Intro to Reinforcement Learning

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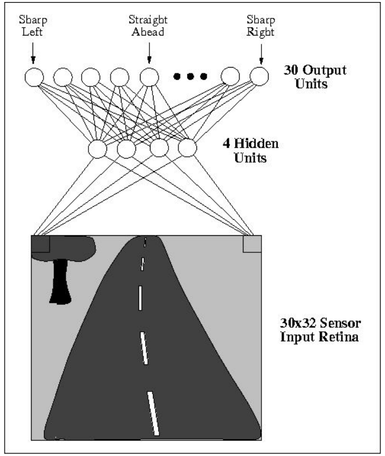
Used Materials

Acknowledgement: Much of the material and slides for this lecture were borrowed from the Deep RL Bootcamp at UC Berkeley organized by Pieter Abbeel, Yan Duan, Xi Chen, and Andrej Karpathy, as well as Katerina Fragkiadaki and Ruslan Salakhutdinov’s 10-703 course at CMU, who in turn borrowed much from Rich Sutton’s class and David Silver’s class on Reinforcement Learning, and with inspiration from Chun Kai Ling and Hai Pham’s slides.

Contents

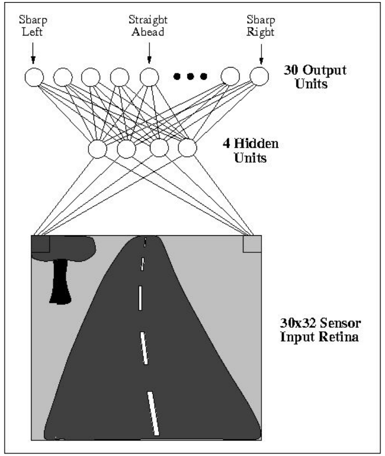
- Introduction to RL
- Markov Decision Processes (MDPs)
- Solving known MDPs using value and policy iteration
- Solving unknown MDPs using function approximation and Q-learning

Reinforcement Learning



ALVINN, 1989

Reinforcement Learning



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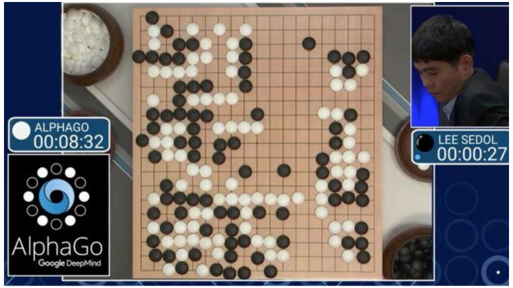

AlphaGo, 2016

DQN, 2015

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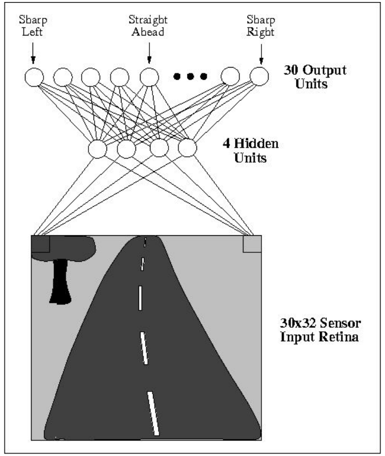

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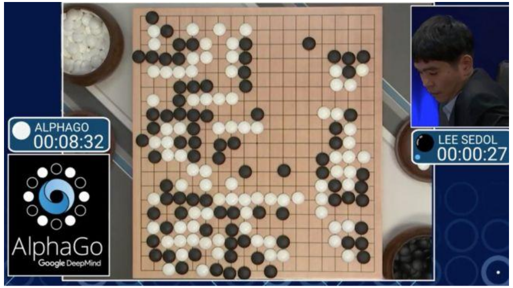

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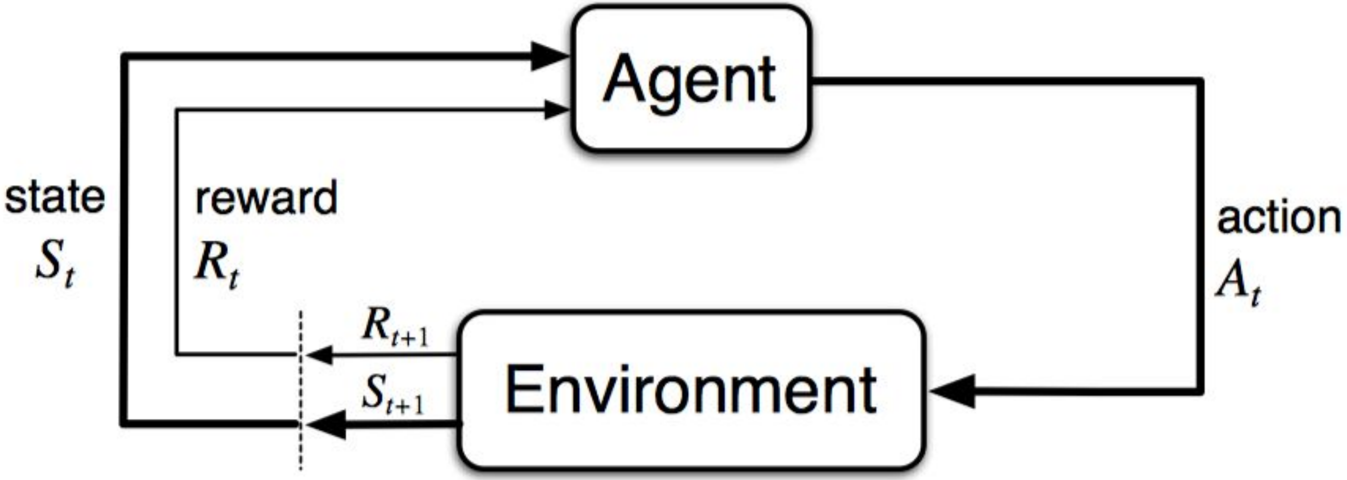
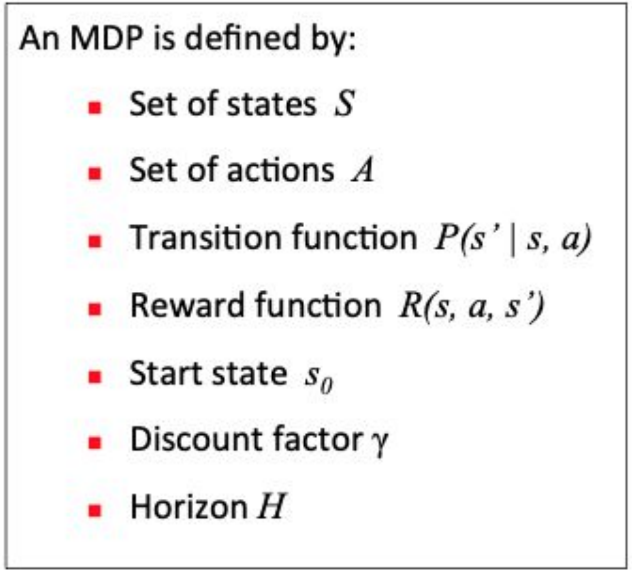


Trajectory

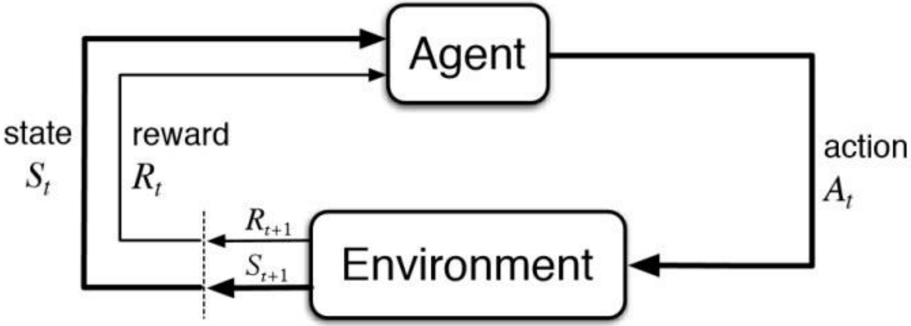
$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \ldots$$

Markov Decision Process (MDPs)



An MDP is defined by:

- Set of states S
- Set of actions A
- Transition function $P(s' \mid s, a)$
- Reward function $R(s, a, s')$
- Start state s_0
- Discount factor γ
- Horizon H



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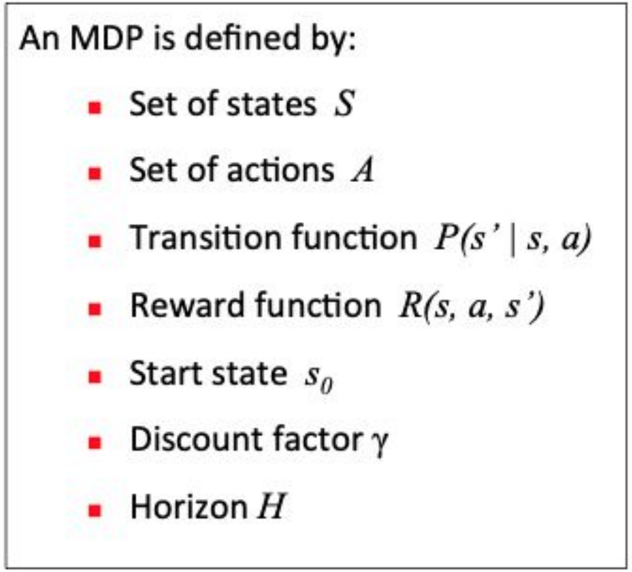

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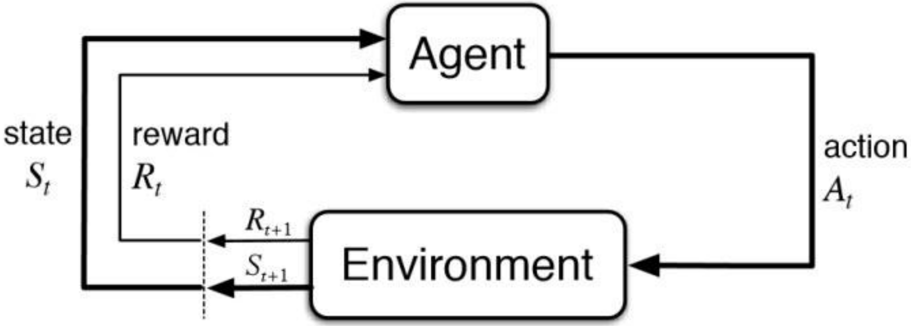

Trajectory

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \ldots$$

Markov assumption + Fully observable

A state should summarize all past information and have the **Markov property.**

$$\mathbb{P}[R_{t+1} = r, S_{t+1} = s' | S_0, A_0, R_1, ..., S_{t-1}, A_{t-1}, R_t, S_t, A_t] = \mathbb{P}[R_{t+1} = r, S_{t+1} = s' | S_t, A_t]$$

for all $s' \in \mathcal{S}, r \in \mathcal{R}$, and all histories

- We should be able to throw away the history once state is known

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If some information is only partially observable: Partially Observable MDP (POMDP)

Return

In continuing tasks, we often use simple *total discounted reward*:

$$G_t = R_{t+1} + \gamma R_{t+2} + ... = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

γ close to 0 leads to "myopic" evaluation

γ close to 1 leads to "far-sighted" evaluation

Policy

Definition: A policy is a distribution over actions given states,

$$\pi(a \mid s) = \mathbf{Pr}(A_t = a \mid S_t = s), \forall t$$

- A policy fully defines the behavior of an agent
- The policy is stationary (time-independent)
- During learning, the agent changes his policy as a result of experience

Special case: deterministic policies

$$\pi(s) = \textit{the action taken with prob} = 1 \textit{ when } S_t = s$$

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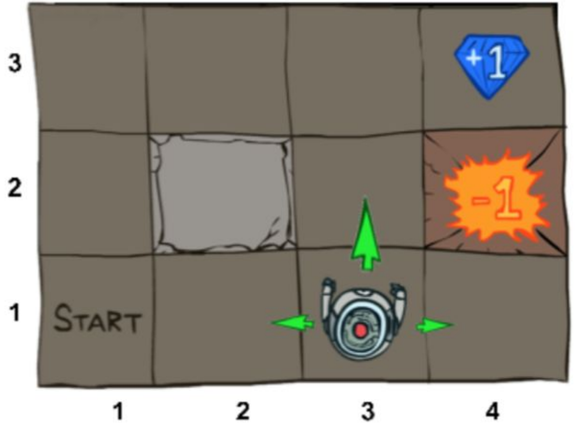
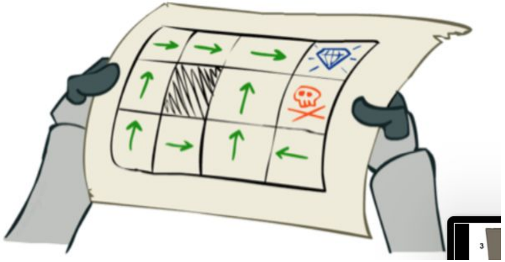
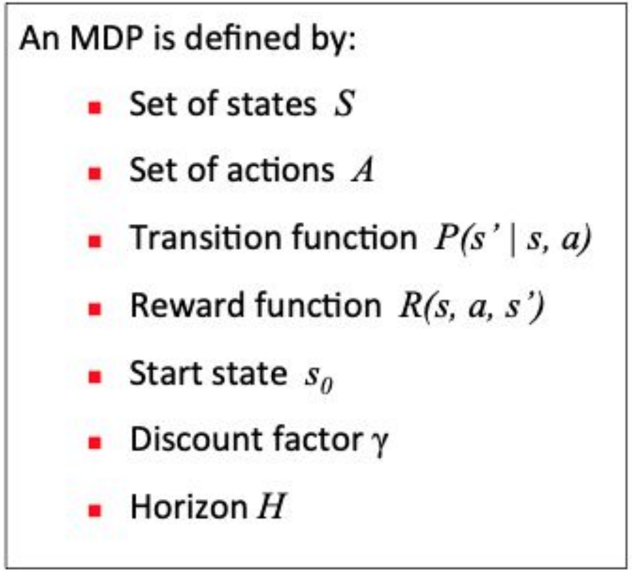


π:



Learn the optimal policy to maximize return



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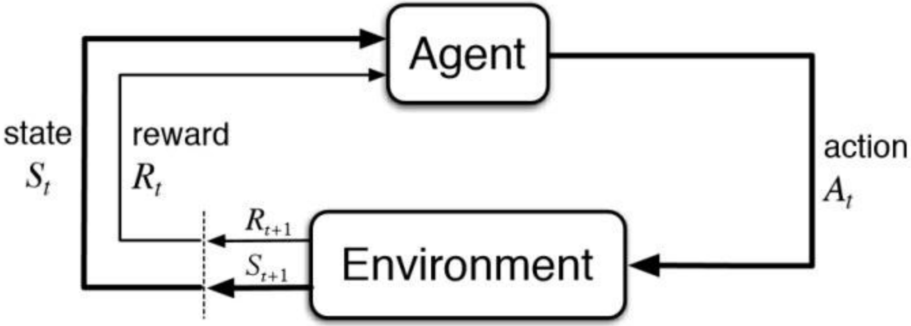
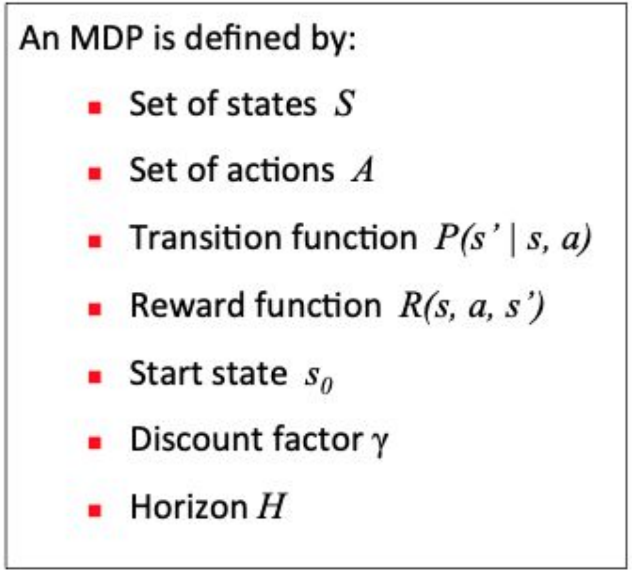


Return:

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Goal: $$\arg\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{H} \gamma^t R_t | \pi\right]$$

Learn the optimal policy to maximize return



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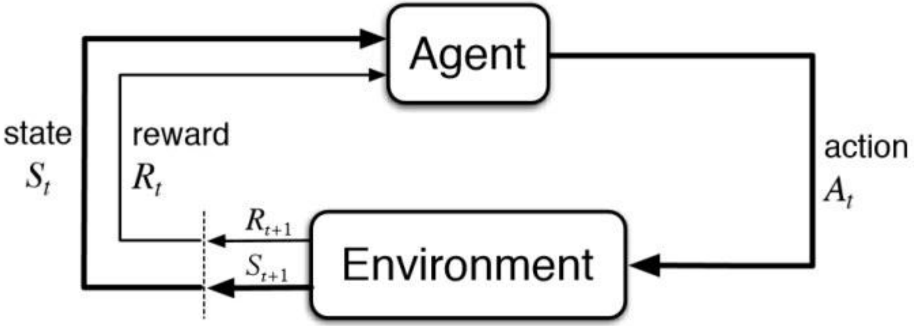

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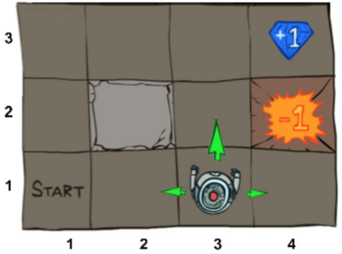

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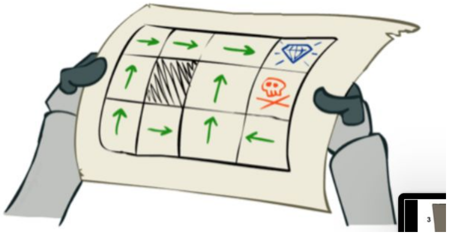
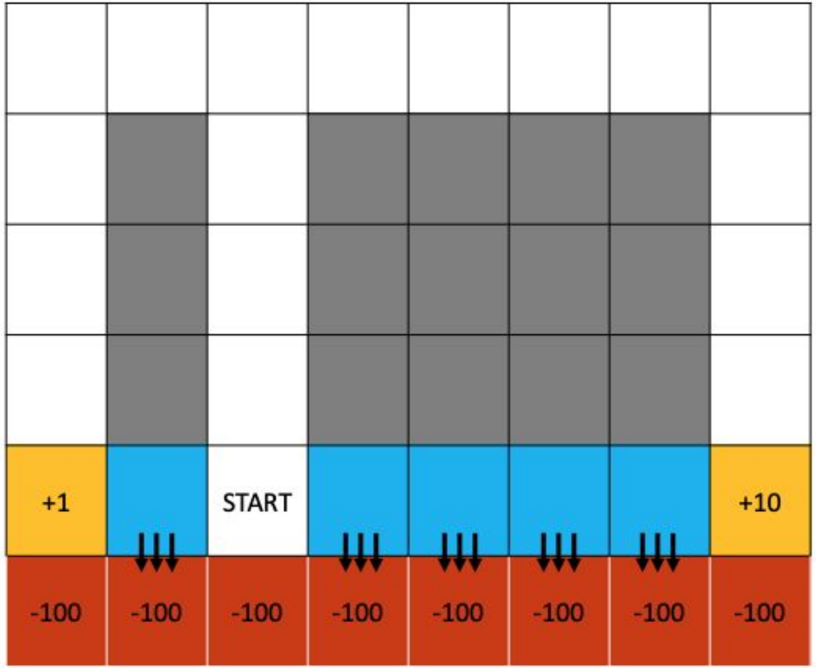


Goal: $$\arg\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{H} \gamma^t R_t \middle| \pi\right]$$

Simple Example

M4-1 Quiz 2



Legend:

Grey: walls, Red: cliff (terminal state), Orange: gold (terminal state), Blue: slippery slope

Actions:

- up, down, left, right
- Taking any action on blue tiles causes you to fall down with probability p.

Q: Which of the settings for gamma and p result in an optimal agent's first action to be "Left"?

A: $\gamma = 0.1,\ p = 0$

B: $\gamma = 0.99999,\ p = 0$

C: $\gamma = 0.1,\ p = 0.2$

D: $\gamma = 0.99999,\ p = 0.2$

Reinforcement Learning vs Supervised Learning

Reinforcement Learning

- Sequential decision making

Supervised Learning

- One-step decision making

Reinforcement Learning vs Supervised Learning

Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward

Supervised Learning

- One-step decision making
- Maximize immediate reward

Reinforcement Learning vs Supervised Learning

Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward
- Sparse rewards

Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision

Reinforcement Learning vs Supervised Learning

Reinforcement Learning

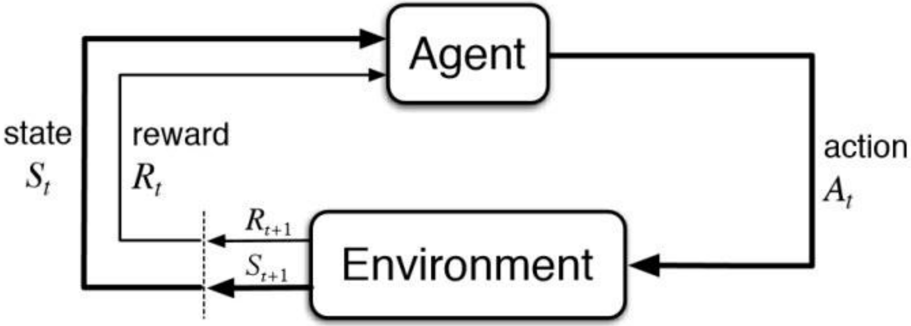
- Sequential decision making
- Maximize cumulative reward
- Sparse rewards
- Environment maybe unknown

Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision
- Environment always known

Intersection between RL and supervised learning

Imitation learning!



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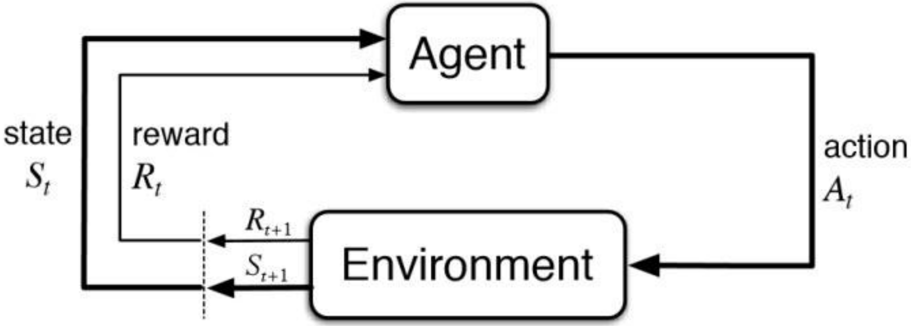

Obtain expert trajectories (e.g. human driver/video demonstrations):

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \ldots$$

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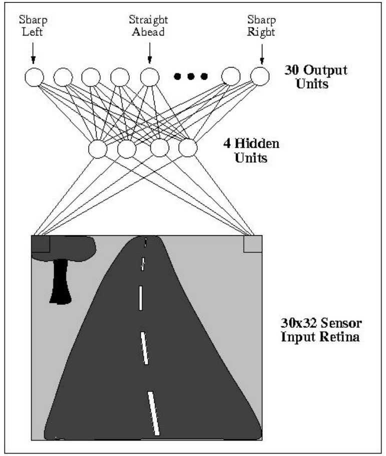

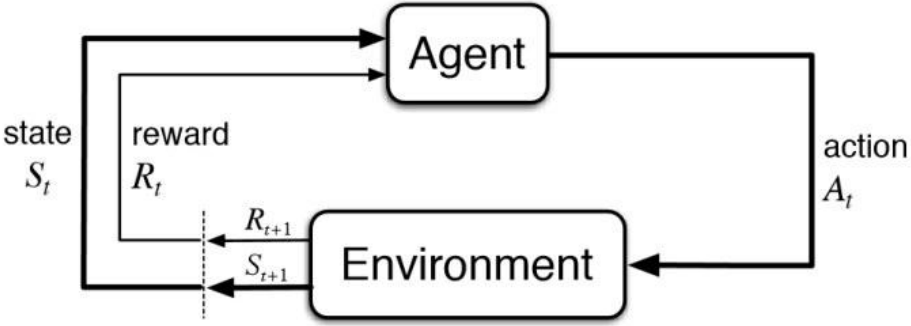

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Perform supervised learning by predicting expert action

D = {(s0, a*0), (s1, a*1), (s2, a*2), …}

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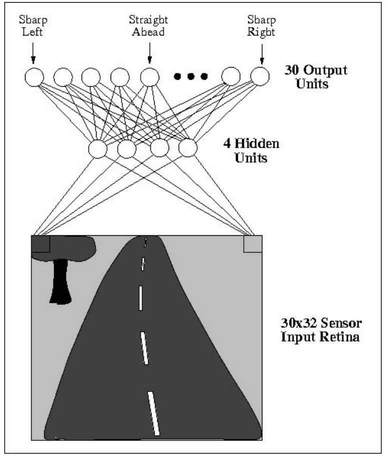

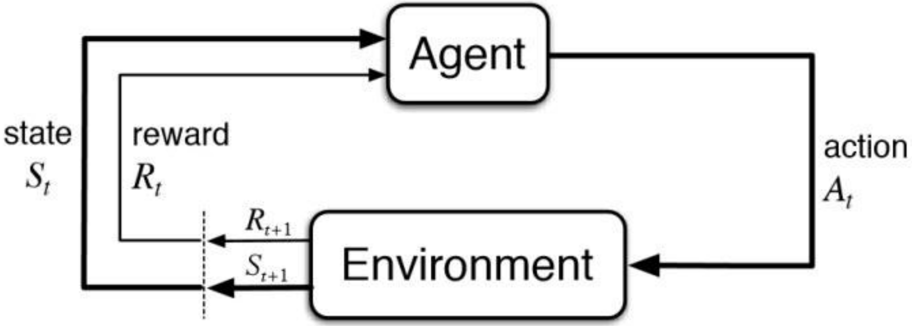

Obtain expert trajectories (e.g. human driver/video demonstrations):

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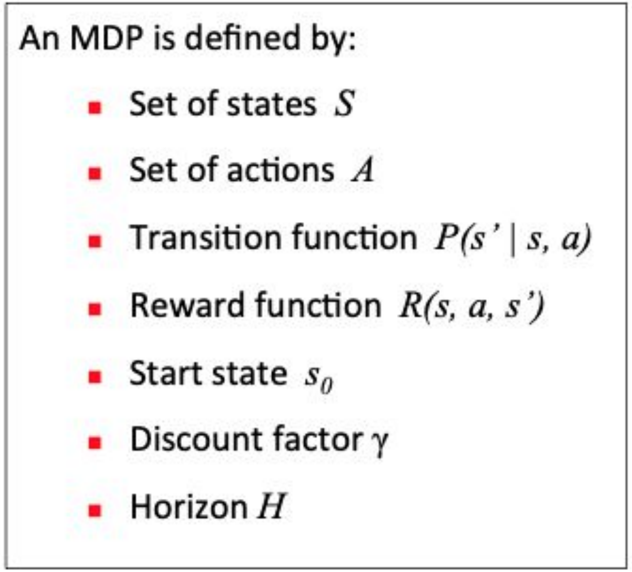
Perform supervised learning by predicting expert action

D = {(s0, a*0), (s1, a*1), (s2, a*2), …}

But: distribution mismatch between training and testing
Hard to recover from sub-optimal states
Sometimes not safe/possible to collect expert trajectories

Learn the optimal policy to maximize return



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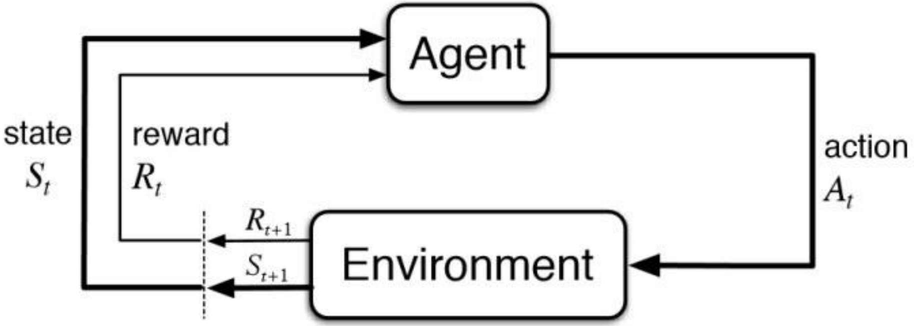

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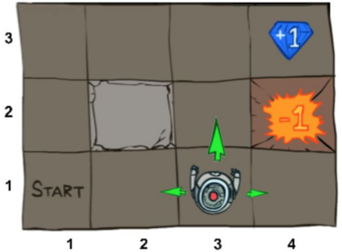

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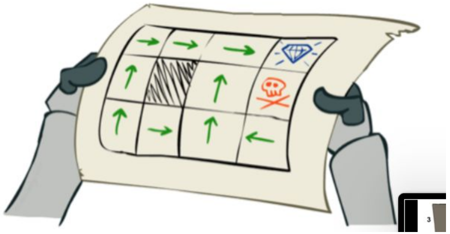
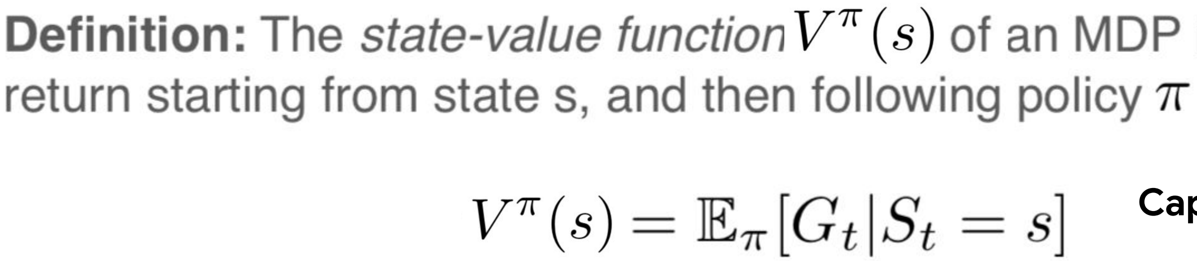


Goal:

$$\arg\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{H} \gamma^t R_t | \pi\right]$$

State and action value functions

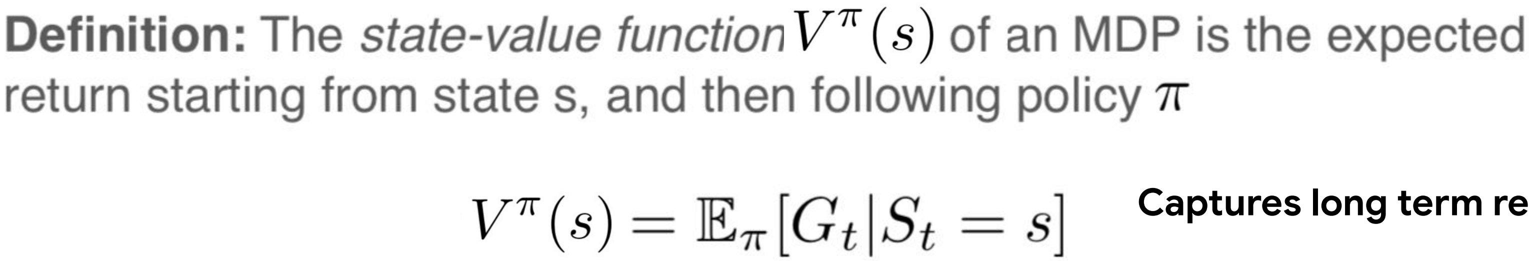


Definition: The *state-value function* $V^{\pi}(s)$ of an MDP is the expected return starting from state s, and then following policy π

$$V^{\pi}(s) = \mathbb{E}_{\pi}[G_t | S_t = s]$$

Captures long term reward

State and action value functions



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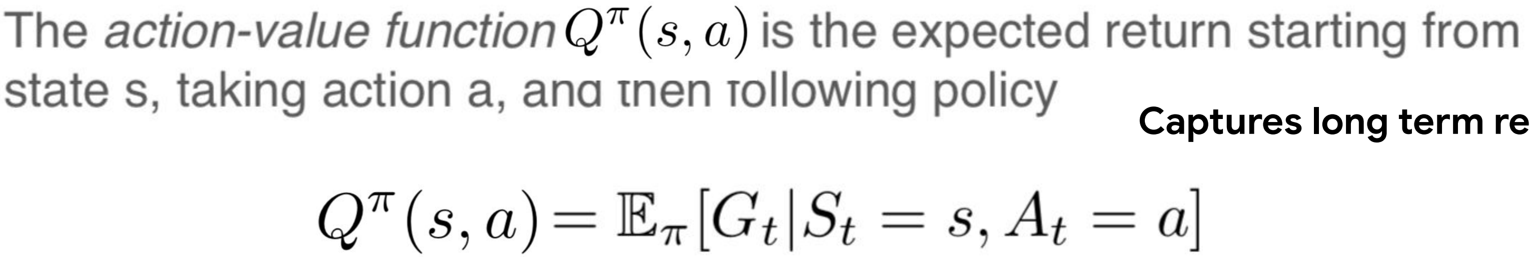
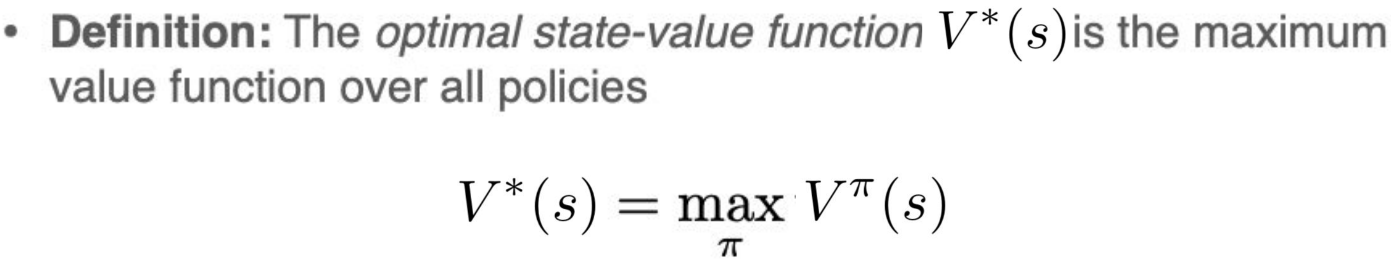


The *action-value function* $Q^{\pi}(s, a)$ is the expected return starting from state s, taking action a, and then following policy

$$Q^{\pi}(s, a) = \mathbb{E}_{\pi}[G_t | S_t = s, A_t = a]$$

Captures long term reward

Optimal state and action value functions



- **Definition:** The *optimal state-value function* $V^*(s)$ is the maximum value function over all policies

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

Optimal state and action value functions

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- The *optimal action-value function*$Q^*(s,a)$is the maximum action-value function over all policies

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

Solving MDPs

- **Prediction**: Given an MDP $(\mathcal{S}, \mathcal{A}, T, r, \gamma)$ and a policy

$$\pi(a|s) = \mathbb{P}[A_t = a | S_t = s] \quad \boxed{V^\pi(s) \quad Q^\pi(s, a)}$$

find the state and action value functions.

Solving MDPs

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find the state and action value functions.

- **Optimal control**: given an MDP $(\mathcal{S}, \mathcal{A}, T, r, \gamma)$, find the optimal policy (aka the planning problem). Compare with the learning problem with missing information about rewards/dynamics.

$$\boxed{V^*(s) \quad Q^*(s,a)}$$

Value functions

- **Value functions** measure the goodness of a particular state or state/action pair: how good is for the agent to be in a particular state or execute a particular action at a particular state, for a given policy.
- **Optimal value functions** measure the best possible goodness of states or state/action pairs *under all possible policies.*

	state values	action values
prediction	v_{π}	q_{π}
control	v_{*}	q_{*}

Relationships between state and action values

State value functions | **Action value functions**

$$\begin{tikzcd}
\boxed{V^\pi(s)} \arrow[d, swap, "V^*(s) = \max_\pi V^\pi(s)"] & & \boxed{Q^\pi(s, a)} \\
\boxed{V^*(s)} & & \boxed{Q^*(s, a)}
\end{tikzcd}$$

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Obtaining the optimal policy

Optimal policy can be found by maximizing over Q*(s,a)

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \; Q^*(s,a) \\ 0, & \text{else} \end{cases}$$

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Optimal policy can also be found by maximizing over V*(s’) with **one-step look ahead**

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right] \\ 0, & \text{else} \end{cases}$$

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$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \left[\sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma V^*(s')) \right] \\ 0, & \text{else} \end{cases}$$

So, how do we find Q*(s,a) and V*(s)?

Recursively:

$$\begin{aligned} G_t &= r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \ldots \\ &= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \ldots \right) \\ &= r_{t+1} + \gamma G_{t+1} \end{aligned}$$

Bellman expectation

Recursively:

$$
\begin{aligned}
G_t &= r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \ldots \\
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Bellman expectation

Recursively:

$$\begin{aligned} G_t &= r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \ldots \\ &= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \ldots \right) \\ &= r_{t+1} + \gamma G_{t+1} \end{aligned}$$

By taking expectations:

$$\begin{aligned} V^\pi(s) &= \mathbb{E}_\pi \left[G_t | S_t = s \right] \\ &= \mathbb{E}_\pi \left[r_{t+1} + \gamma G_{t+1} | S_t = s \right] \\ &= \mathbb{E}_\pi \left[r_{t+1} + \gamma V^\pi(S_{t+1}) | S_t = s \right] \end{aligned}$$

Bellman expectation

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Bellman expectation

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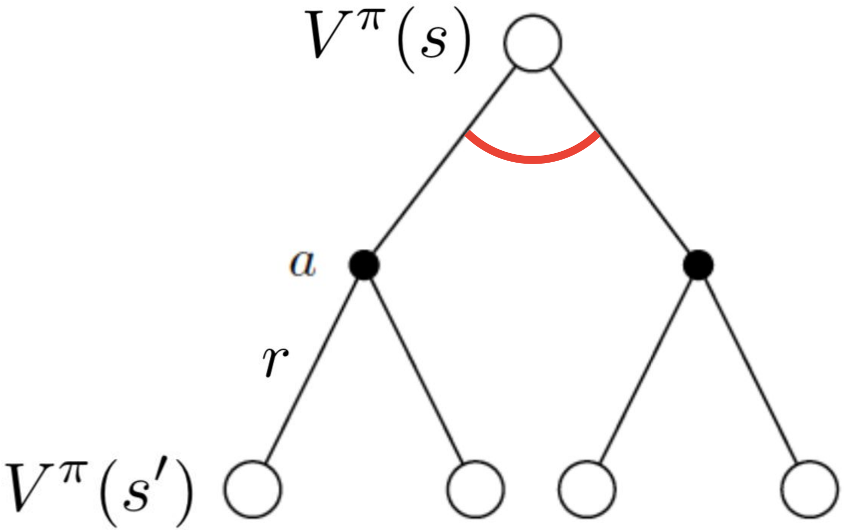
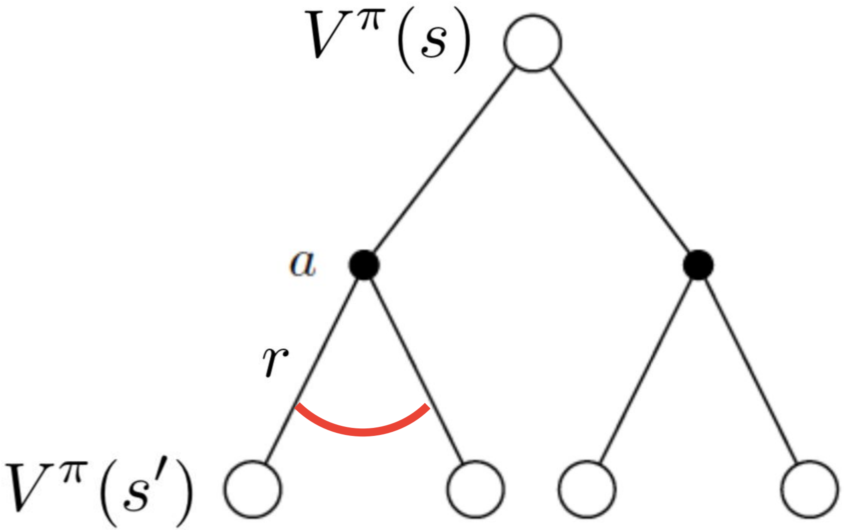
Bellman expectation for state value functions



$$V^\pi(s) = \sum_a \pi(a|s)$$

Bellman expectation for state value functions



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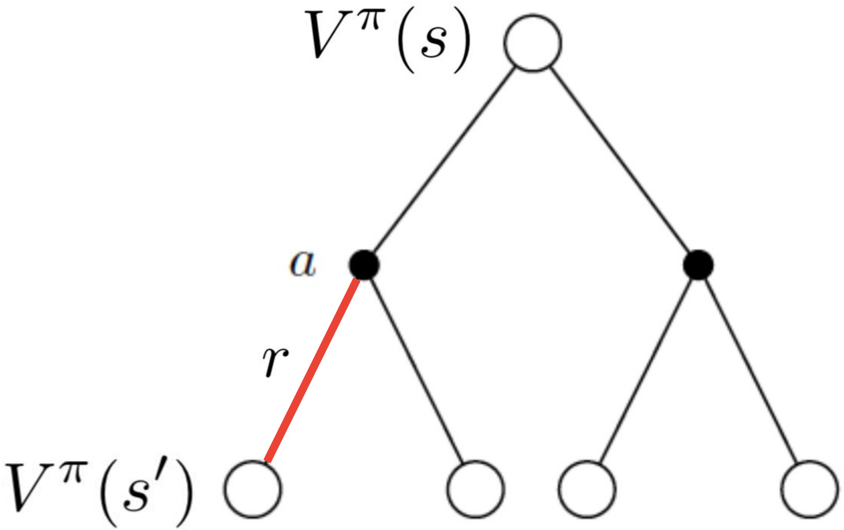
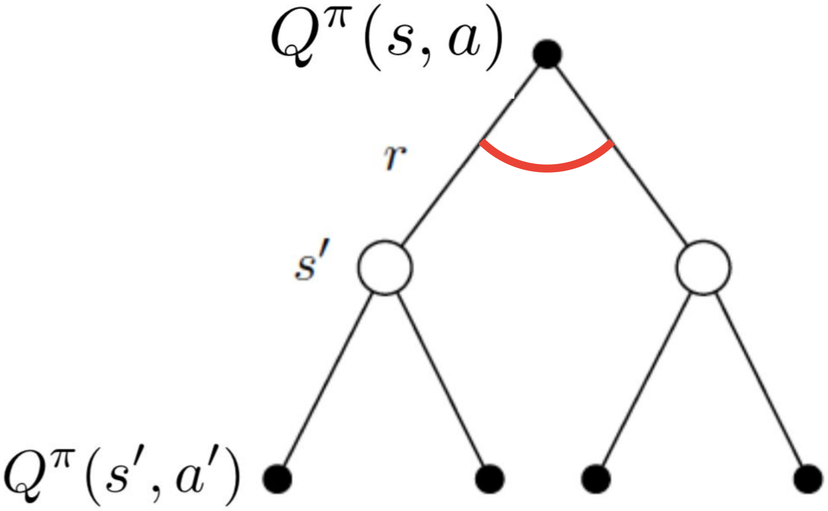
Bellman expectation for state value functions



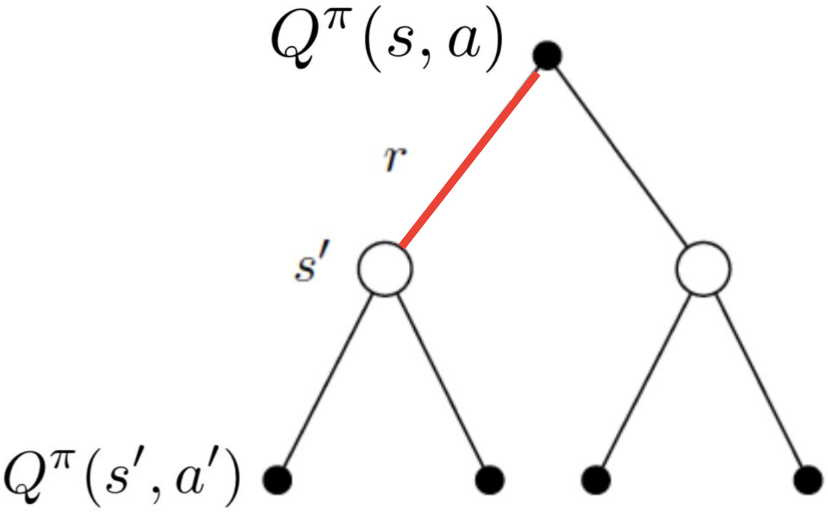
$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V^{\pi}(s') \right]$$

Bellman expectation for action value functions



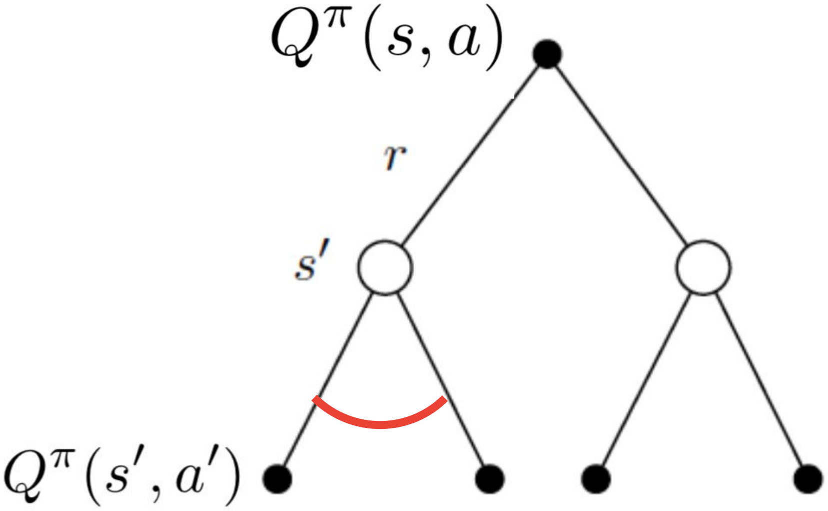
$$Q^\pi(s,a) = \sum_{s'} p(s'|s,a)$$

Bellman expectation for action value functions



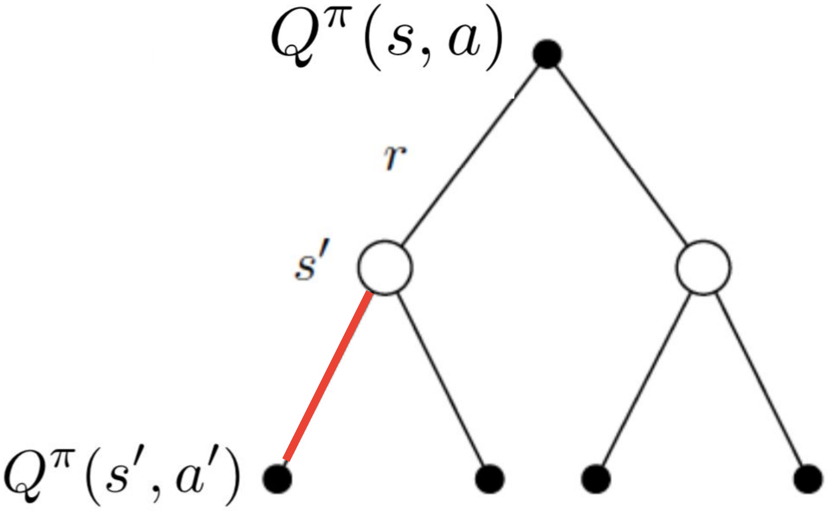
$$Q^\pi(s,a) = \sum_{s'} p(s'|s,a) \Bigg(r(s,a,s')$$

Bellman expectation for action value functions



$$Q^\pi(s,a) = \sum_{s'} p(s'|s,a) \left(r(s,a,s') + \gamma \sum_{a'} \pi(a'|s') \right.$$

Bellman expectation for action value functions



$$Q^\pi(s,a) = \sum_{s'} p(s'|s,a) \left(r(s,a,s') + \gamma \sum_{a'} \pi(a'|s') Q^\pi(s',a') \right)$$

Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V^{\pi}(s') \right]$$

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Solve the linear system

variables: $V^{\pi}(s)$ for all s

constants: p(s’|s,a), r(s,a,s’)

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Solve by iterative methods

$$V^{\pi}_{[k+1]}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V^{\pi}_{[k]}(s')\right]$$

Policy Evaluation

Policy evaluation
Iterate until convergence:

$$V^{\pi}_{[k+1]}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V^{\pi}_{[k]}(s') \right]$$

Policy Iteration

1. Policy evaluation

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2. Policy Improvement

Find the best action according to one-step look ahead

$$\pi_{[k+1]}(a|s) = \arg\max_{a} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V^{\pi}_{[k]}(s') \right]$$

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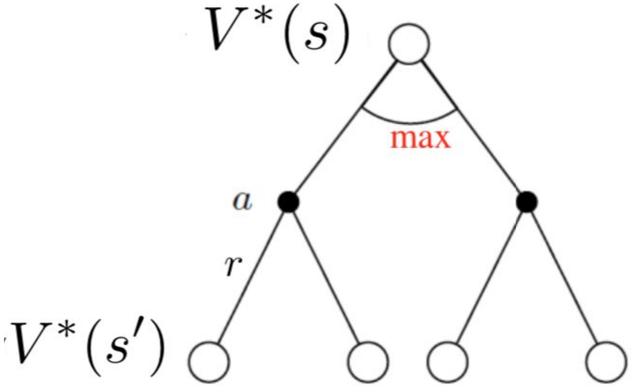
Bellman optimality for state value functions

The value of a state under an optimal policy must equal the expected return for the best action from that state



$$V^*(s) = \max_a Q^*(s,a)$$
$$= \max_a \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right]$$

For the Bellman expectation equations we summed over all leaves, here we choose the **best** branch

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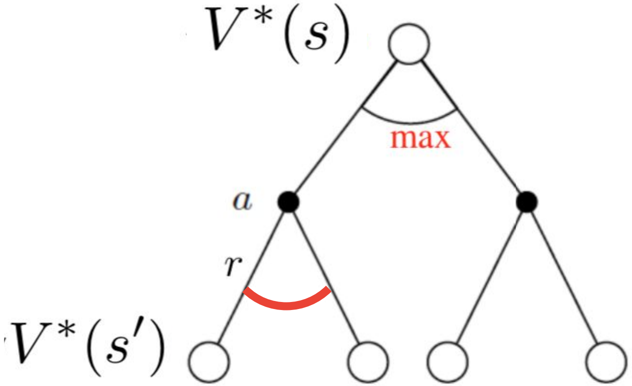

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\end{aligned}
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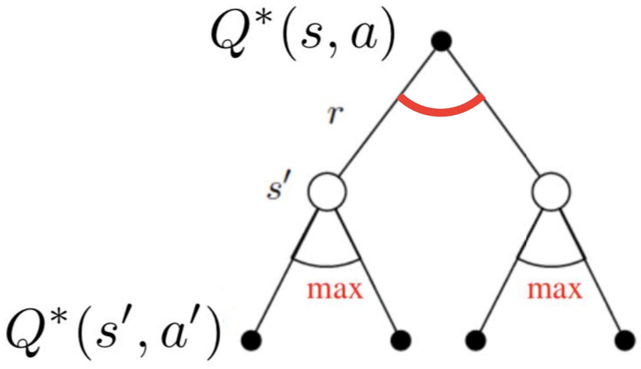
Bellman optimality for action value functions



$$\begin{aligned} Q^*(s,a) &= \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right] \\ &= \mathbb{E}_{s'}\left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right] \end{aligned}$$

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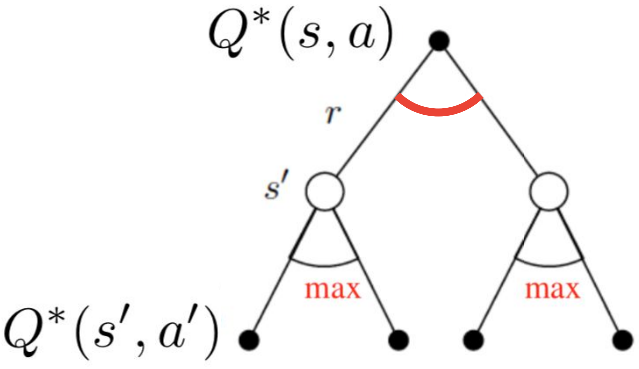
Bellman optimality for action value functions



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$$

Solving the Bellman optimality equations

$$V^*(s) = \max_a \left[\sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma V^*(s')) \right]$$

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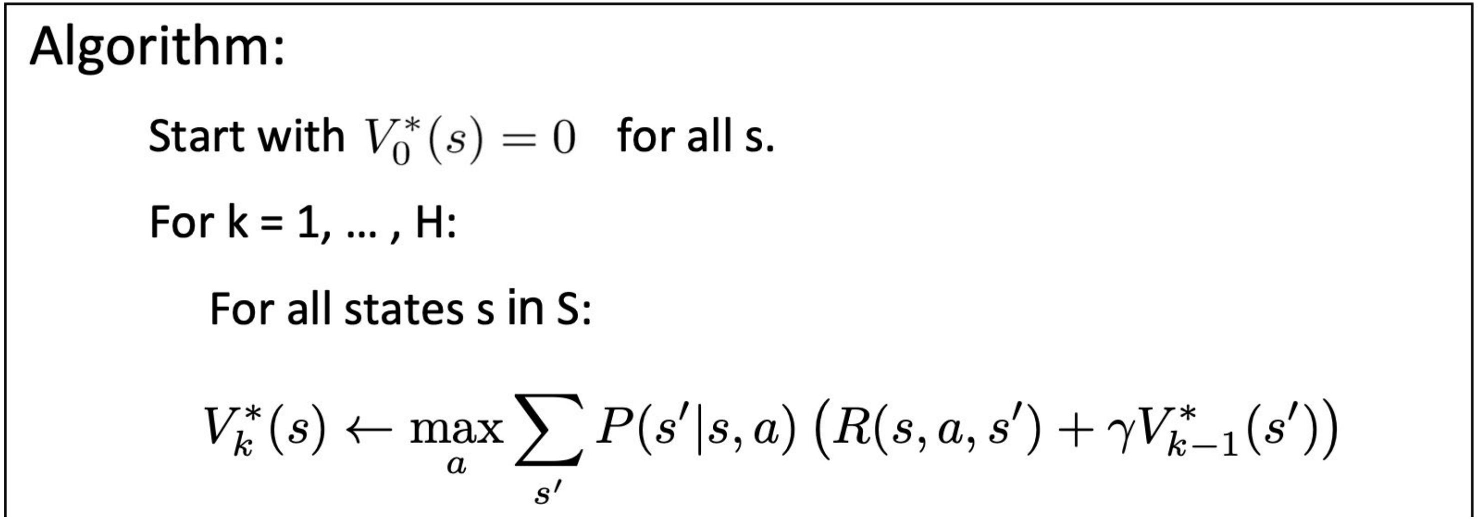
$$V^*_{[k+1]}(s) = \max_a \left[\sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma V^*_{[k]}(s')) \right]$$

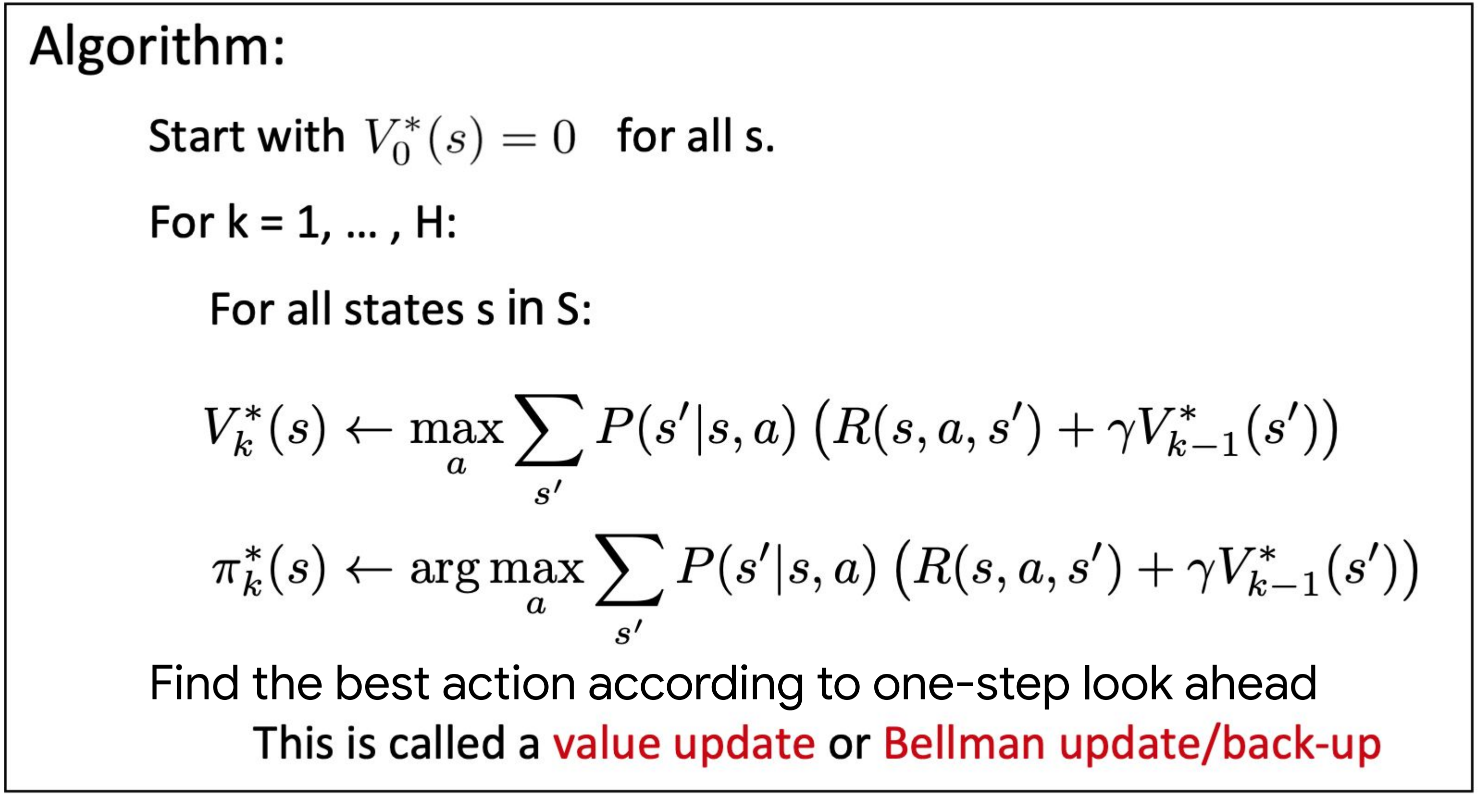
Value Iteration



Algorithm:

```
Start with $V_0^*(s) = 0$ for all s.
For k = 1, … , H:
    For all states s in S:
        $V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)\left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$
```

Value Iteration



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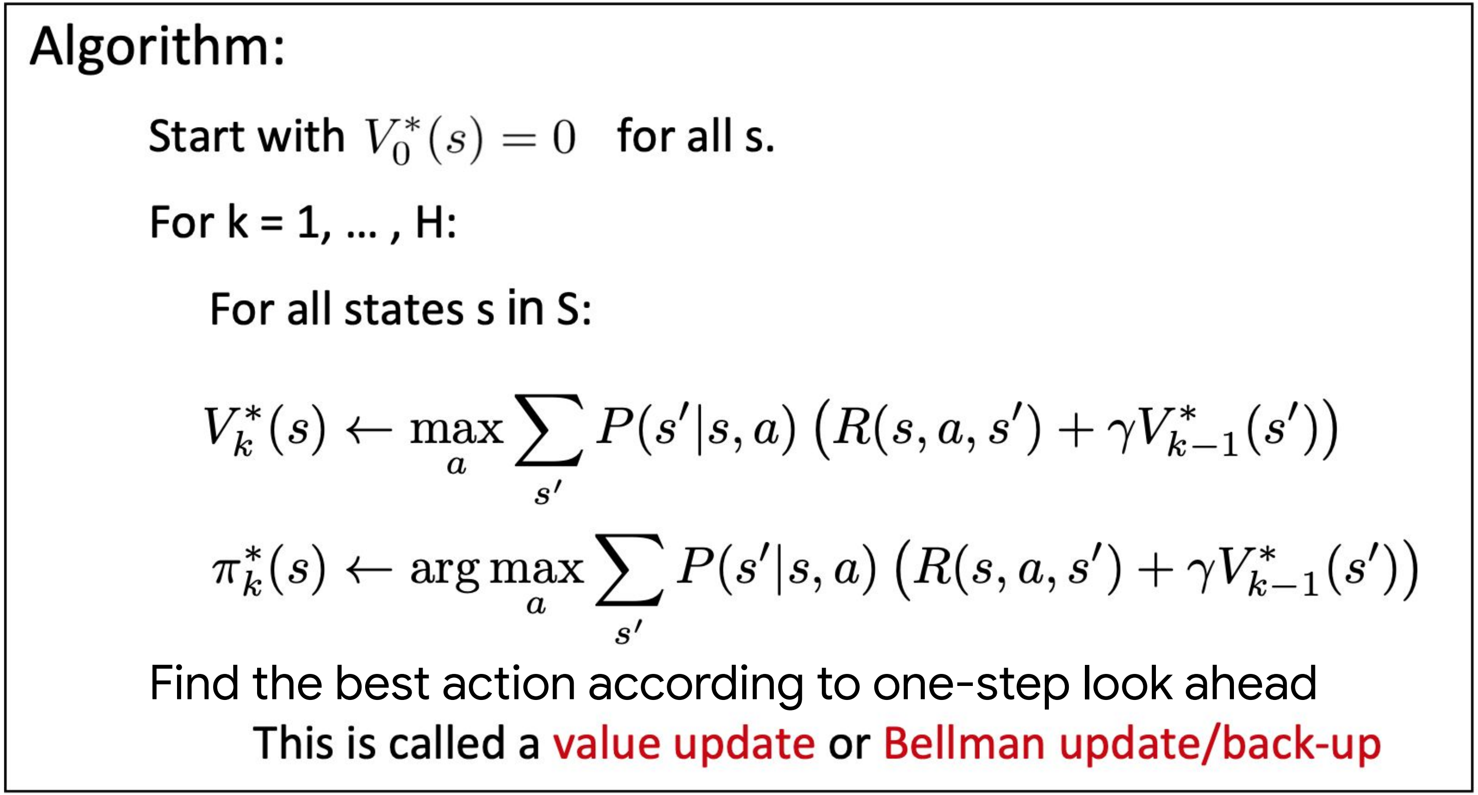
```

Find the best action according to one-step look ahead

This is called a value update or Bellman update/back-up

Value Iteration

Repeat until policy converges. Guaranteed to converge to optimal policy.



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Q-Value Iteration

$Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q^*_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*_k(s',a'))$$

Summary: Exact methods

Fully known MDP

states
transitions
rewards

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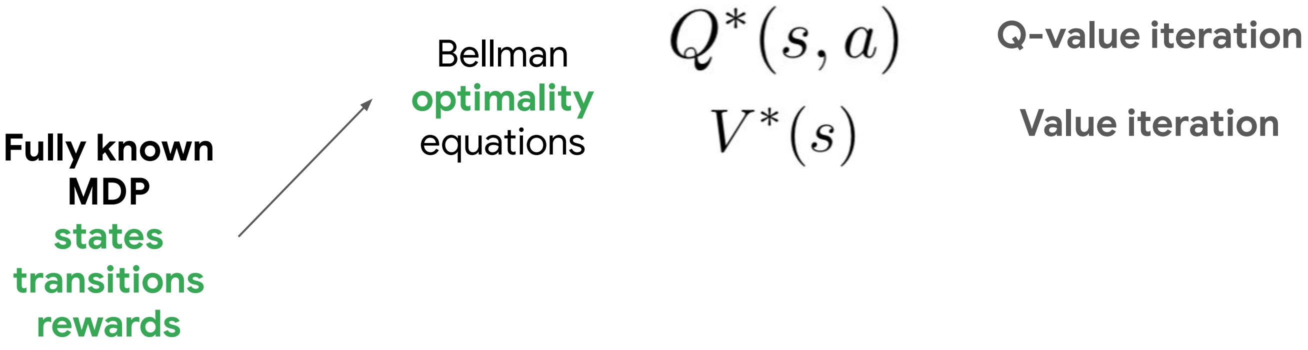

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Bellman optimality equations

$Q^*(s, a)$ Q-value iteration

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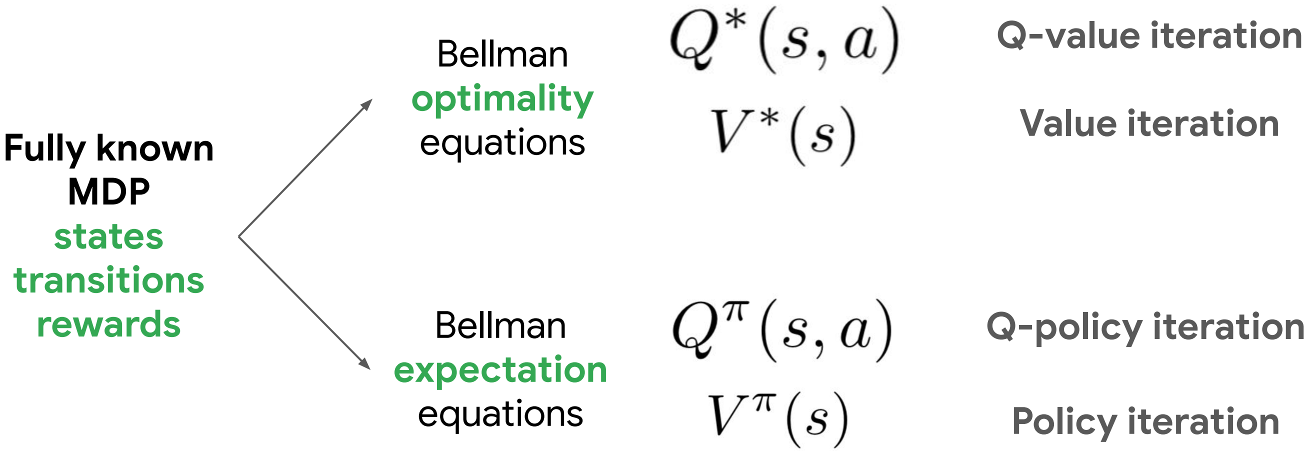

Fully known MDP
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Bellman
optimality
equations

$Q^*(s, a)$ — **Q-value iteration**

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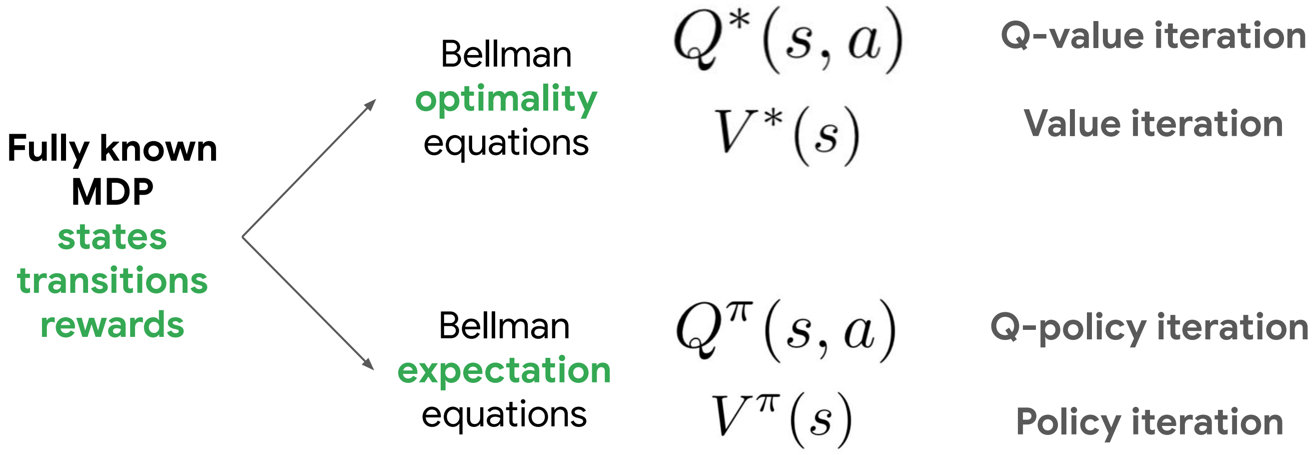

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Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations:
Iterate over and storage for all states and actions: requires small, discrete state and action space
Update equations require fully observable MDP and known transitions

Solving unknown MDPs using function approximation

Recap: Q-value iteration

$Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

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$$Q^*_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*_k(s',a'))$$

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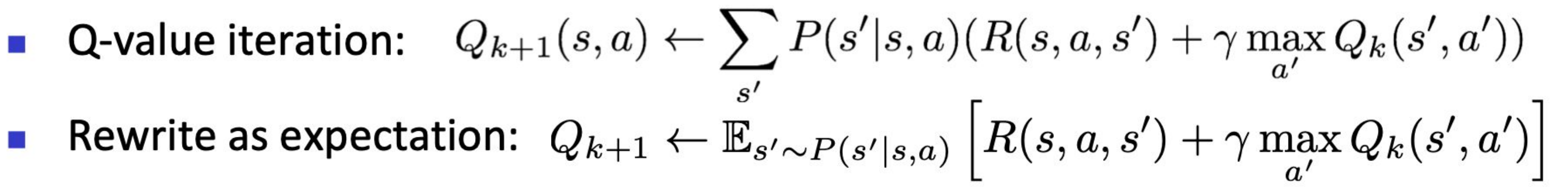
This is problematic when do not know the transitions

Tabular Q-learning



- **Q-value iteration:** $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- **Rewrite as expectation:** $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$

Tabular Q-learning

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- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$
- (Tabular) Q-Learning: replace expectation by samples

Tabular Q-learning

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- (Tabular) Q-Learning: replace expectation by samples
  - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$ simulation and exploration
  - Consider your old estimate: $Q_k(s,a)$
  - Consider your new sample estimate:

$$\mathrm{target}(s') = r(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

$$\mathrm{error}(s') = \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a)\right)$$

Tabular Q-learning update

$$\begin{aligned}
Q_{k+1}(s,a) &= Q_k(s,a) + \overset{\substack{\text{learning rate}\\ \big\downarrow}}{\alpha}\ \mathrm{error}(s') \\
&= Q_k(s,a) + \alpha\left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a)\right)
\end{aligned}$$

Key idea: implicitly estimate the transitions via simulation

Tabular Q-learning

Bellman optimality

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$

Algorithm:

```
Start with $Q_0(s,a)$ for all s, a.
Get initial state s
For k = 1, 2, … till convergence
    Sample action a, get next state s’
    If s’ is terminal:
        $\text{target} = r(s,a,s')$
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```

- Choose random actions?
- Choose action that maximizes $Q_k(s, a)$ (i.e. greedily)?
- ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

Epsilon-greedy

Poor estimates of Q(s,a) at the start:

Bad initial estimates in the first few cases can drive policy into sub-optimal region, and never explore further.

$$\pi(s) = \begin{cases} \max_a \hat{Q}(s,a) & \text{with probability } 1-\epsilon \\ \text{random action} & \text{otherwise} \end{cases}$$

Gradually decrease epsilon as policy is learned.

Tabular Q-learning

Algorithm:

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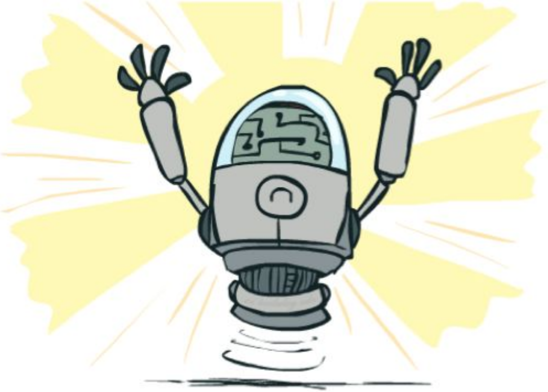
    Start with $Q_0(s, a)$ for all s, a.
    Get initial state s
    For k = 1, 2, … till convergence
        Sample action a, get next state s’
        If s’ is terminal:
            $\text{target} = r(s, a, s')$
            Sample new initial state s’
        else:
            $\text{target} = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$
        $Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left( r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$
        $s \leftarrow s'$
```

- ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

Convergence

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called off-policy learning
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly



Tabular Q-learning

Tabular: keep a |S| x |A| table of Q(s,a)
Still requires small and discrete state and action space
How can we generalize to unseen states?

```
Algorithm:
    Start with $Q_0(s, a)$ for all s, a.
    Get initial state s
    For k = 1, 2, … till convergence
        Sample action a, get next state s’
        If s’ is terminal:
            $\text{target} = r(s, a, s')$
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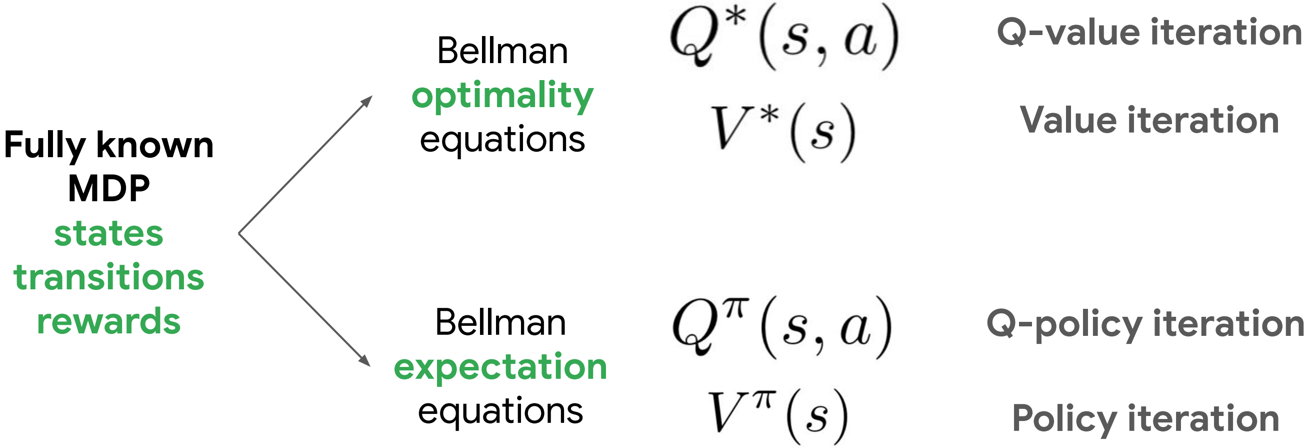

Fully known MDP
states
transitions
rewards

Bellman **optimality** equations

$Q^*(s,a)$ **Q-value iteration**

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Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations:
Iterate over and storage for all states and actions: requires small, discrete state and action space
Update equations require fully observable MDP and known transitions

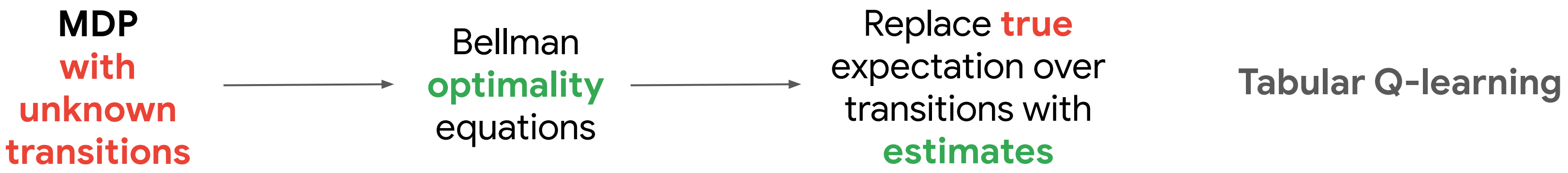
Summary: Tabular Q-learning

MDP with unknown transitions → Bellman **optimality** equations → Replace **true** expectation over transitions with **estimates** → **Tabular Q-learning**

$s' \sim P(s'|s,a)$ **simulation and exploration, epsilon greedy is important!**

$$\underbrace{Q^*(s,a)}_{\text{old estimate}} = \mathbb{E}_{s'}\left[\underbrace{r(s,a,s') + \gamma \max_{a'} Q^*(s',a')}_{\text{target}}\right]$$

Summary: Tabular Q-learning



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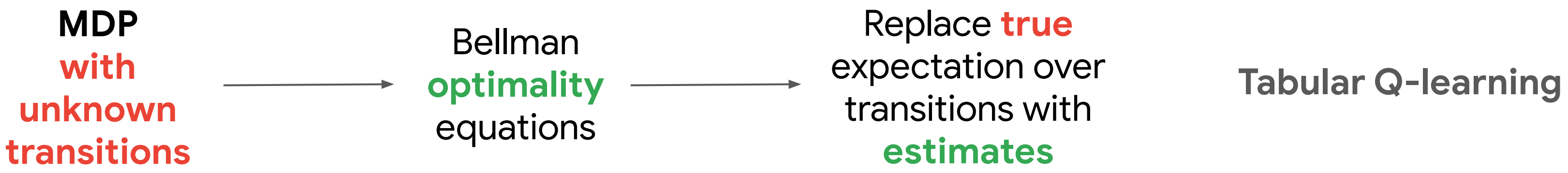

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Deep Q-learning

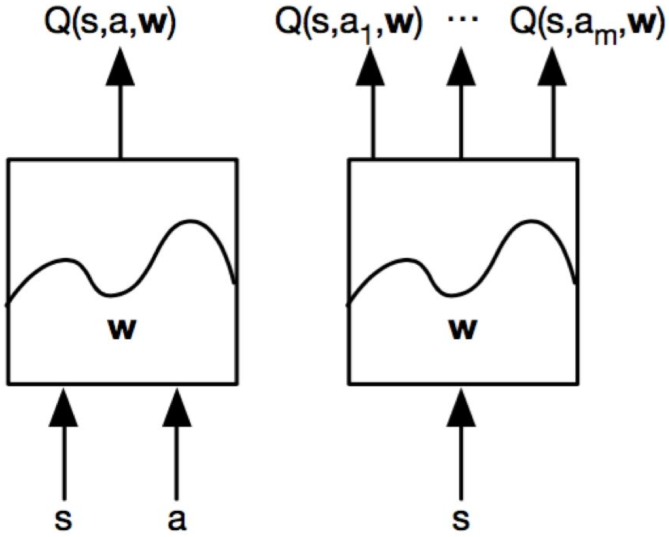
Q-learning with function approximation to **extract informative features** from **high-dimensional** input states.

DQN, 2015

Deep Q-learning

Represent value function by Q-network with weights w

$$Q(s, a, \mathbf{w}) \approx Q^*(s, a)$$



Deep Q-learning

- Optimal Q-values should obey Bellman equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s',a')^* \mid s,a\right]$$

- Treat right-hand $r + \gamma \max_{a'} Q(s',a',\mathbf{w})$ as a target

- Minimize MSE loss by stochastic gradient descent

$$l = \left(r + \gamma \max_{a} Q(s',a',\mathbf{w}) - Q(s,a,\mathbf{w})\right)^2$$

- Remember VFA lecture: Minimize mean-squared error between the true action-value function $q_\pi(S,A)$ and the approximate Q function:

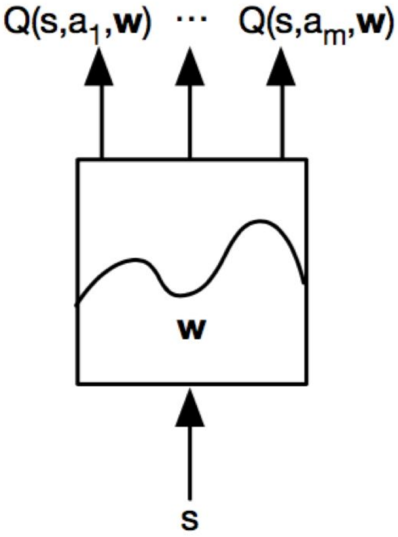
$$J(\mathbf{w}) = \mathbb{E}_\pi\left[(q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))^2\right]$$

Deep Q-learning

- Minimize MSE loss by stochastic gradient descent

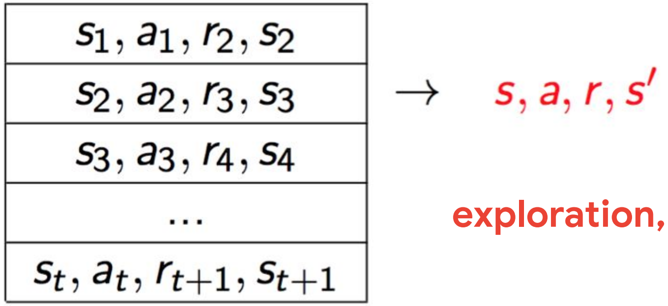
$$I = \left(r + \gamma \max_{a} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w})\right)^2$$

- Converges to Q* using table lookup representation
- But diverges using neural networks due to:
  - Correlations between samples
  - Non-stationary targets



Experience replay

- To remove correlations, build data-set from agent’s own experience



s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\dots$
$s_t, a_t, r_{t+1}, s_{t+1}$

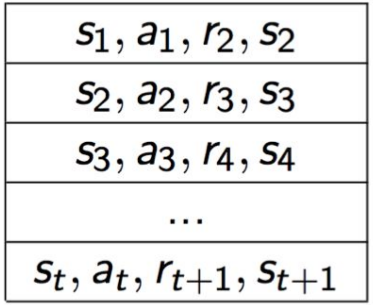
$\rightarrow \quad s, a, r, s'$

exploration, epsilon greedy is important!

- Sample random mini-batch of transitions (s,a,r,s′) from D

Fixed Q-targets

- Sample random mini-batch of transitions (s,a,r,s′) from D
- Compute Q-learning targets w.r.t. old, fixed parameters w−



s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\dots$
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- Optimize MSE between Q-network and Q-learning targets

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
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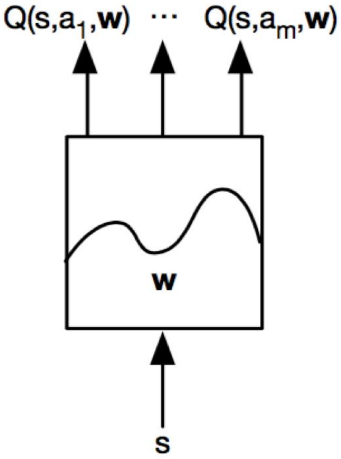
$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s'\sim\mathcal{D}_i}\left[\left(\underbrace{r+\gamma\max_{a'}Q(s',a';w_i^-)}_{\text{Q-learning target}} - \underbrace{Q(s,a;w_i)}_{\text{Q-network}}\right)^2\right]$$



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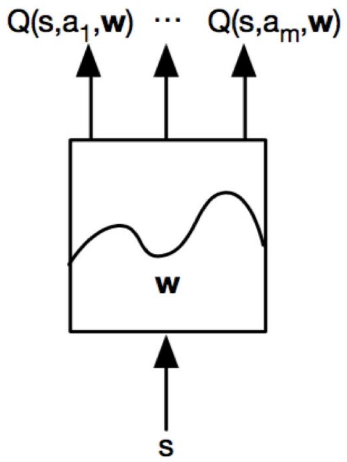
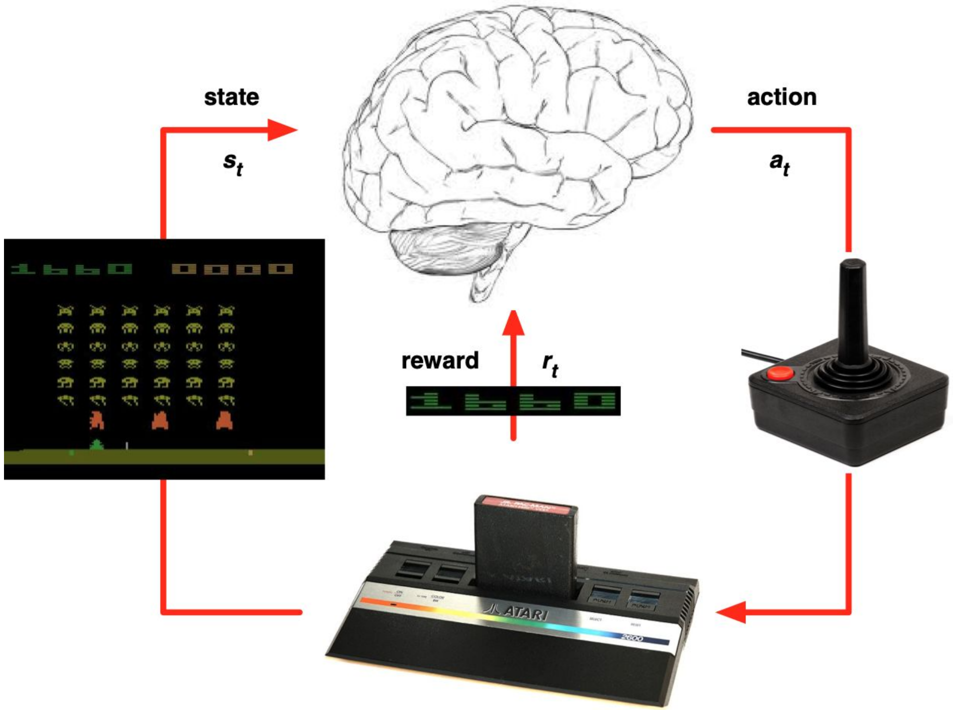


- Use stochastic gradient descent

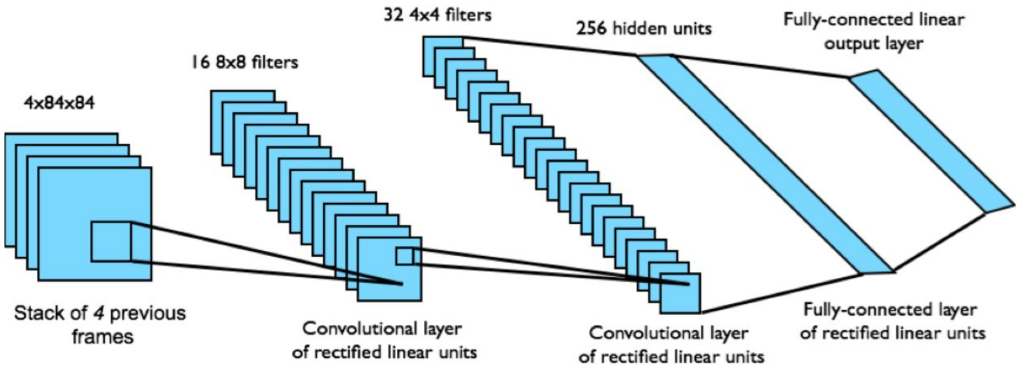
Update **w-** with updated **w** every ~1000 iterations

Deep Q-learning for Atari



Deep Q-learning for Atari

- End-to-end learning of values Q(s,a) from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is Q(s,a) for 18 joystick/button positions
- Reward is change in score for that step


32 4x4 filters
256 hidden units
Fully-connected linear output layer
16 8x8 filters
4x84x84
Stack of 4 previous frames
Convolutional layer of rectified linear units
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- Network architecture and hyperparameters fixed across all games

Mnih et.al., Nature, 2014

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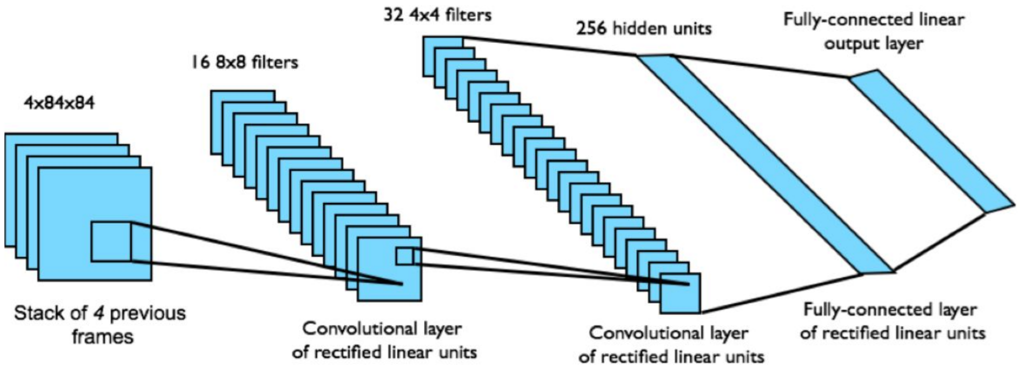
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Encourage Markov property

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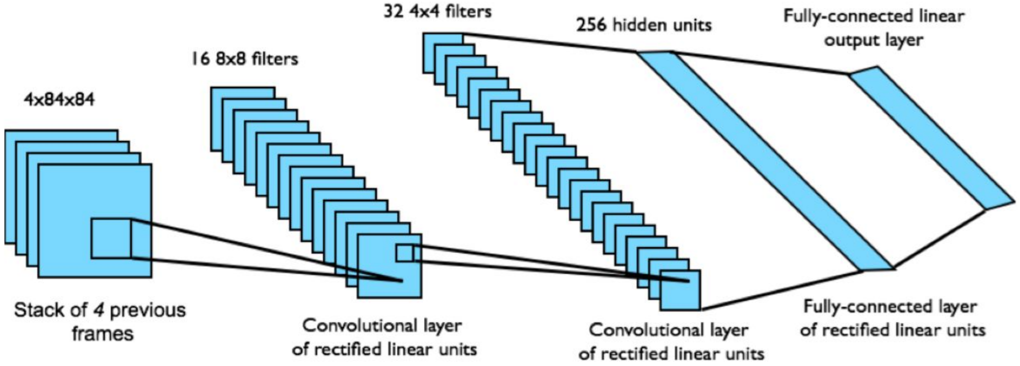
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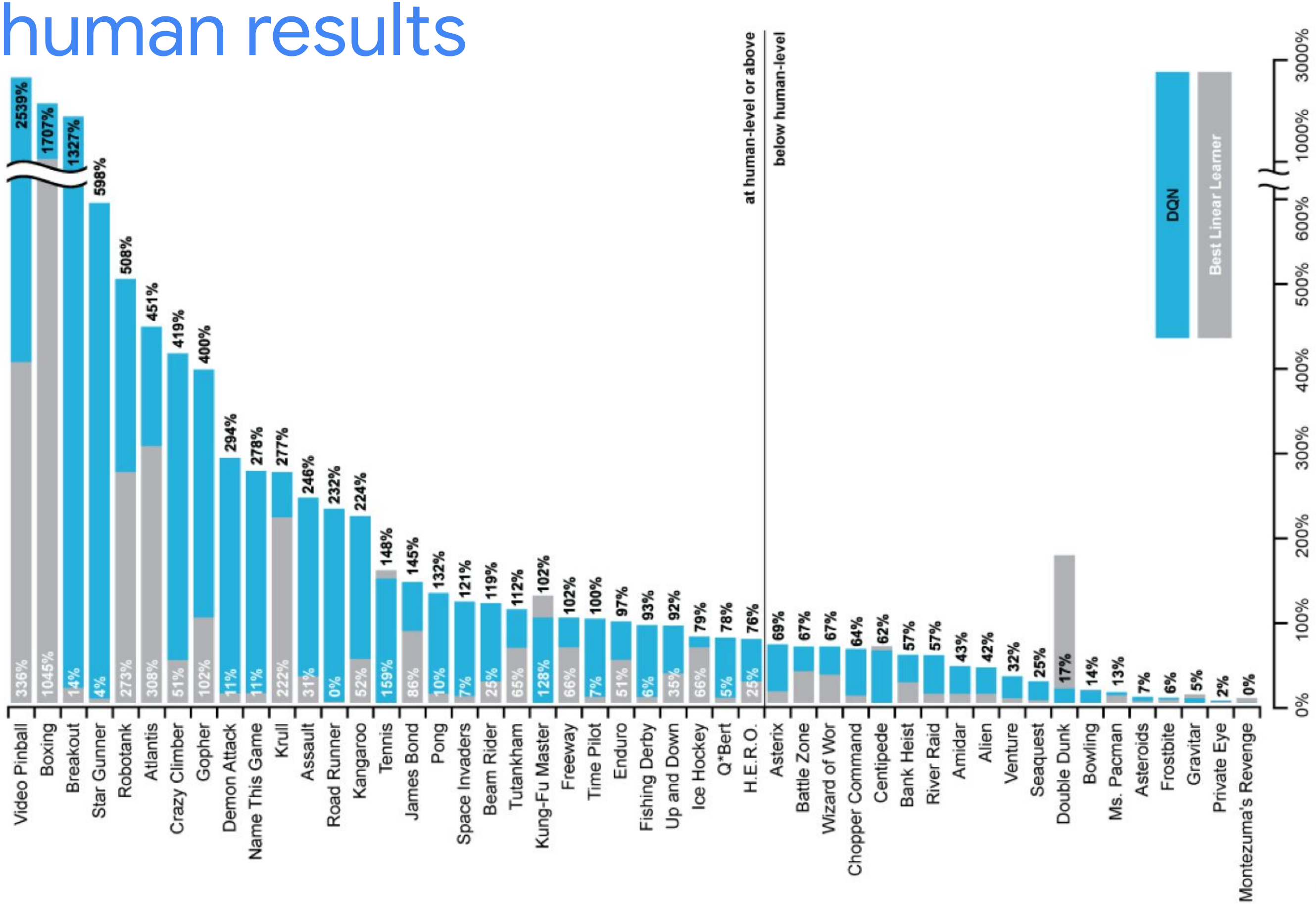
Superhuman results



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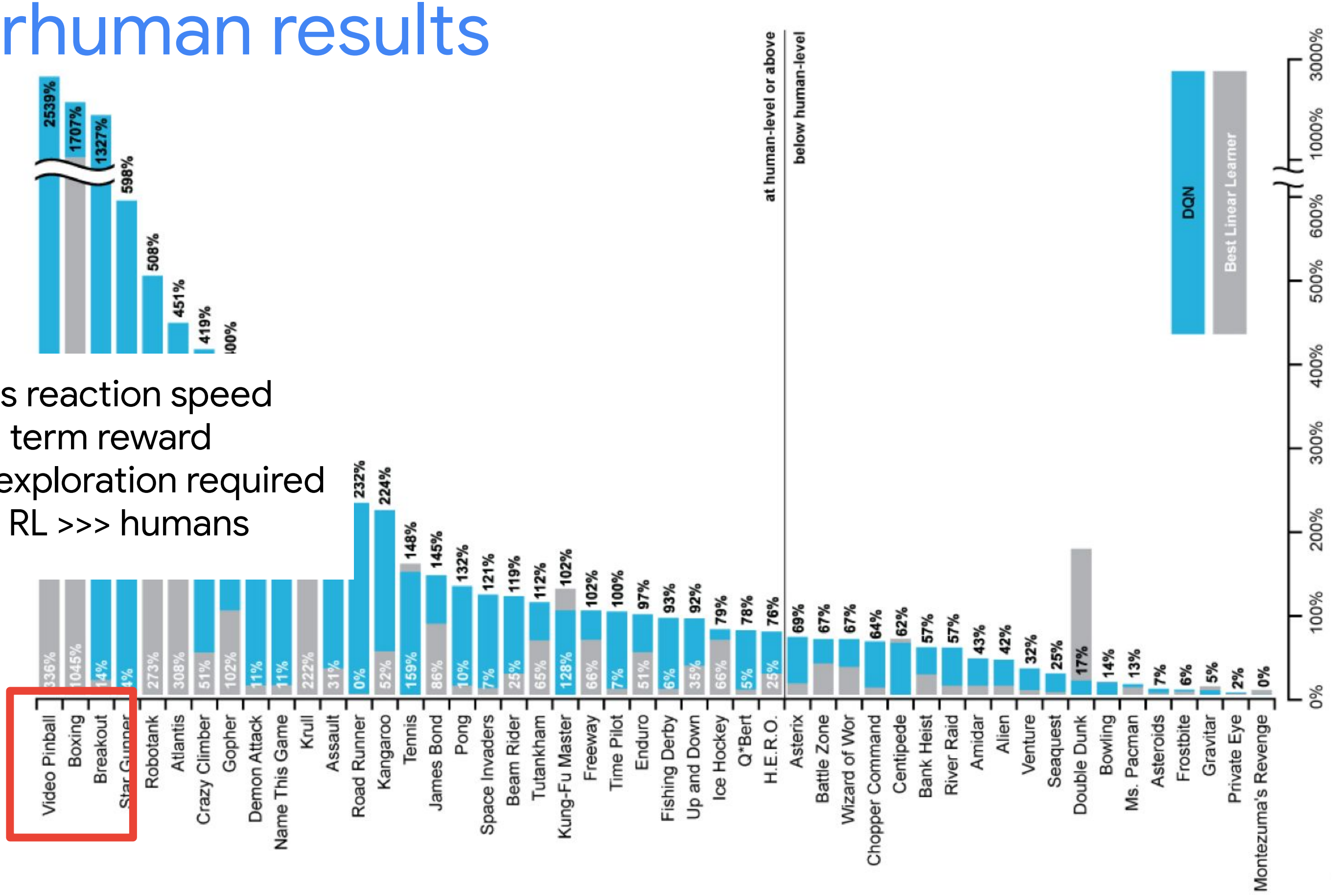
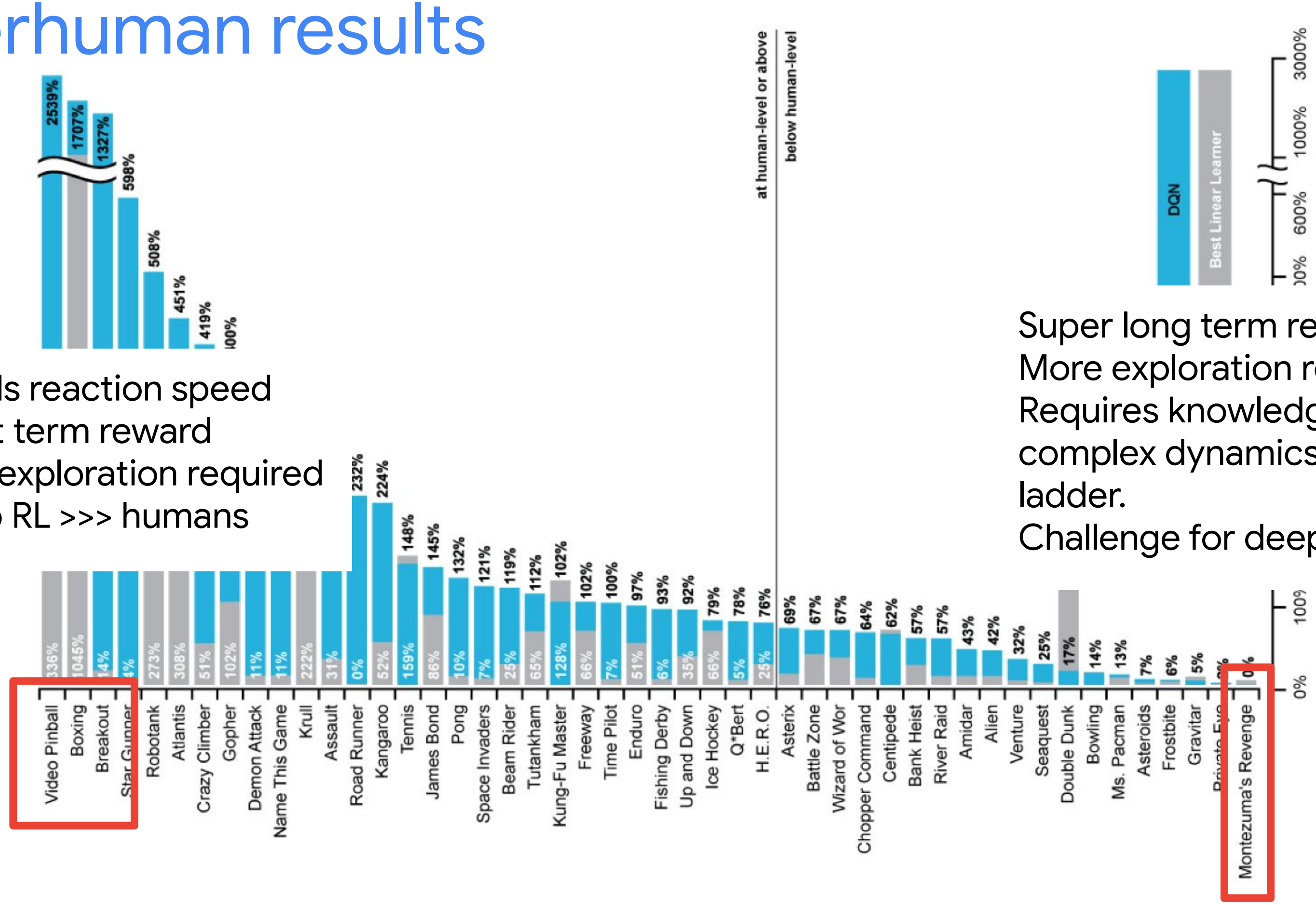


Needs reaction speed
Short term reward
Less exploration required
Deep RL >>> humans

Superhuman results



Needs reaction speed
Short term reward
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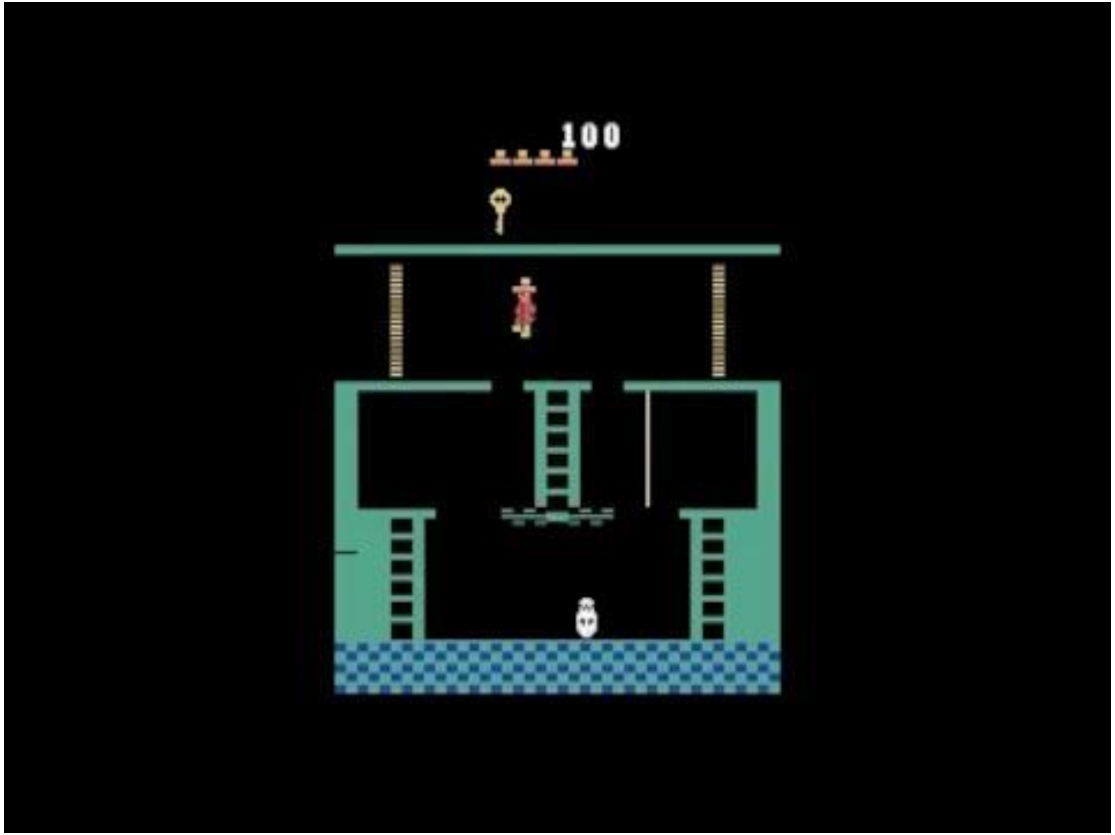
Deep RL >>> humans

Super long term reward
More exploration required
Requires knowledge of complex dynamics e.g. key, ladder.
Challenge for deep RL

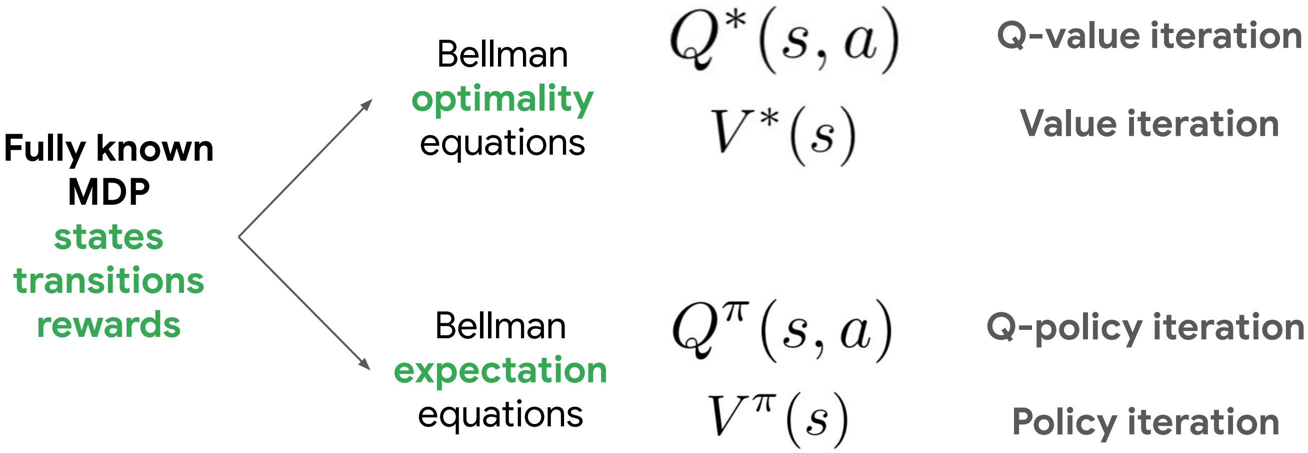
Superhuman results on Montezuma’s Revenge



Encourages agent to explore its environment by maximizing **curiosity.**
I.e. how well can I predict my environment?
1. Less training data
2. Stochastic
3. Unknown dynamics
So I should explore more.

Burda et. al., ICLR 2019

Summary: Exact methods



Fully known MDP

states

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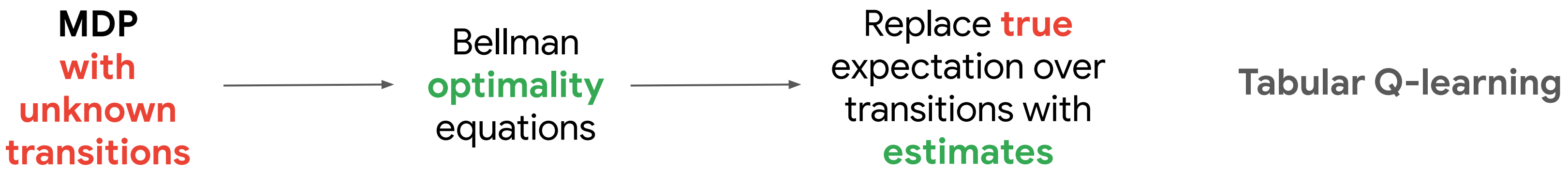

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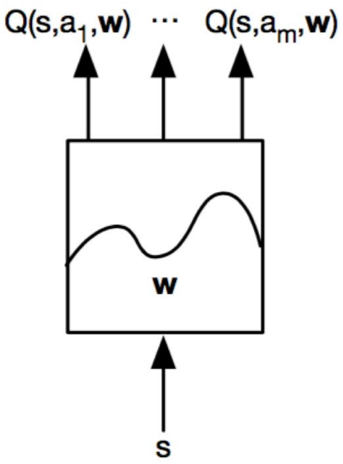

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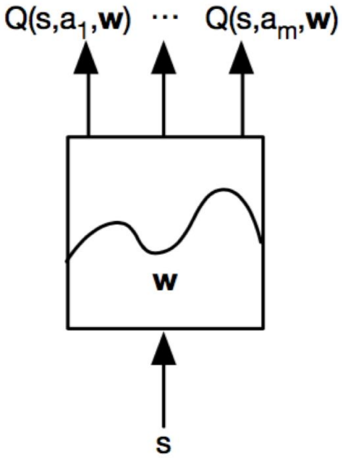

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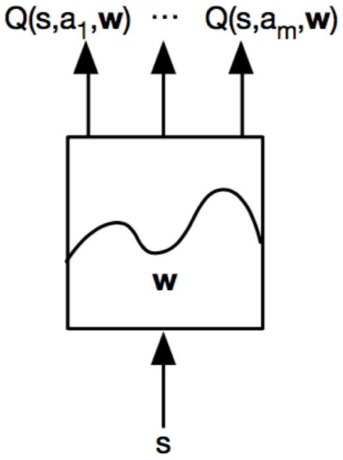

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Stochastic gradient descent + Experience replay + Fixed Q-targets

Works for high-dimensional state and action spaces
Generalizes to unseen states