EP2827: Thermodynamics
Homework Set I*

February 15, 2019

1.

Consider three systems: System A is a paramagnetic solid/lump sample described by thermodynamic coordinates $\mathcal{H}, M$ i.e. the magnetic induction ("H field") and the magnetization (magnetic dipole moment density) respectively; system B, another paramagnetic solid with coordinates $\mathcal{H}', M'$; and system C, a gas described by coordinates P, V. When A and C are in thermal equilibrium (i.e. separated by a diathermic partition), the experiments reveal that following relation between the thermodynamic coordinates of A and C holds,

$$n\,R\,C\,\mathcal{H} - MPV = 0.$$

When B and C are in thermal equilibrium, experiments lead to the following relation among the set of thermodynamic coordinates of B and C,

$$n\,R\,\Theta M' + n\,R\,C'\,\mathcal{H}' - M'PV = 0.$$

Here n, R, C, C', Θ are some constants. What are the three functions (of the three pairs of thermodynamic coordinates) that are equal to each other at thermal equilibrium. These three functions give the temperature expression in terms of the thermodynamic coordinates for each system.
Hint: Recall what we did in class, one can equate,

$$f_{AB} = n\,R\,C\,\mathcal{H} - MPV,\; f_{BC} = n\,R\,\Theta M' + n\,R\,C'\,\mathcal{H}' - M'PV.$$

You need to find out g_A, g_B, g_C.

(5 points)

Solution:

As done in class, we will rewrite the two thermal equilibrium equations in the form, the C system coordinate $V = ...$ To wit,

$$V = \frac{n\,R\,C\,\mathcal{H}}{MP}, \tag{1}$$

$$V = \frac{n\,R\,\Theta M' + n\,R\,C'\,\mathcal{H}}{M'P}. \tag{2}$$

Now equating these two we get an expression leads to elimination of V, and we get,

$$\frac{n\,R\,C\,\mathcal{H}}{MP} = \frac{n\,R\,\Theta M' + n\,R\,C'\,\mathcal{H}}{M'P}.$$

But lo and behold the C system coordinate, P also gets canceled out from both sides, which leads to

$$\frac{C\,\mathcal{H}}{M} = \Theta + \frac{C'\,\mathcal{H}}{M'}. \tag{3}$$

*Due in class on Friday, 15th Feb.

The lhs is purely a function of the coordinates of A while the rhs is purely a function of B. Now, from the first equation (1), we note that,

$$\frac{C\,\mathcal{H}}{M} = \frac{PV}{n\,R}.$$

Thus we have equality of three functions of three coordinate pairs of the three distinct systems (at thermal equilibrium)

$$\frac{C\,\mathcal{H}}{M} = \Theta + \frac{C'\,\mathcal{H}}{M'} = \frac{PV}{nR}. \tag{4}$$

2.

(a) Set each of these functions to the ideal gas scale temperature, θ and write down the equation of state of A, B and C.

(5 points)

Solution:

Equating each function of (4) as θ, the ideal gas scale temperature

$$\frac{C\,\mathcal{H}}{M} = \theta; \quad \underline{M = \frac{C\,\mathcal{H}}{\theta}}.$$

$$\Theta + \frac{C'\,\mathcal{H}}{M'} = \theta; \quad \underline{M' = \frac{C'\,\mathcal{H}}{\theta - \Theta}}.$$

and,

$$\frac{PV}{nR} = \theta; \quad \underline{PV = n\,R\,\theta}.$$

One can identify the first equation as indeed the equation of state of a paramagnet, known as Curie's law and C being the Curie constant. The second equation is equation of state of a paramagnet with a Curie temperature, Θ at and under which there is spontaneous magnetization i.e. $M' \neq 0$ even when $\mathcal{H}' = 0$. The third equation is simply the ideal gas law/ equation of state.

3.

In the table below, the entries in the top row represents pressures of a gas in the bulb of a constant volume gas thermometer when the bulb is immersed in water at triple point. The entries in the bottom row on the other hand represents corresponding readings of pressure when the bulb is placed in an unknown substance at a constant unknown temperature. Calculate the ideal gas temperature of this substance.

P_3, in mm of Hg	1000.0	750.00	500.00	250.00
P, in mm of Hg	1535.3	1151.6	767.82	383.95

(Use of "interpolation" function in Mathematica®is recommended:
https://reference.wolfram.com/language/ref/Interpolation.html
If you prefer Maple®or MATLAB ®that is fine as well. Attach the print out of the code and output with the HW set.)

(5 points)

Solution: Using the equation,

$$\theta = 273.16K \times \frac{P}{P_3}$$

we get the table. (Note: One **must** maintain significant figures throughout).

θ, in K	419.38	419.43	419.48	419.52
P, in mm of Hg	1535.3	1151.6	767.82	383.95

Interpolating using Mathematica®using the following line of code

Interpolation[{{383.95,419.52},{767.82,419.48},{1151.6,419.43},{1535.6,419.38}},**0.00**]

which generates the mathematica output

419.54

i.e, the ideal gas scale temperature of this substance is,

$$\theta = \lim_{P\to 0} 273.16K \times \frac{P}{P_3} = 419.54\,K.$$