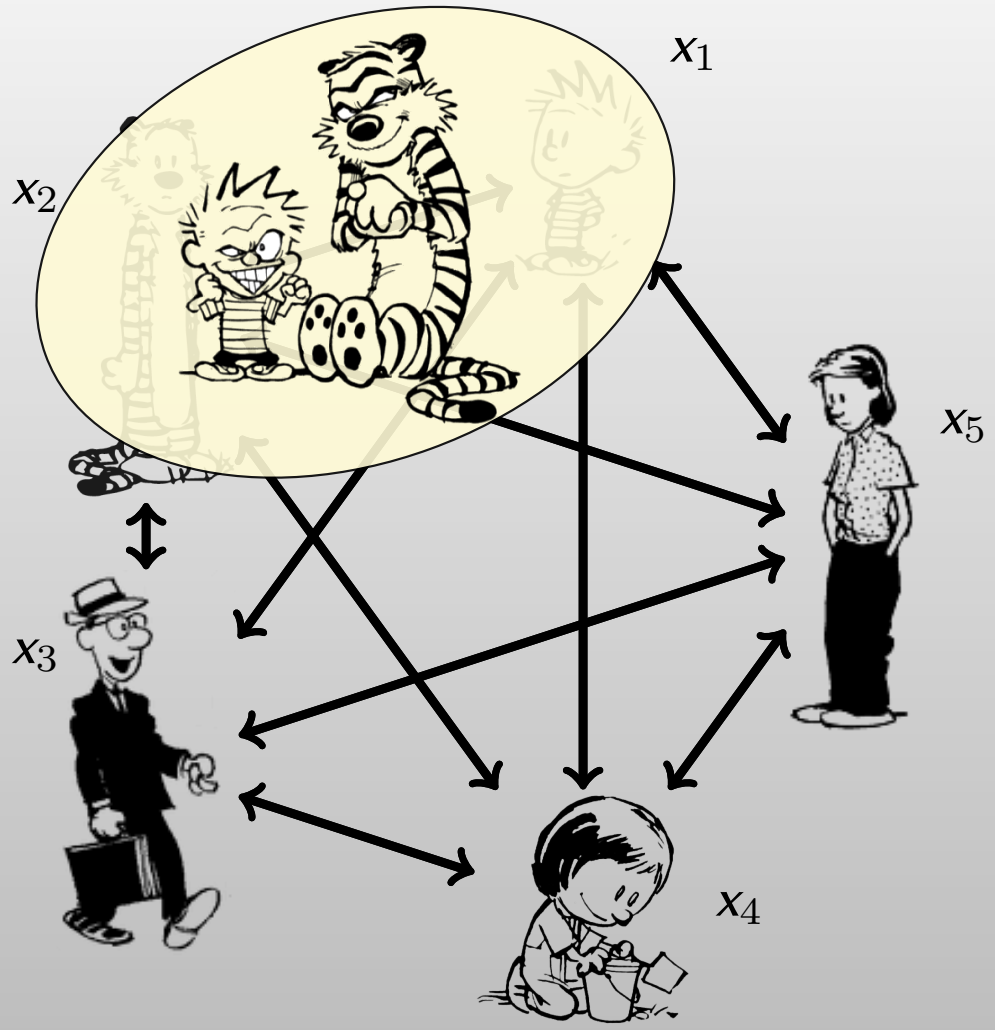
Security guarantees:

- Privacy ("learn no more than" prescribed output)
- Input independence
- Output consistency, etc..

Secure computation



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- *Ex:* election, auction, etc.

Security guarantees:

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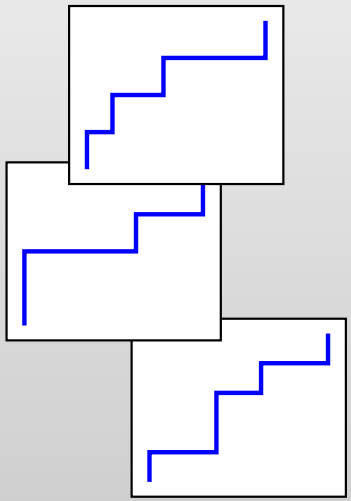
..even if some parties cheat, collude!

Examples: Sugar Beets

Beet Farmers



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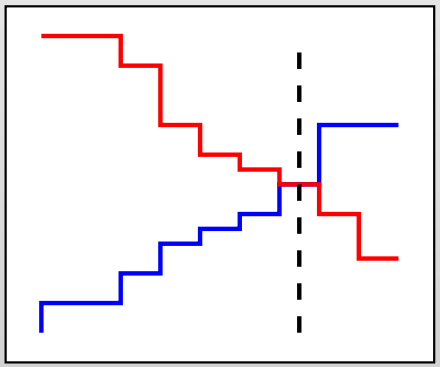
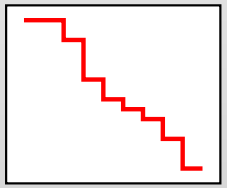




- Farmers make bids ("at price X, I will produce Y amount")
- Purchaser bids ("at price X, I will buy Y amount")
- **Market clearing price** (MCP): price at which total supply = demand
- 2009: MCP (+ bids at that price) computed via secure computation

Examples: Ad conversion

Ad impressions
alice@gmail.com
bob@gmail.com
charlie@gmail.com
dianne@gmail.com
edwin@gmail.com
frank@gmail.com
gina@gmail.com

In-store purchases	
albert@gmail.com	$80K
bob@gmail.com	**$160K**
caroline@gmail.com	$99K
edwin@gmail.com	**$99K**
felipe@gmail.com	$85K
frank@gmail.com	**$77K**
hilda@gmail.com	$113K

```
SELECT  SUM(amount)
FROM    ads, purchases
WHERE   ads.email = purchases.email
```

- Computed with secure computation by Google and its customers

Examples: Wage Equity Study

The New York Times

How Boston Is Trying to Close the Gender Pay Gap

Through pay-negotiation workshops and partnerships with more than 100 companies, the city is trying to help female workers match the salaries of male counterparts.

BOSTON WOMEN'S WORKFORCE COUNCIL REPORT 2017

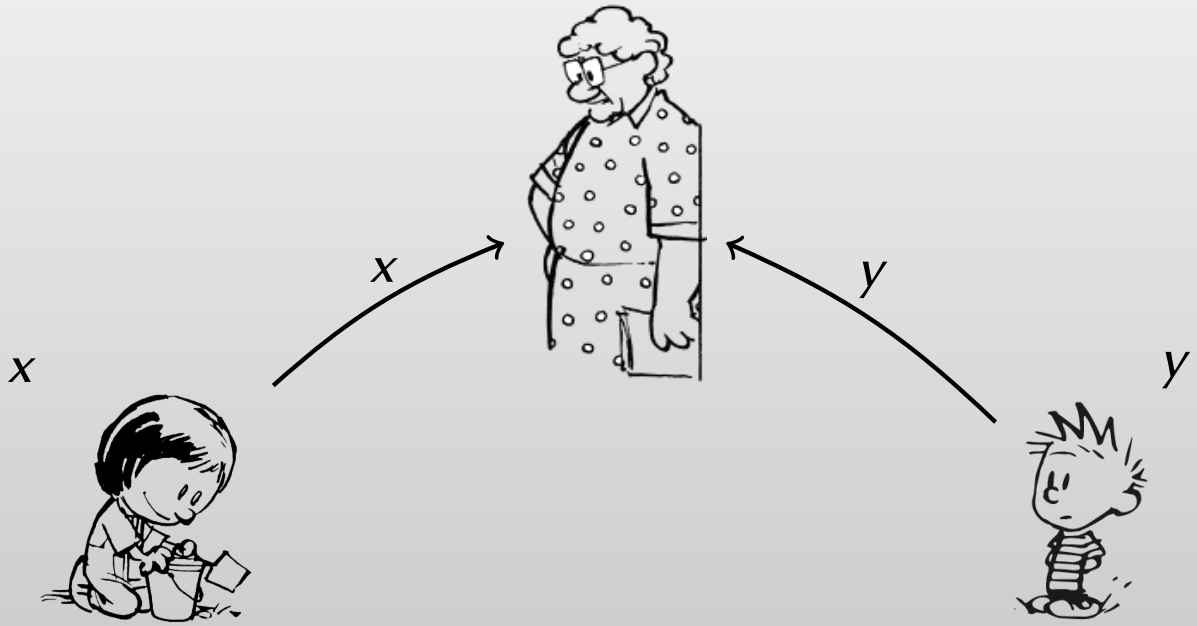
DATA SUBMISSION PROCESS:

Part of the commitment employers make when signing the Boston 100% Talent Compact is to anonymously report employee data to the BWWC biennially. The Software & Application Innovation Lab at Boston University's Rafik B. Hariri Institute of Computing and Computational Science & Engineering, the BWWC's data partner, developed a completely confidential reporting system from which anonymous data from multiple independent sources can be analyzed in the aggregate.

During the submission process, Compact signers submit their wage data in the aggregate form over a unique, web-based software program that employs encryption using a technique known as secure multi-party computation. During this process, individual compensation data never leaves each organization's server. The BWWC then receives aggregate data unconnected to any firm.

What does it mean to "securely" compute f?

Defining security: ideal world



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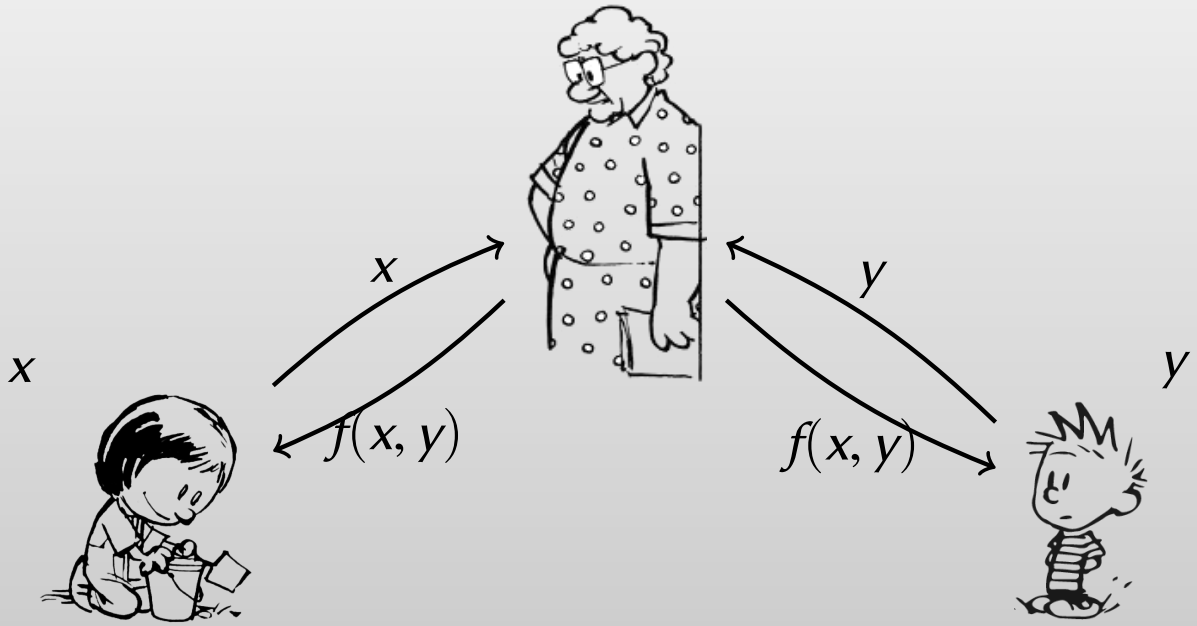

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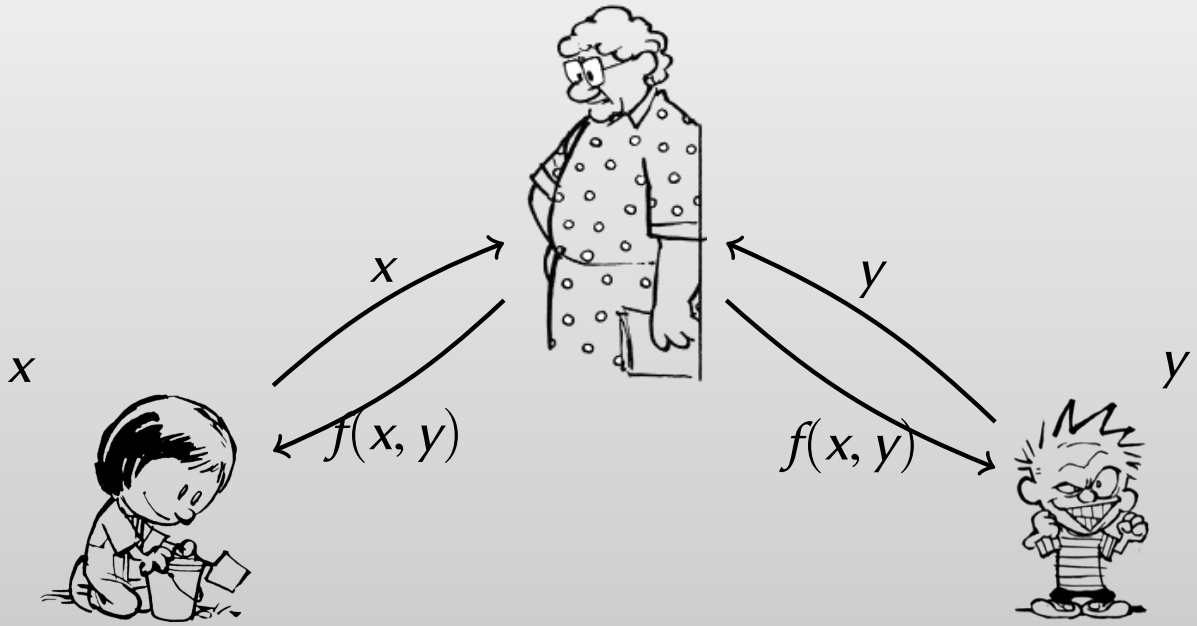
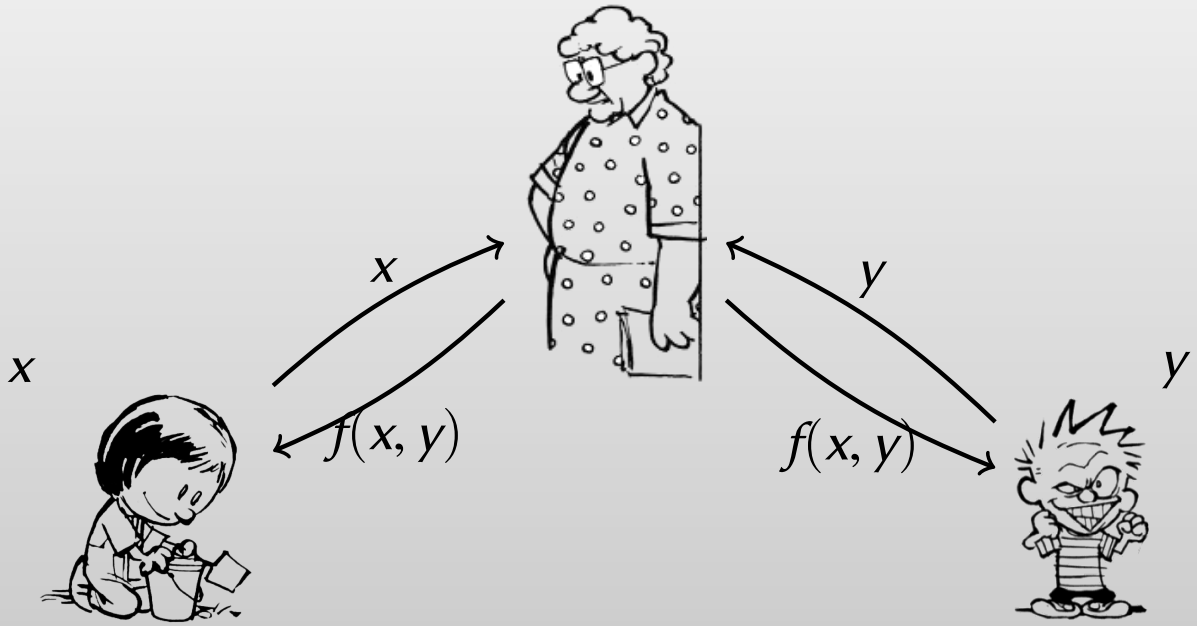


What can a corrupt party do in this **ideal world**?

Defining security: ideal world



What can a corrupt party do in this **ideal world**?

- Choose any input y (independent of x)
- Learn only $f(x, y)$, and nothing more
- Cause honest party to learn $f(x, y)$

Real-ideal paradigm [GoldwasserMicali84]

Security goal: real protocol interaction is ***as secure as*** *the ideal-world interaction*

Roadmap

1. **Secure computation:** Concepts & definitions
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Warm-up: garbled truth table

Alice does the following:

1. Write truth table of function f

1	1	$f(1,1)$
1	2	$f(1,2)$
1	3	$f(1,3)$
1	4	$f(1,4)$
2	1	$f(2,1)$
2	2	$f(2,2)$
2	3	$f(2,3)$
2	4	$f(2,4)$
3	1	$f(3,1)$
3	2	$f(3,2)$
3	3	$f(3,3)$
3	4	$f(3,4)$
4	1	$f(4,1)$
4	2	$f(4,2)$
4	3	$f(4,3)$
4	4	$f(4,4)$

Warm-up: garbled truth table

Alice does the following:

1. Write truth table of function f
2. For each possible input, choose random **cryptographic key**

A_1	B_1	$f(1,1)$
A_1	B_2	$f(1,2)$
A_1	B_3	$f(1,3)$
A_1	B_4	$f(1,4)$
A_2	B_1	$f(2,1)$
A_2	B_2	$f(2,2)$
A_2	B_3	$f(2,3)$
A_2	B_4	$f(2,4)$
A_3	B_1	$f(3,1)$
A_3	B_2	$f(3,2)$
A_3	B_3	$f(3,3)$
A_3	B_4	$f(3,4)$
A_4	B_1	$f(4,1)$
A_4	B_2	$f(4,2)$
A_4	B_3	$f(4,3)$
A_4	B_4	$f(4,4)$

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Alice does the following:

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3. Encrypt each output with corresponding keys

$$
\begin{array}{|l|}
\hline
\mathbb{E}_{A_1, B_1}(f(1,1)) \\
\mathbb{E}_{A_1, B_2}(f(1,2)) \\
\mathbb{E}_{A_1, B_3}(f(1,3)) \\
\mathbb{E}_{A_1, B_4}(f(1,4)) \\
\mathbb{E}_{A_2, B_1}(f(2,1)) \\
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\mathbb{E}_{A_3, B_2}(f(3,2)) \\
\mathbb{E}_{A_3, B_3}(f(3,3)) \\
\mathbb{E}_{A_3, B_4}(f(3,4)) \\
\mathbb{E}_{A_4, B_1}(f(4,1)) \\
\mathbb{E}_{A_4, B_2}(f(4,2)) \\
\mathbb{E}_{A_4, B_3}(f(4,3)) \\
\mathbb{E}_{A_4, B_4}(f(4,4)) \\
\hline
\end{array}
$$

Warm-up: garbled truth table

Alice does the following:

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4. Randomly permute ciphertexts, send to Bob

$$
\begin{array}{|l|}
\hline
\mathbb{E}_{A_3,B_4}(f(3,4)) \\
\mathbb{E}_{A_4,B_3}(f(4,3)) \\
\mathbb{E}_{A_3,B_3}(f(3,3)) \\
\mathbb{E}_{A_2,B_3}(f(2,3)) \\
\mathbb{E}_{A_4,B_2}(f(4,2)) \\
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\mathbb{E}_{A_4,B_4}(f(4,4)) \\
\mathbb{E}_{A_1,B_4}(f(1,4)) \\
\mathbb{E}_{A_2,B_2}(f(2,2)) \\
\mathbb{E}_{A_1,B_2}(f(1,2)) \\
\mathbb{E}_{A_2,B_1}(f(2,1)) \\
\mathbb{E}_{A_1,B_3}(f(1,3)) \\
\mathbb{E}_{A_4,B_1}(f(4,1)) \\
\mathbb{E}_{A_3,B_1}(f(3,1)) \\
\mathbb{E}_{A_1,B_1}(f(1,1)) \\
\mathbb{E}_{A_3,B_2}(f(3,2)) \\
\hline
\end{array}
$$

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?? **Somehow** Bob obtains "correct" A_x, B_y ??

$$\mathbb{E}_{A_3,B_4}(f(3,4))$$
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Through trial decryption, Bob learns only $f(x, y)$

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$\mathbb{E}_{A_2, B_3}(f(2,3))$
$\mathbb{E}_{A_4, B_2}(f(4,2))$
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$\mathbb{E}_{A_3, B_1}(f(3,1))$
$\mathbb{E}_{A_1, B_1}(f(1,1))$
$\mathbb{E}_{A_3, B_2}(f(3,2))$

Security of warm-up protocol

Suffices to show that Bob's view in the protocol can be **simulated** given just Bob's ideal input/output.

Bob's view (real): $\approx$ **Simulated view:**

A_4, B_2	A^*, B^*
$\mathbb{E}_{A_3, B_4}(f(3,4))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_4, B_3}(f(4,3))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_3, B_3}(f(3,3))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_2, B_3}(f(2,3))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_4, B_2}(f(4,2))$	$\mathbb{E}_{A^*, B^*}(f(x,y))$
$\mathbb{E}_{A_2, B_4}(f(2,4))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_4, B_4}(f(4,4))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_1, B_4}(f(1,4))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_2, B_2}(f(2,2))$	$\mathbb{E}_{A_?, B_?}(0)$
$\mathbb{E}_{A_1, B_2}(f(1,2))$	$\mathbb{E}_{A_?, B_?}(0)$
$\vdots$	$\vdots$

Extending warm-up protocol

Problem: Cost scales with the truth table size of f!

Problem: How does Bob magically learn "correct" A_x, B_y?

Extending warm-up protocol

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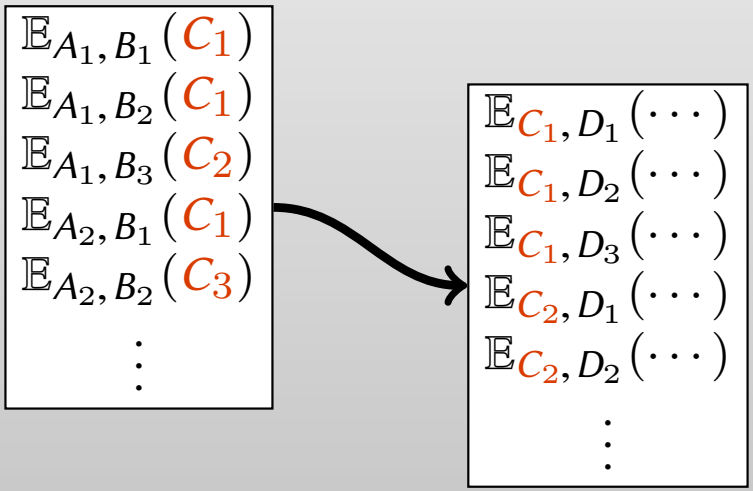
- Idea: instead of encrypting outputs, encrypt **keys to yet more garbled tables**



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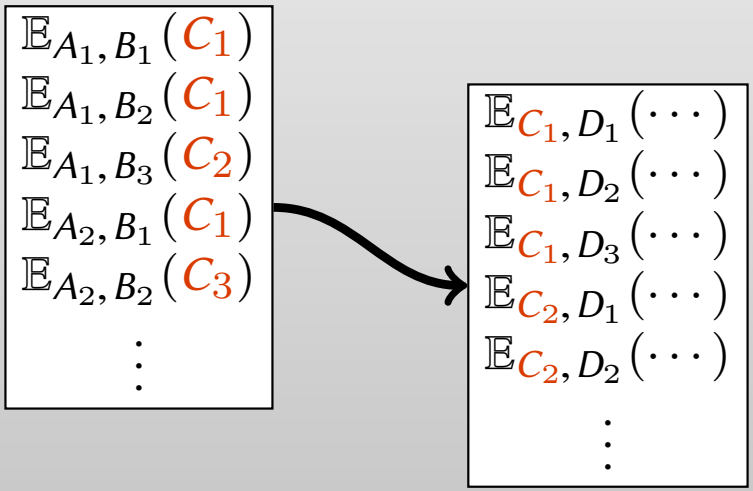
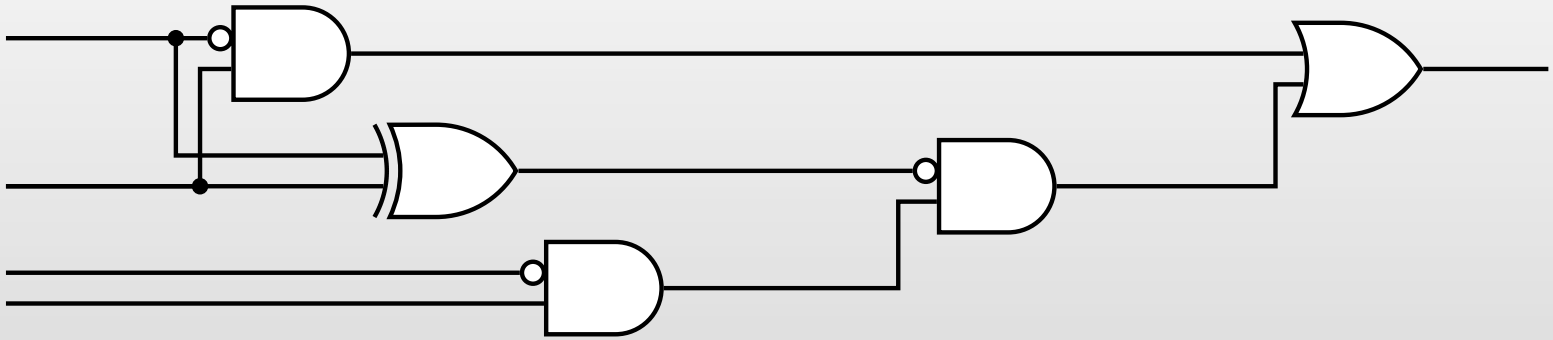


Problem: How does Bob magically learn "correct" A_x, B_y?

- Discuss later (oblivious transfer)

Garbled circuit framework [Yao86]



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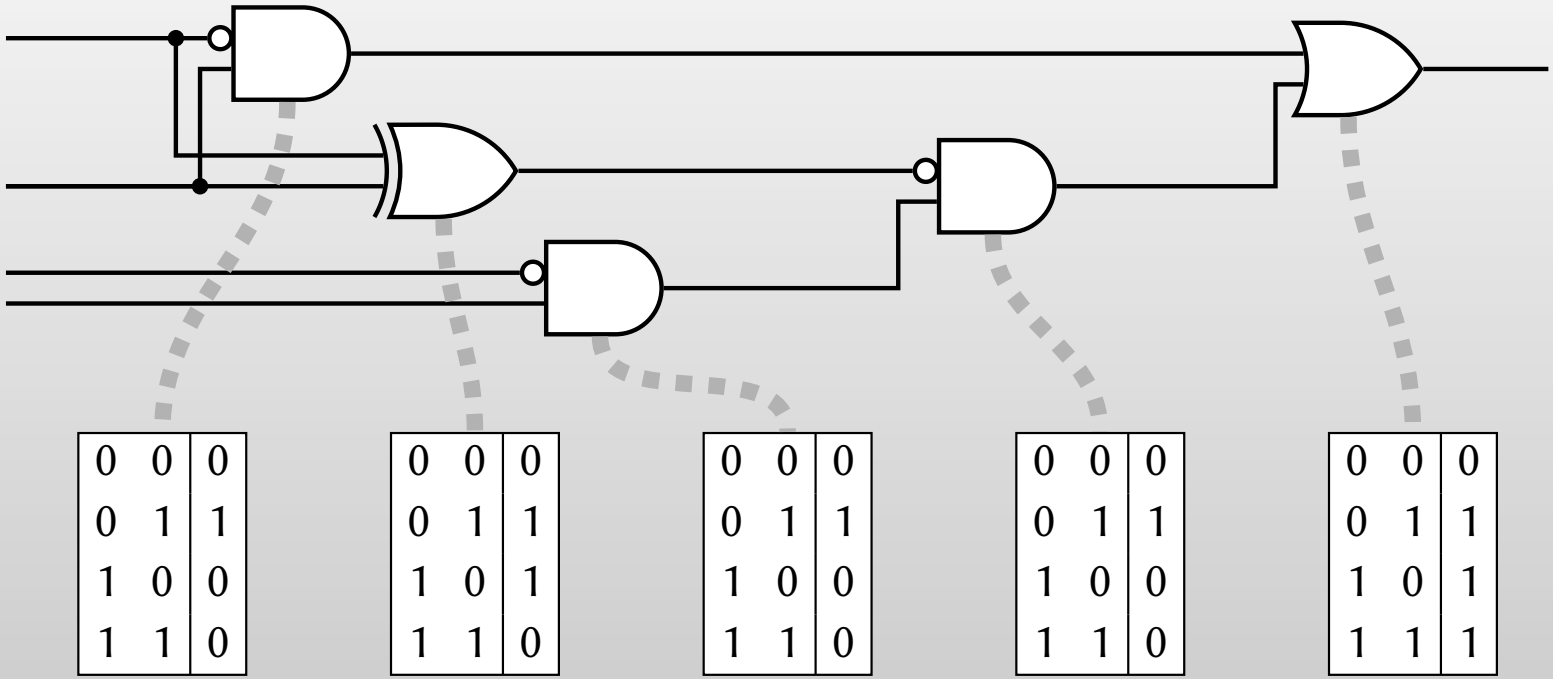

0	0	0
0	1	1
1	0	0
1	1	0

0	0	0
0	1	1
1	0	1
1	1	0

0	0	0
0	1	1
1	0	0
1	1	0

0	0	0
0	1	1
1	0	0
1	1	0

0	0	0
0	1	1
1	0	1
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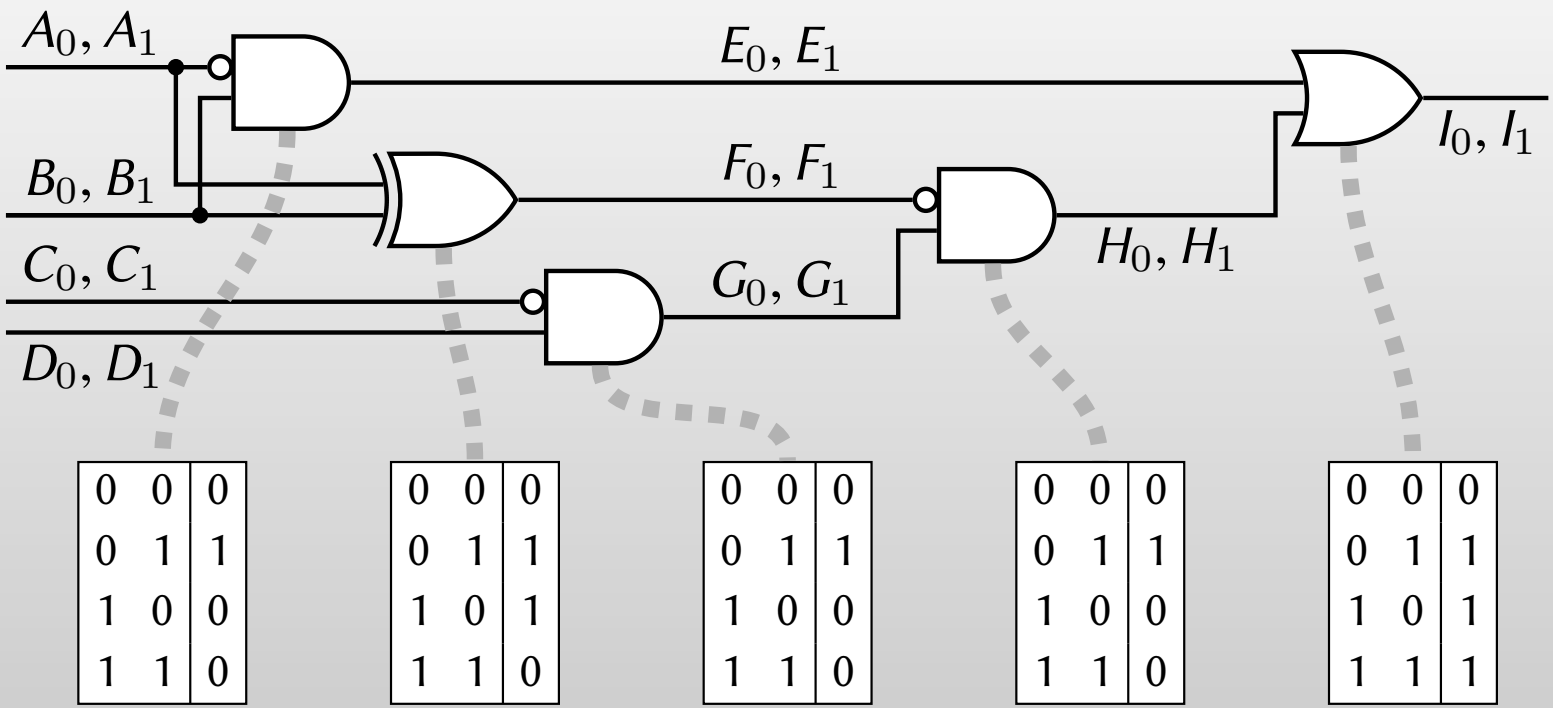

Garbling a circuit:

- Pick random **labels** W_0, W_1 on each wire

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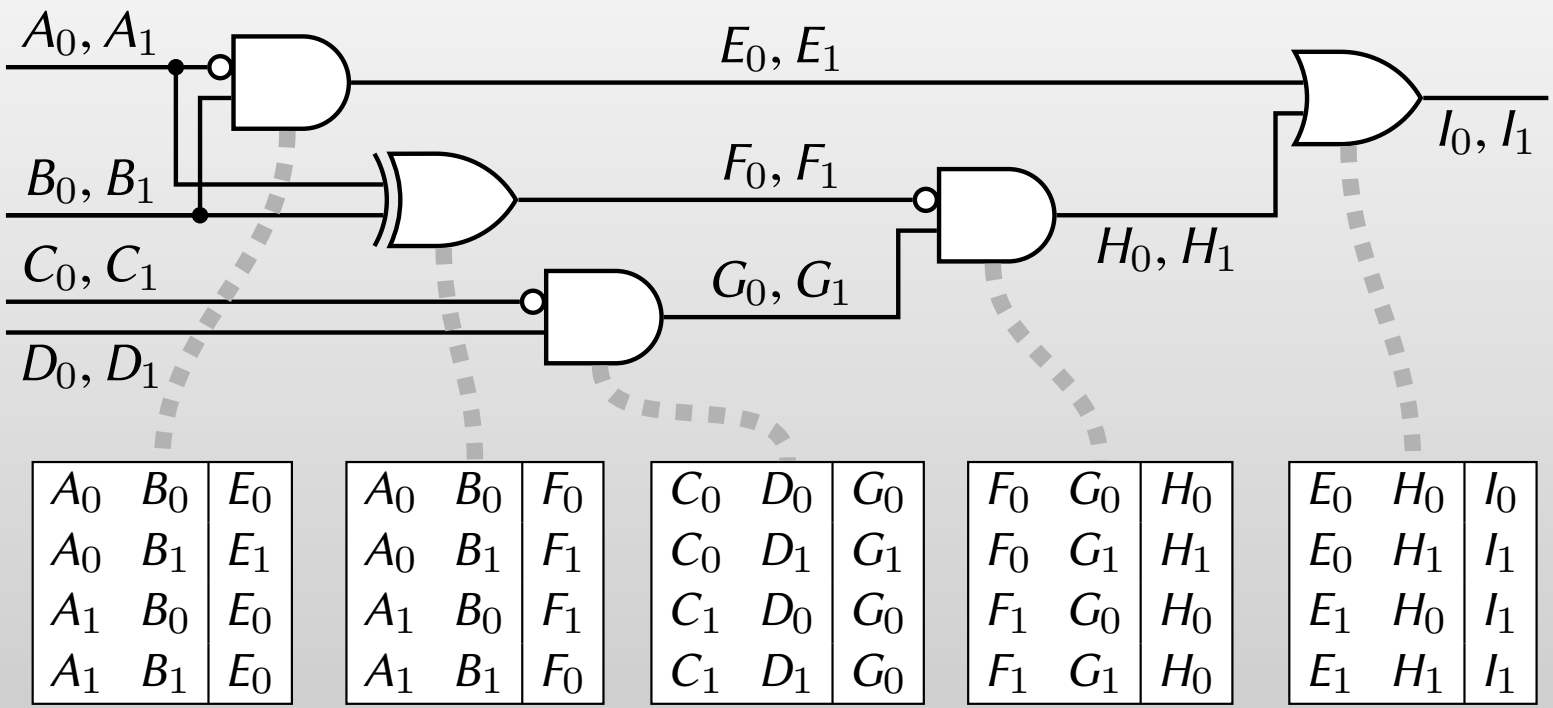

A_0	B_0	E_0
A_0	B_1	E_1
A_1	B_0	E_0
A_1	B_1	E_0

A_0	B_0	F_0
A_0	B_1	F_1
A_1	B_0	F_1
A_1	B_1	F_0

C_0	D_0	G_0
C_0	D_1	G_1
C_1	D_0	G_0
C_1	D_1	G_0

F_0	G_0	H_0
F_0	G_1	H_1
F_1	G_0	H_0
F_1	G_1	H_0

E_0	H_0	I_0
E_0	H_1	I_1
E_1	H_0	I_1
E_1	H_1	I_1

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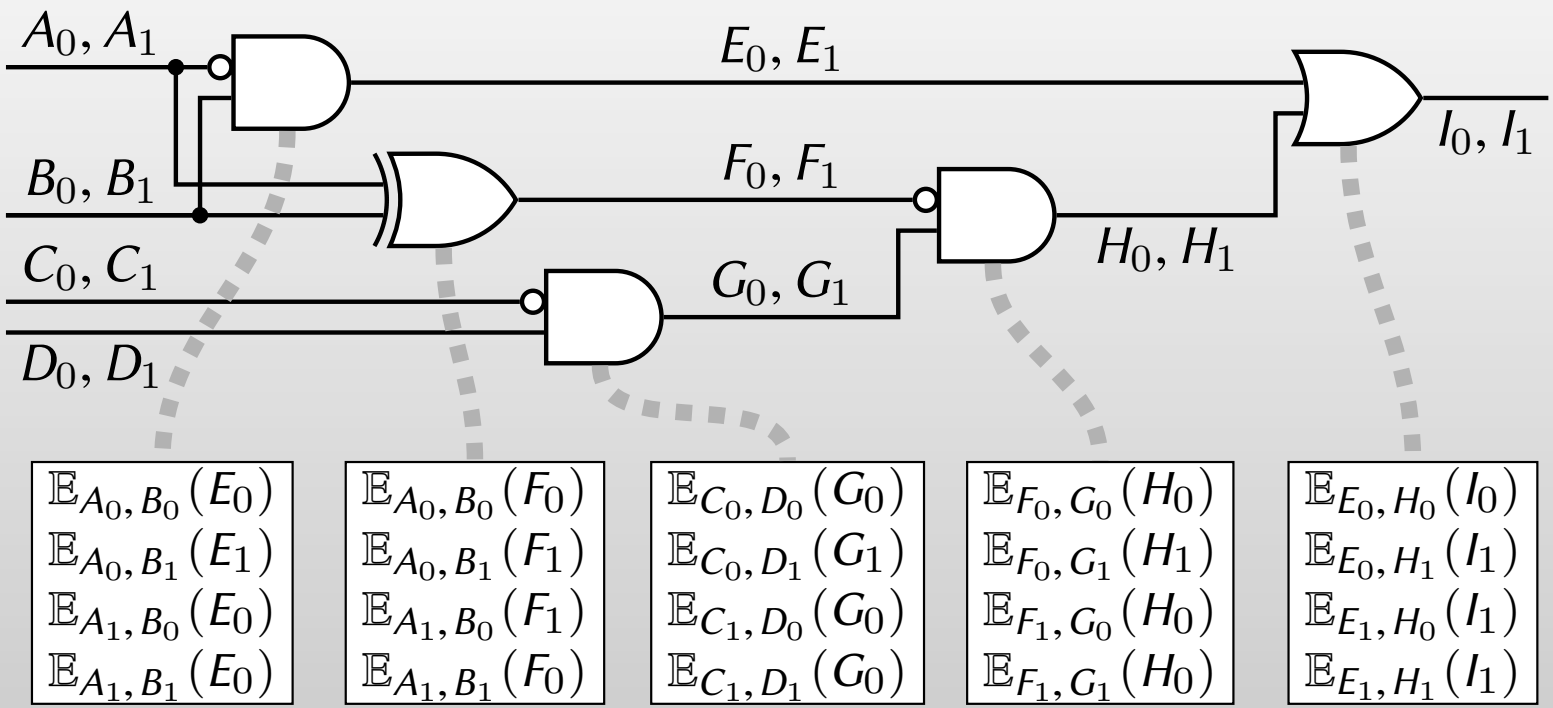

Garbling a circuit:

- Pick random **labels** W_0, W_1 on each wire
- "Encrypt" truth table of each gate

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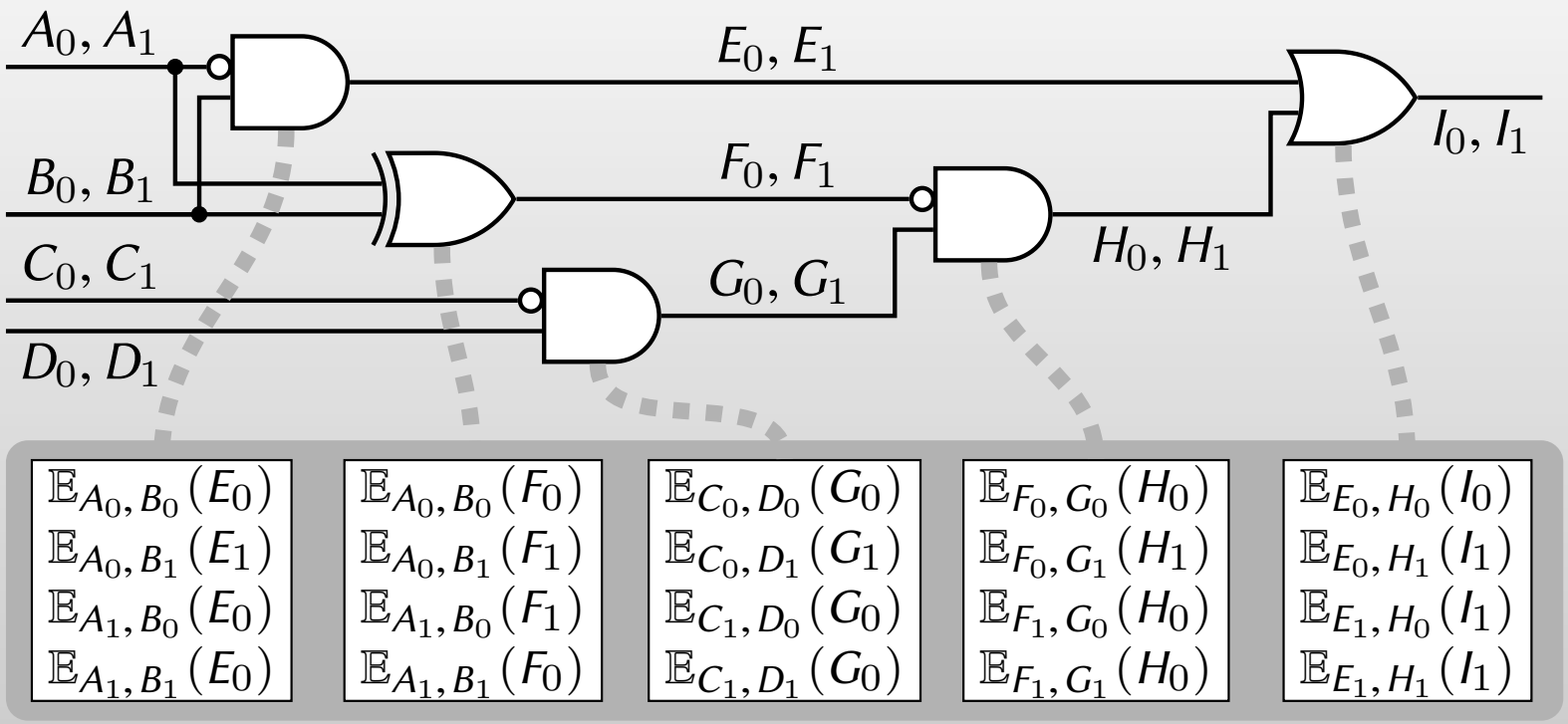

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- **Garbled circuit** ≡ all encrypted gates

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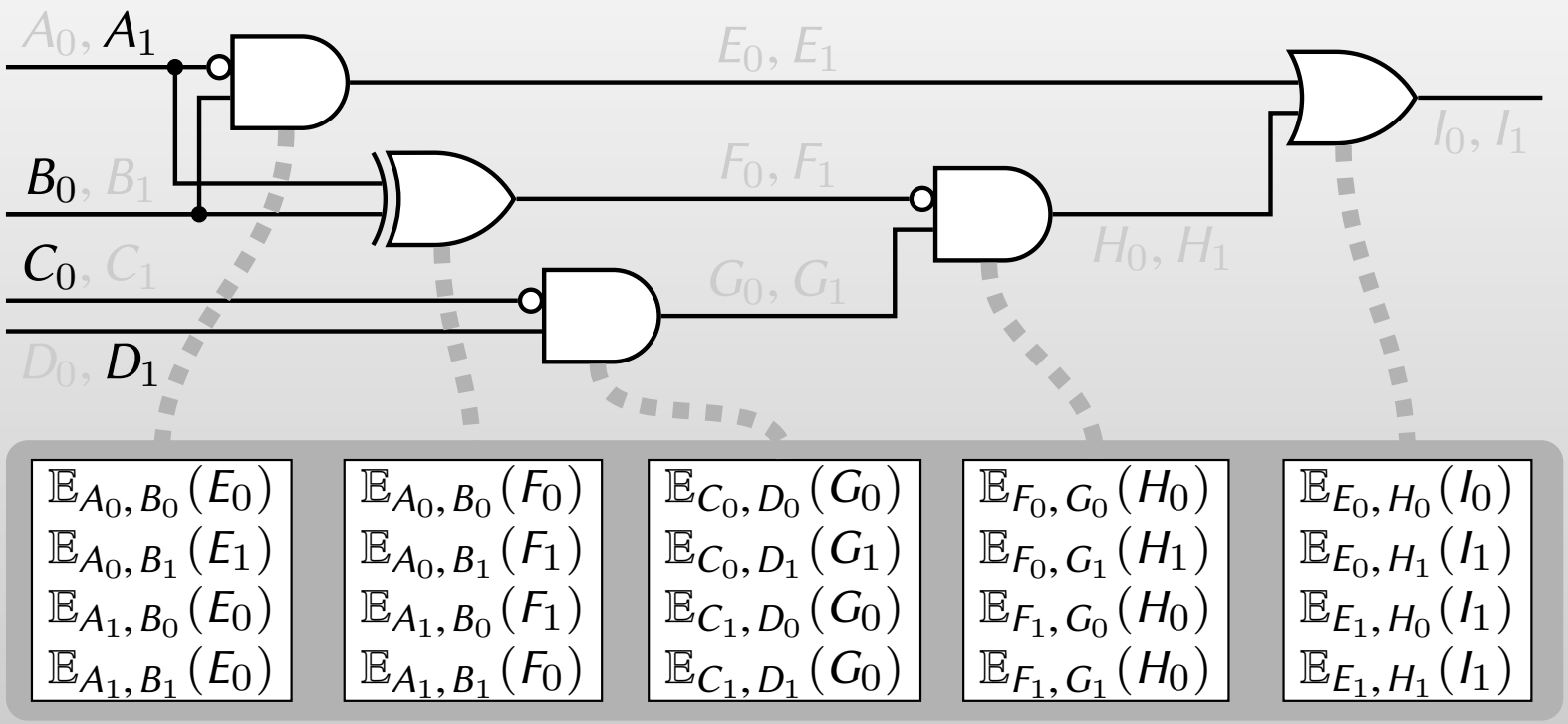

Garbling a circuit:

- Pick random **labels** W_0, W_1 on each wire
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- **Garbled circuit** ≡ all encrypted gates
- **Garbled encoding** ≡ one label per wire

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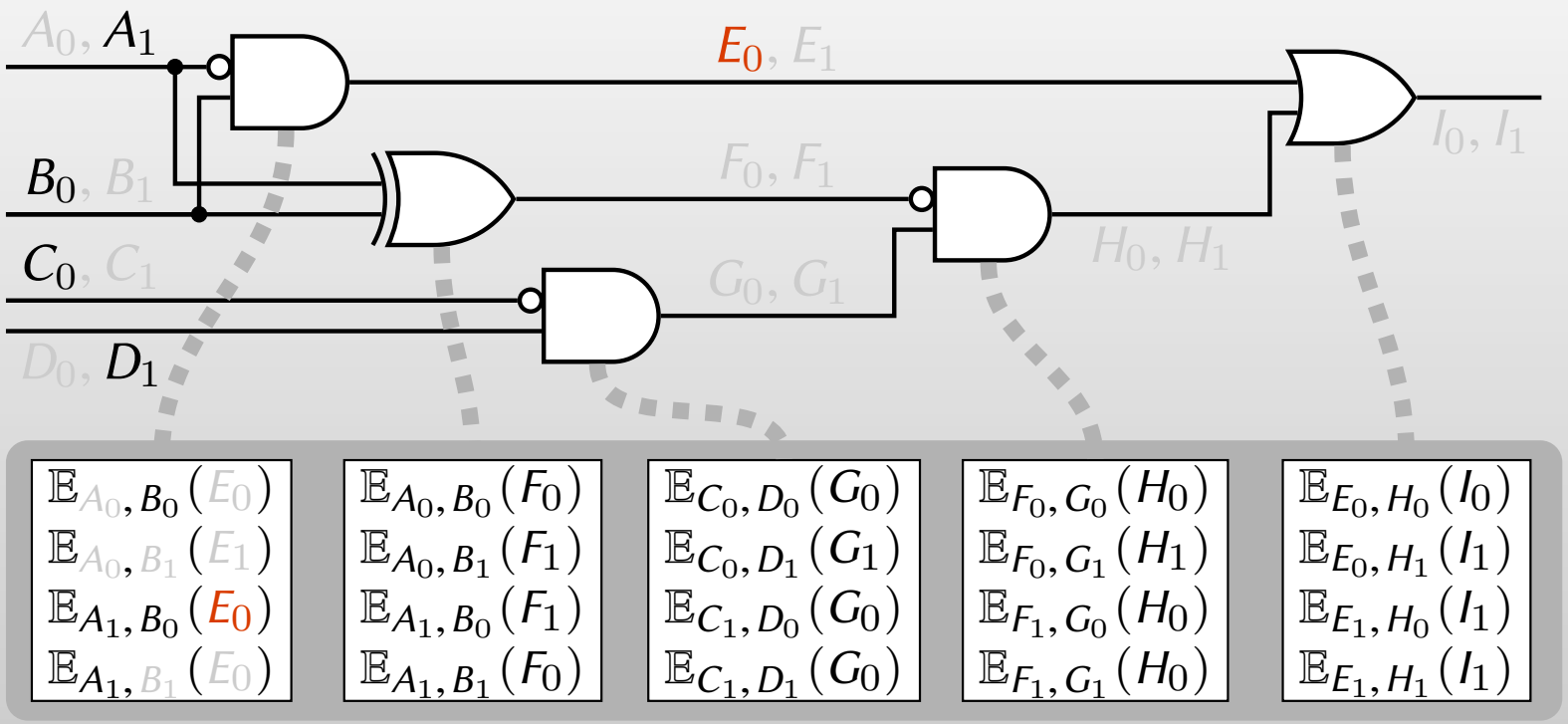

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Garbled evaluation:

- Only one ciphertext per gate is decryptable

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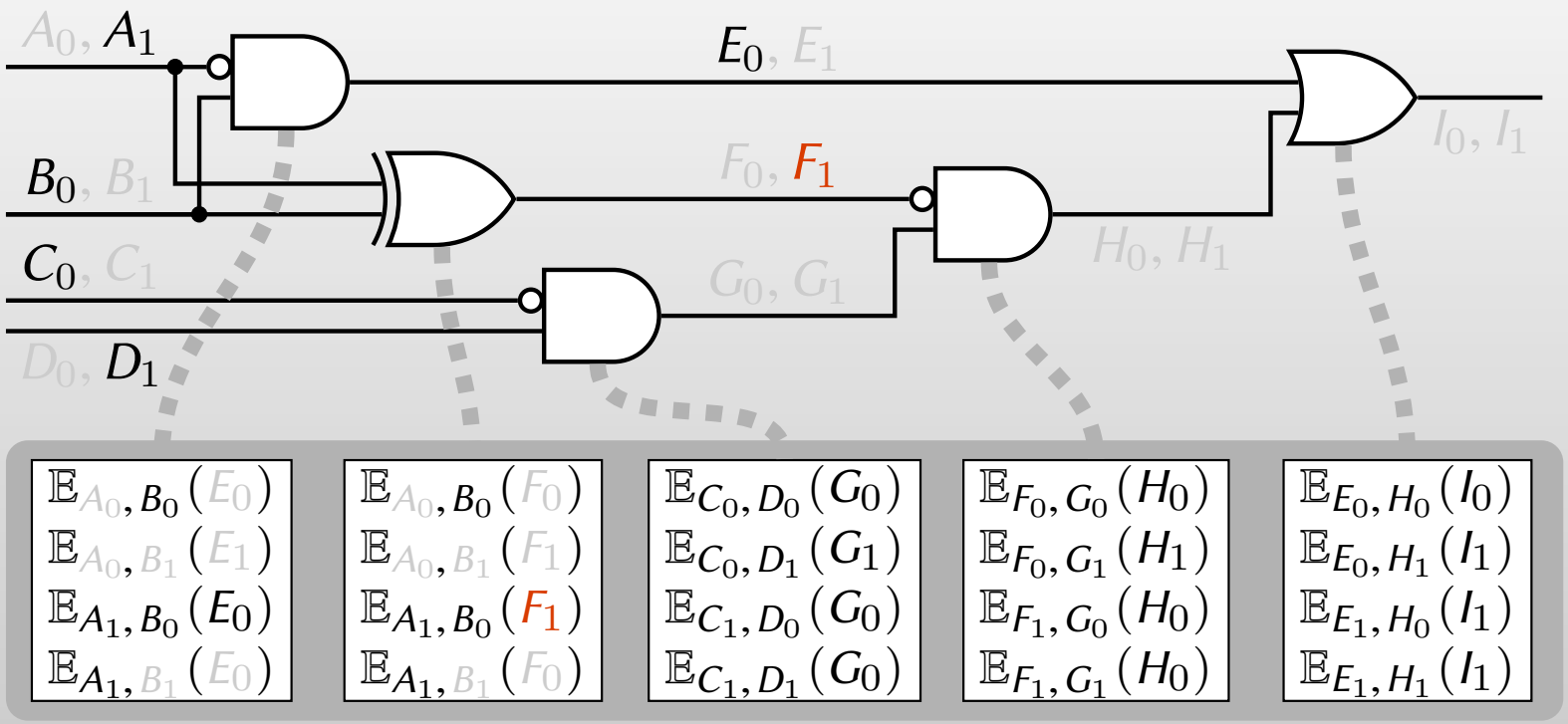

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Garbled evaluation:

- Only one ciphertext per gate is decryptable
- Result of decryption = value on outgoing wire

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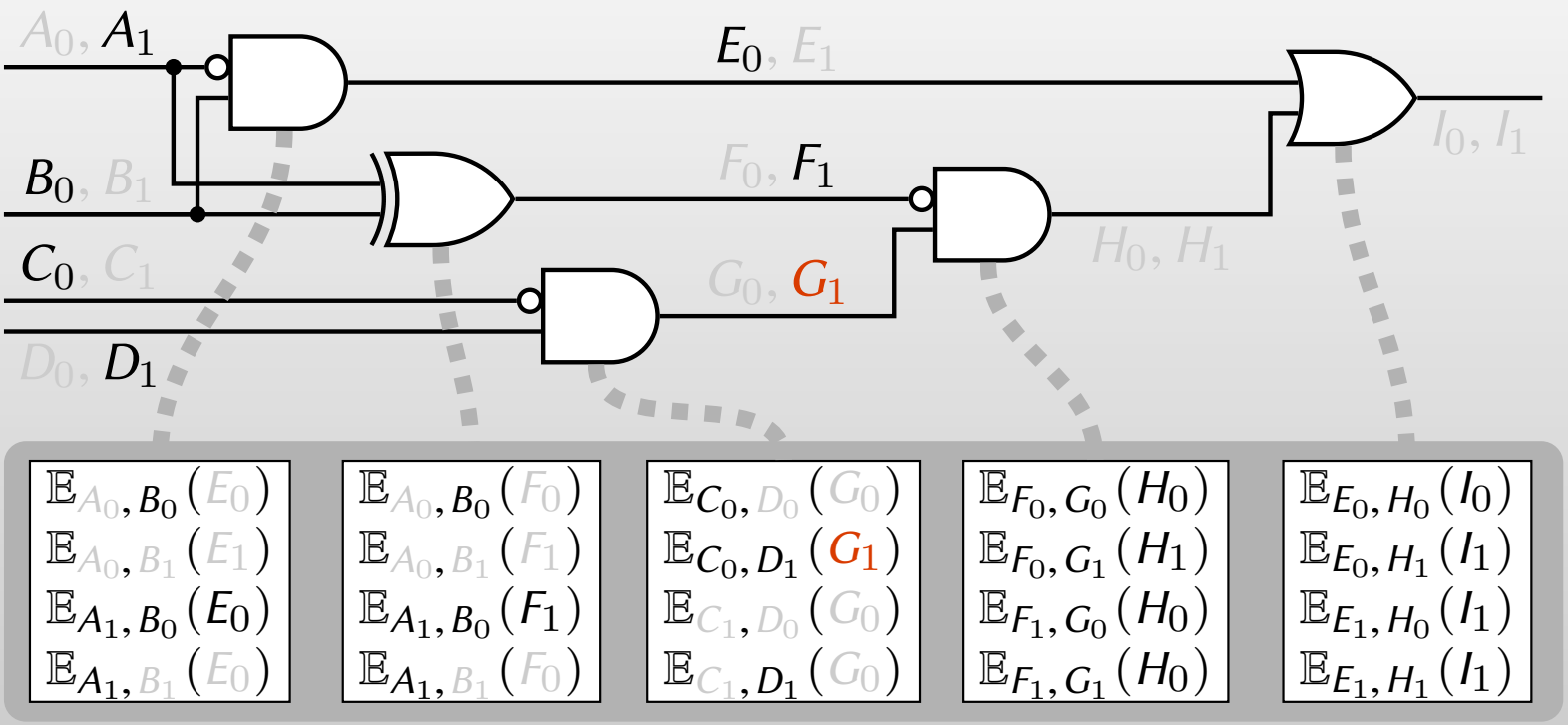

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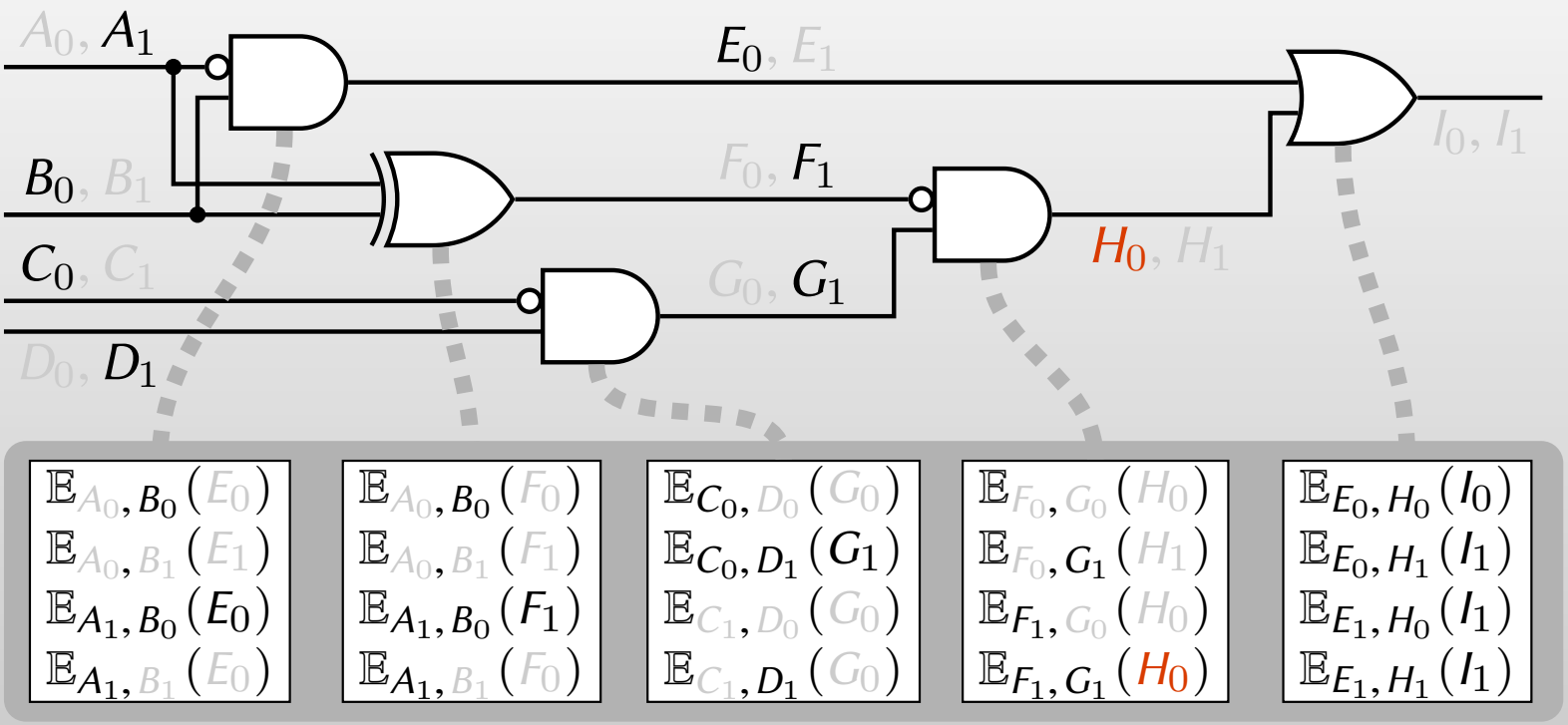

Garbling a circuit:

- Pick random **labels** W_0, W_1 on each wire
- "Encrypt" truth table of each gate
- **Garbled circuit** ≡ all encrypted gates
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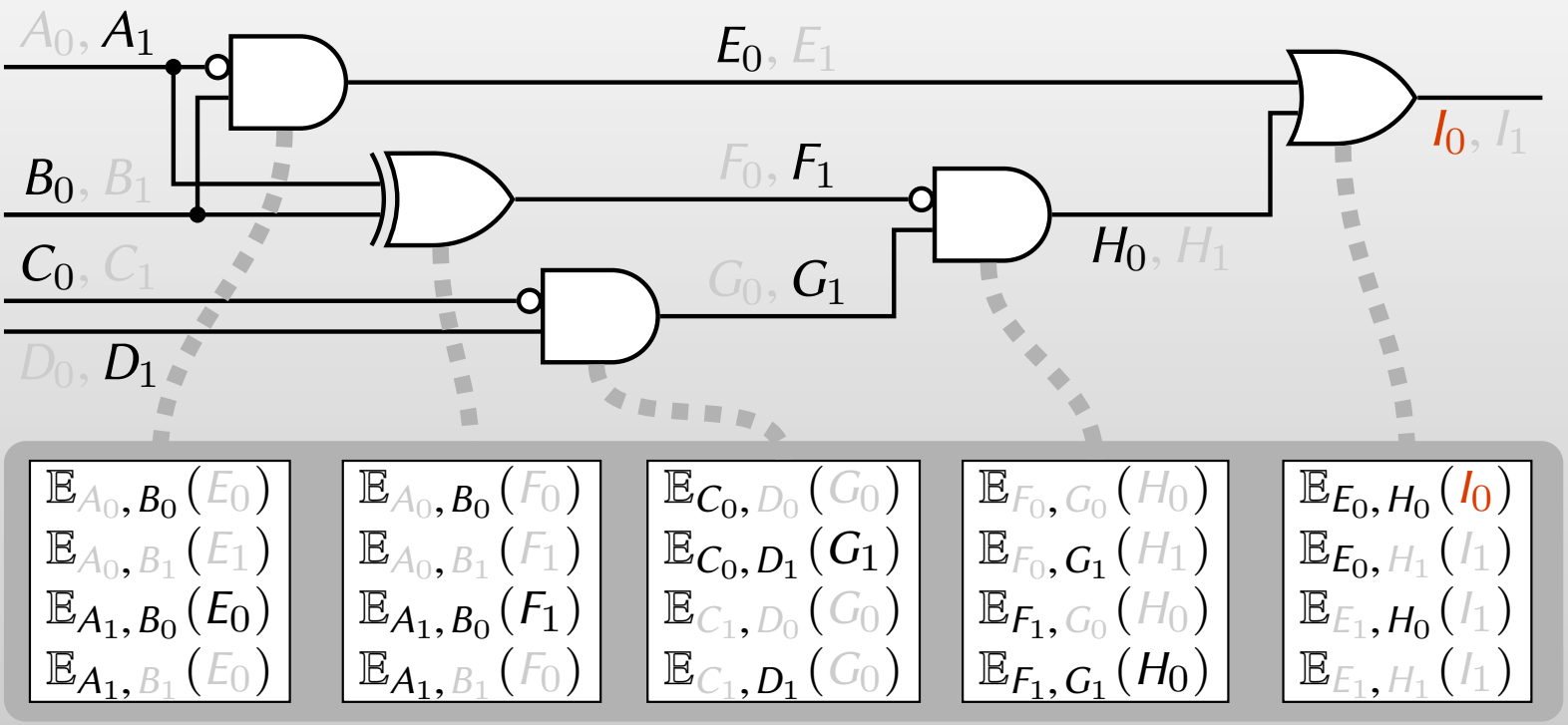

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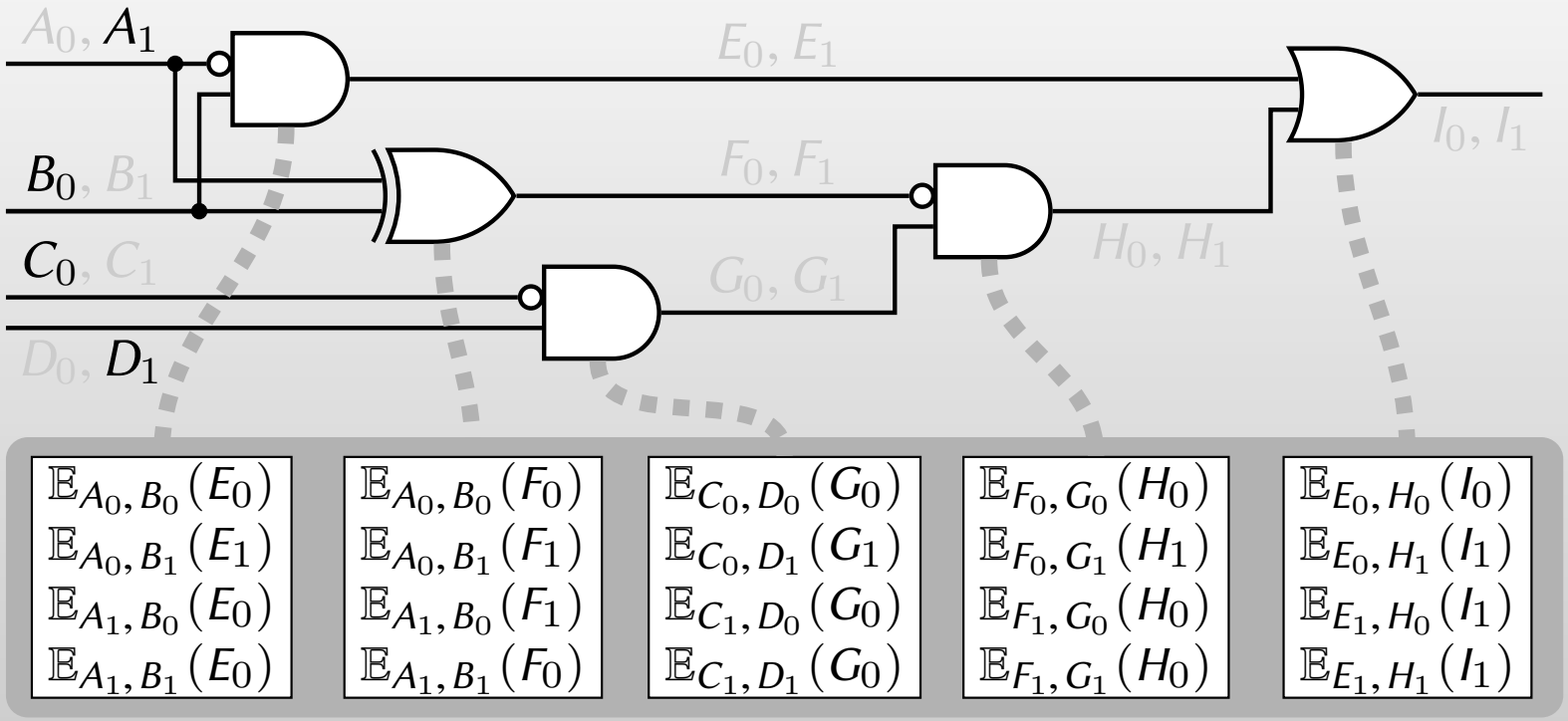

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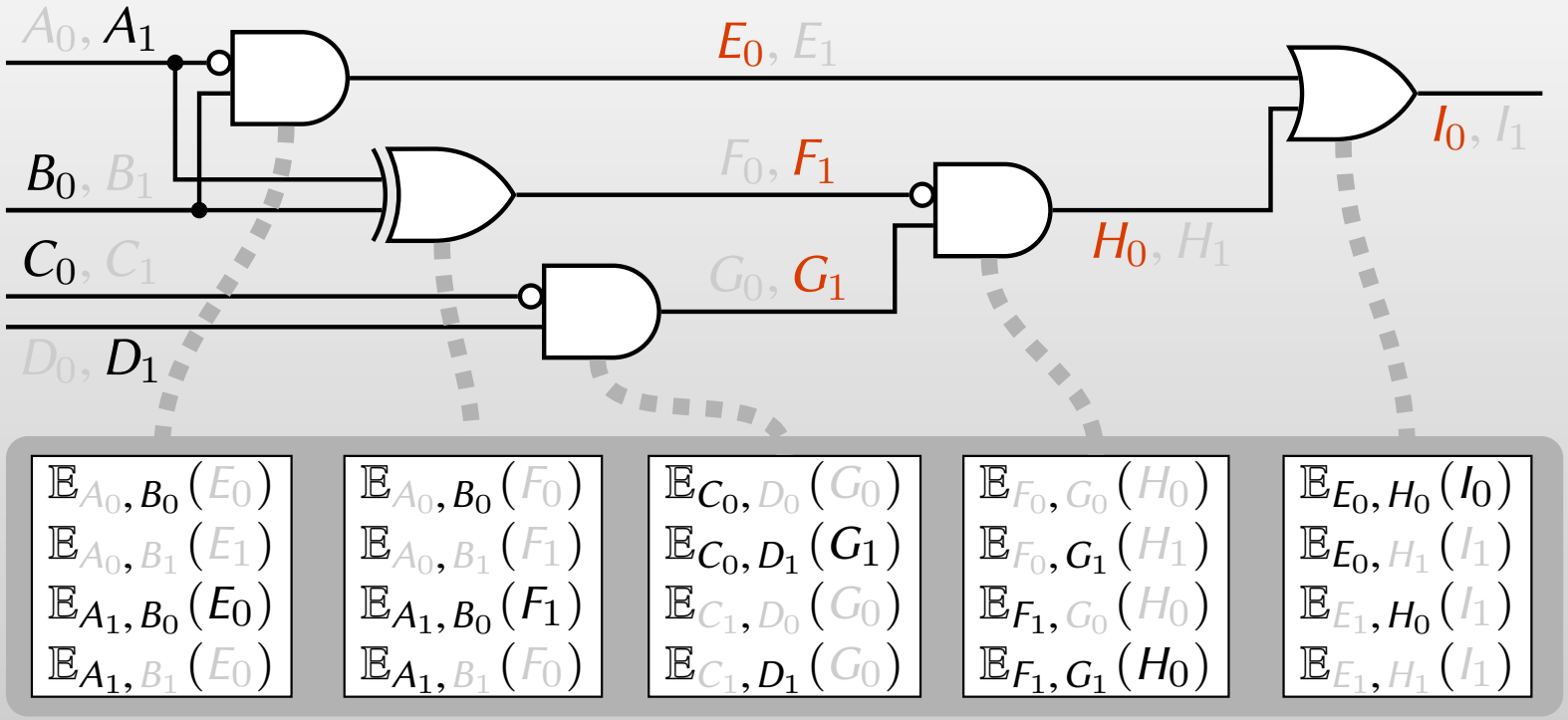

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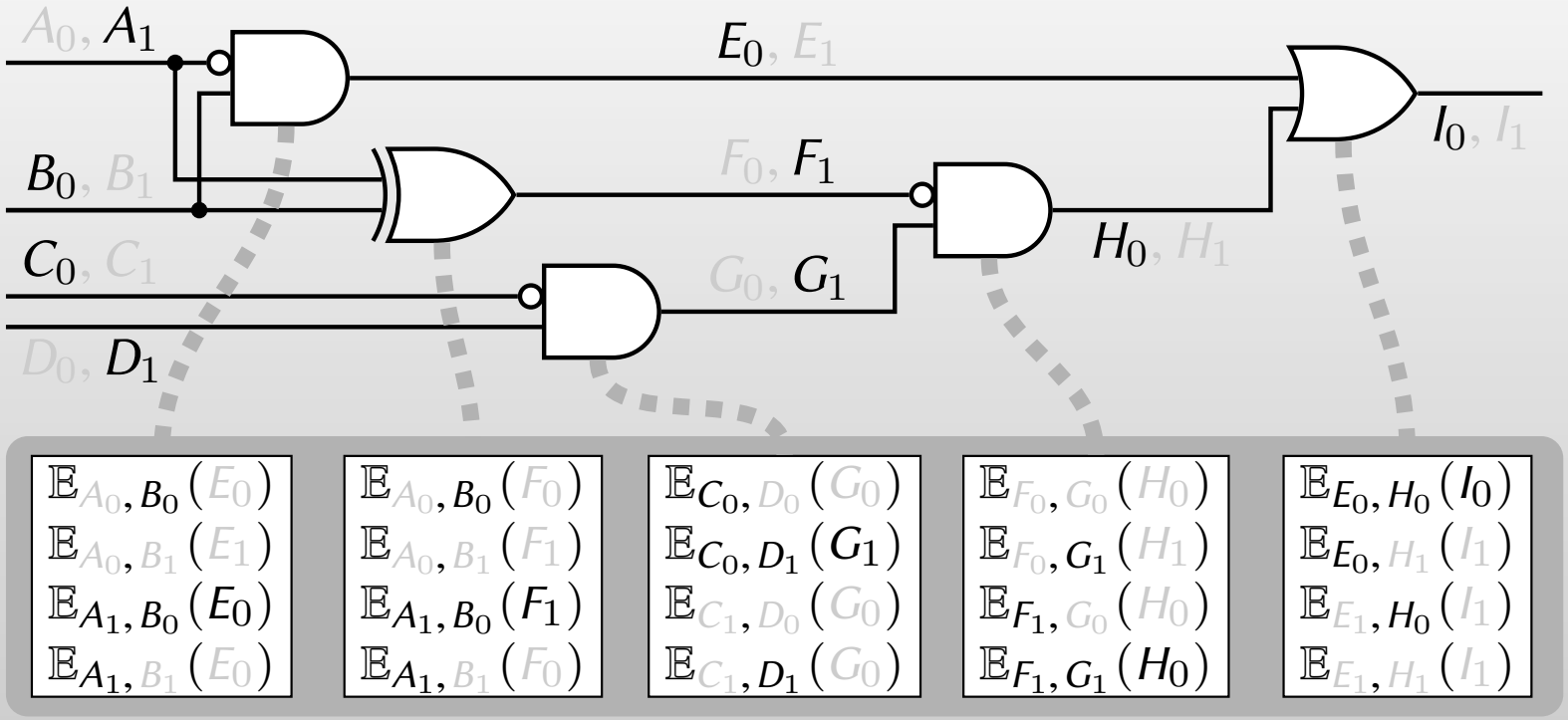
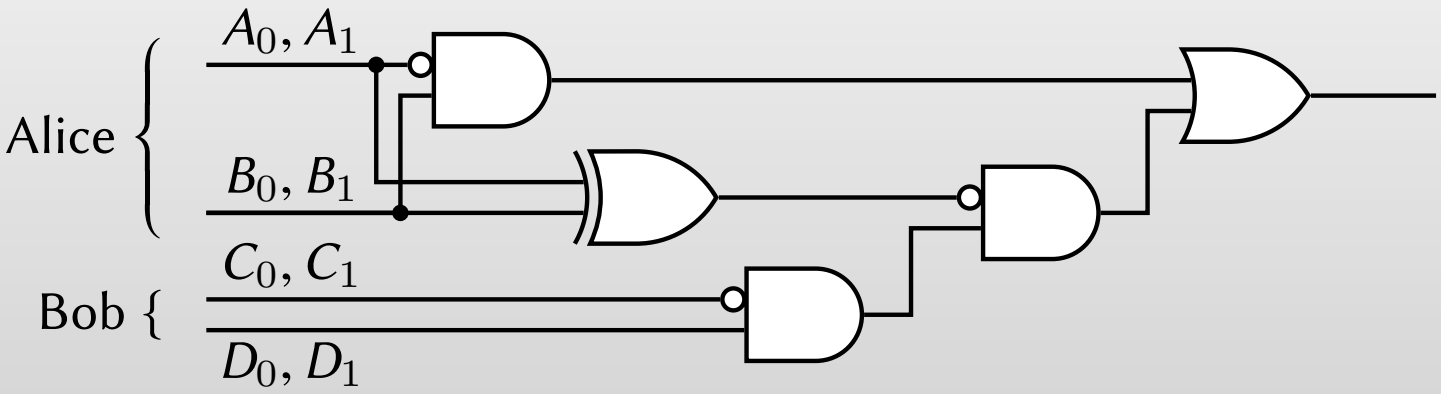


Key idea: Given garbled circuit + garbled input . . .

- . . . Only thing you can do is **(blindly) evaluate circuit** on that input
- Learn only 1 label per wire: hard to guess "complementary" label
- Seeing a single label hides logical value on wire, although . . .
- Revealing both labels on *output wires* leaks *only* circuit output

Oblivious transfer

How does evaluator (Bob) get the garbled input?



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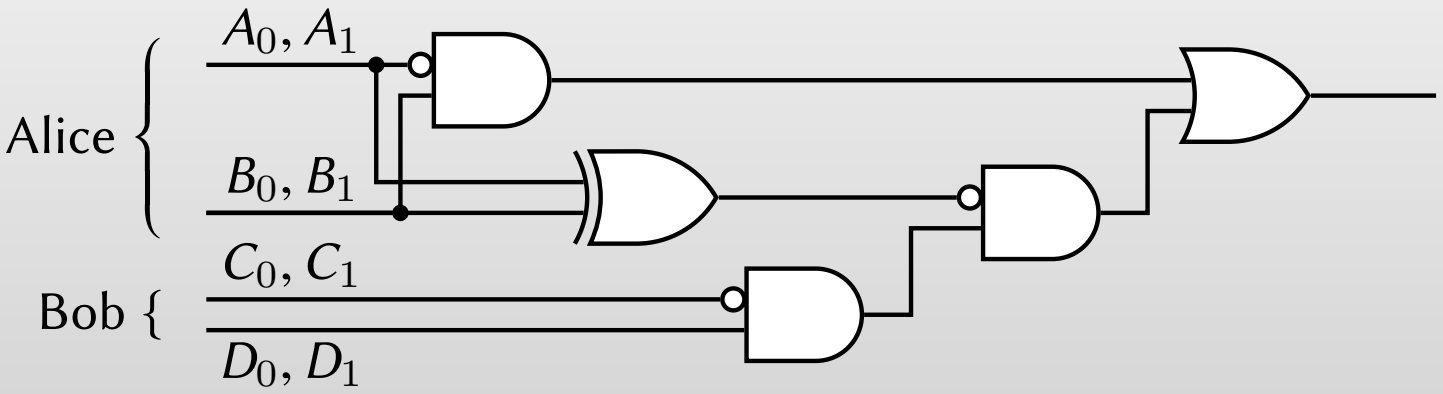

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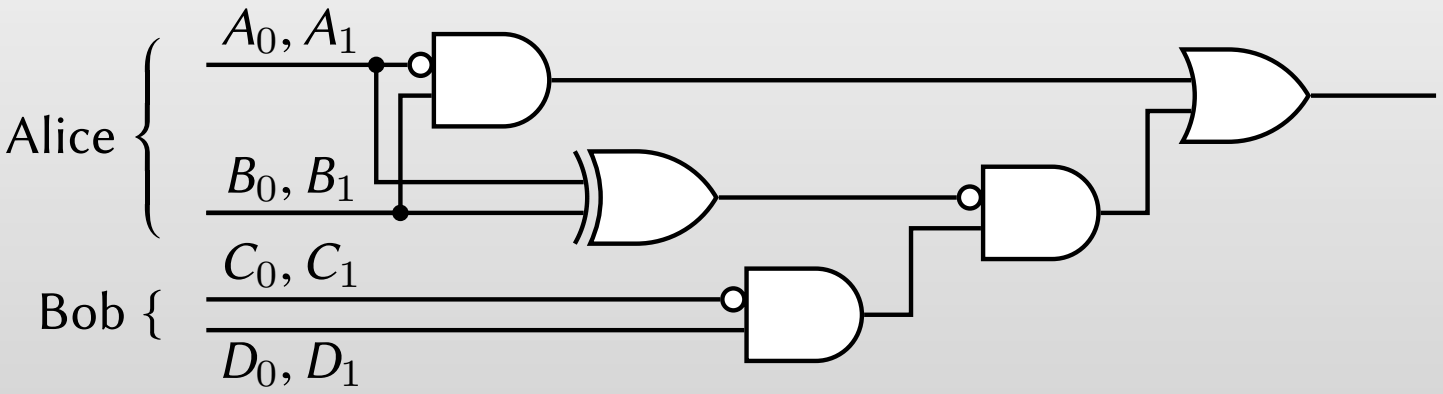

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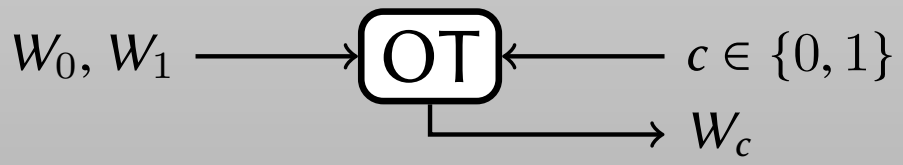
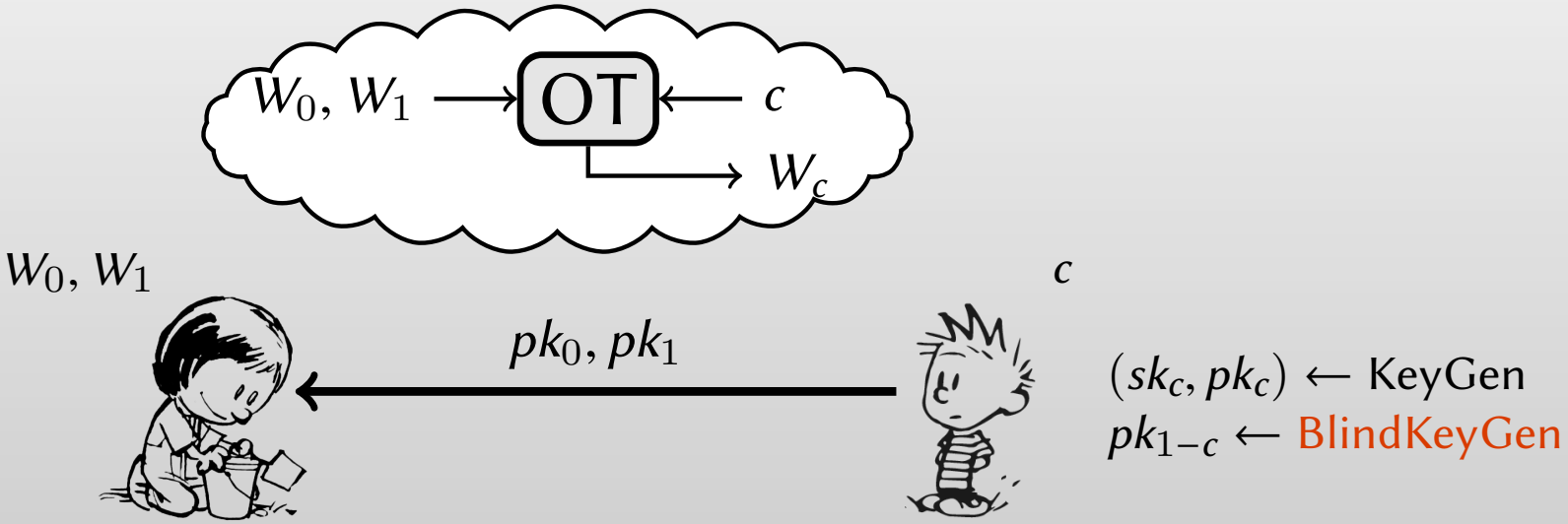
Evaluator's inputs: We need the following "gadget" (oblivious transfer):



How to construct OT?



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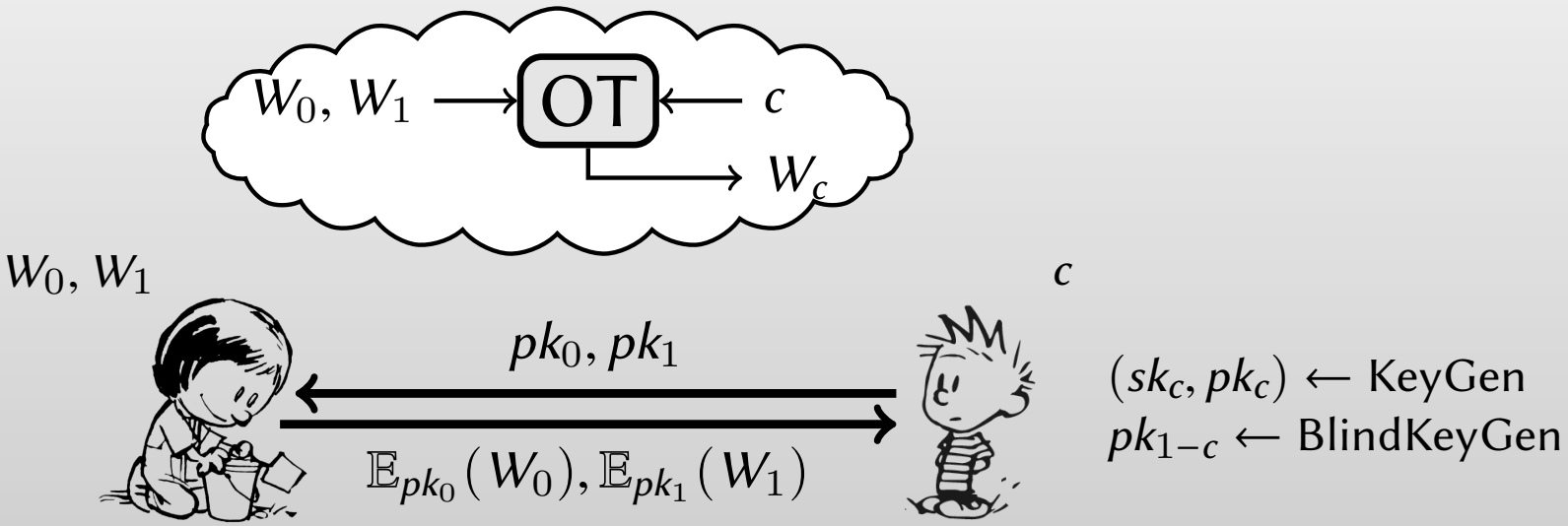

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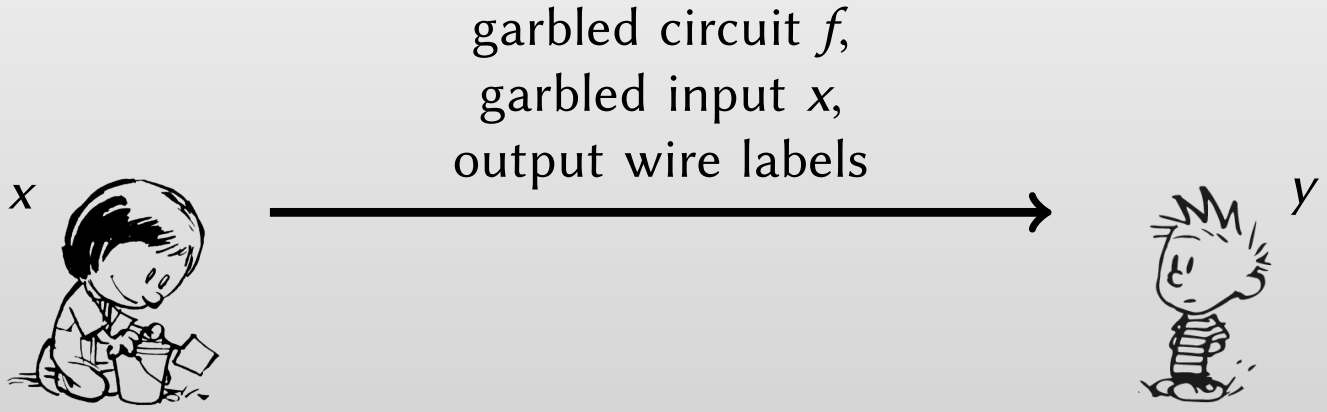

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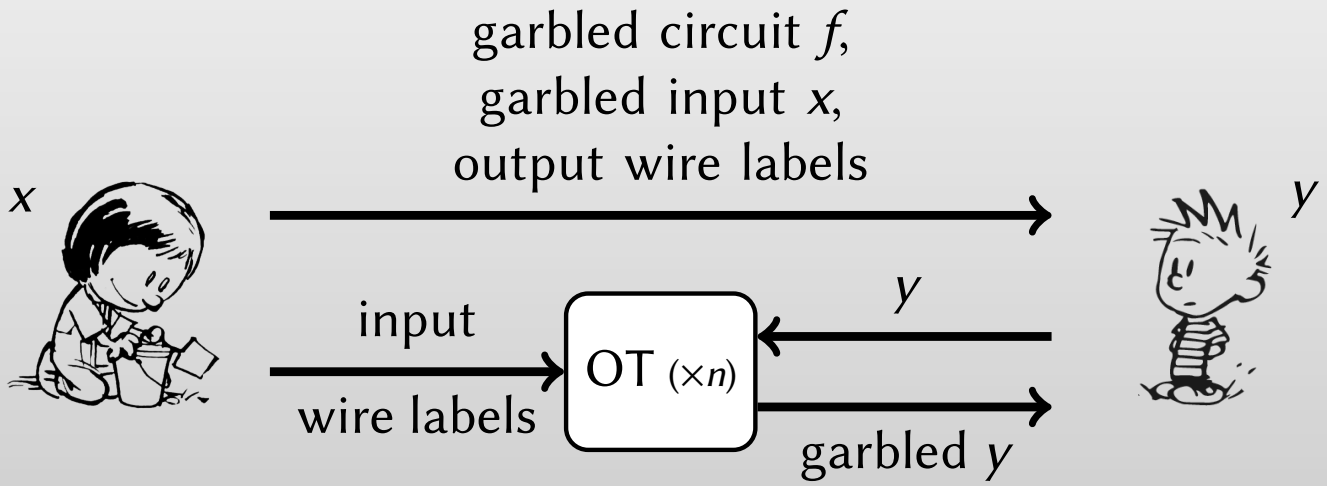

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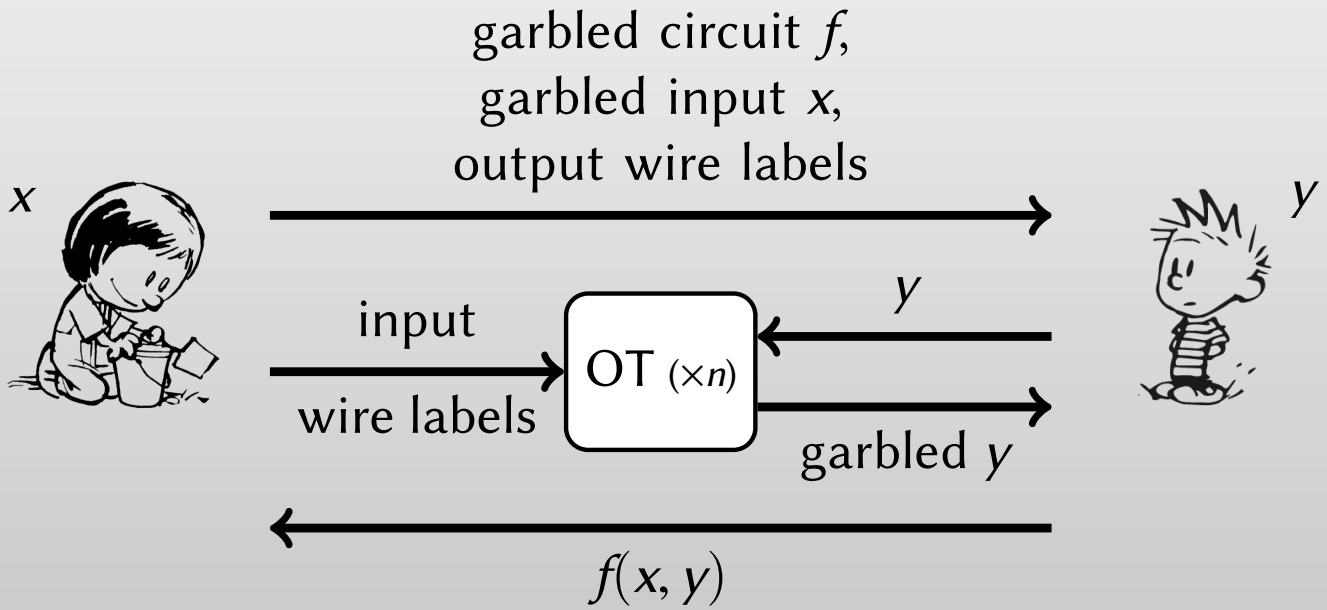

- Given garbled f + garbled inputs + all output labels $\Rightarrow$ Bob learns **only** $f(x, y)$

Summary so far

Secure Computation allows parties to perform a computation on private input, learning only the output.

- market clearing price, advertising revenue, . . .

Security: every attack against the protocol can be "simulated" in an ideal world interaction.

Yao's protocol:

- Garbled lookup table for each gate of boolean circuit
- Oblivious transfer for each input wire