Mathematical Methods-II

(PH41008)

Spring-2020, IIT KGP

Problem Set 02

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Instructions:

- These exercises are meant to complement the classroom lectures and to help clarify important concepts.
- If you have taken help of your friends or teachers, please mention it; this will be viewed positively.
- If you manage to copy solutions from articles, books or lecture notes, then please provide the references. Make sure that you understand it and are not copying it blindly. Something similar, with a good twist, can be served to you in the exams.
- Solution set should be in legible handwriting. If I'm forced to guess, I will guess an empty set $\emptyset$ and award zero consistently.
- Submission timing is non-negotiable. Submissions will not be accepted after the due date and time.
- Questions given below are not my original creation, they have been taken from various books and may have been suitably reworded or modified.

Q1. For each of the following questions, identify if the operator L is a linear operator. Also, give justification for your choice.

(a) $Lu = u_x + xu_y$
(b) $Lu = u_x + uu_y$
(c) $Lu = u_x + u_y^2$
(d)$Lu = u_x + u_y + 1$
(e) $Lu = \sqrt{1+x^2}(\cos y)u_x + u_{yxy} - [\arctan(x/y)]u$

Q2. For the partial differential equations listed below, find (i) the order and (ii) determine the type, whether they are: linear (homogeneous or inhomogeneous) or nonlinear. Provide justification.

(a) $u_t - u_{xx} + 1 = 0$
(b)$u_t - u_{xx} + xu = 0$
(c)$u_t - u_{xxt} + uu_x = 0$
(d)$u_{tt} - u_{xx} + x^2 = 0$
(e)$iu_t - u_{xx} + u/x = 0$
(f)$u_x(1+u_x^2)^{-1/2} + u_y(1+u_y^2)^{-1/2} = 0$
(g) $u_x + e^y u_y = 0$
(h)$u_t + u_{xxxx} + \sqrt{1+u} = 0$

Q3. Show that the difference of two solutions of an inhomogeneous linear equation $Lu = g$ with the same g is a solution of the homogeneous equation $Lu = 0$.

Q4. Solve the following first order partial differential equations:

(i) $2u_t + 3u_x = 0$ given the auxiliary condition $u = \sin x$ at $t = 0$.

(ii) $3u_y + u_{xy} = 0$.

(iii) $(1 + x^2)u_x + u_y = 0$. Also, sketch some of the characteristic curves.

(iv) $xu_x + yu_y = 0$.

(v) $\sqrt{1 - x^2}u_x + u_y = 0$ given the auxiliary condition $u(0, y) = y$.

(vi) a. $yu_x + xu_y = 0$ given the auxiliary data $u(0, y) = e^{-y^2}$.
(vi) b. Identify the region of the xy plane in which the solution is uniquely determined.

(vii) $au_x + bu_y + cu = 0$.

(viii) $u_x + u_y = 1$.

(ix) $u_x + u_y + u = e^{x+2y}$ given $u(x, 0) = 0$.

Q5. Solve $au_x + bu_y = f(x, y)$, where $f(x, y)$ is a given function. If $a \neq 0$, write the solution in the form

$$u(x, y) = (a^2 + b^2)^{-1/2} \int_L f ds + g(bx - ay,)$$

where g is an arbitrary function of one variable, L is the characteristic line segment from the y axis to the point (x, y), and the integral is a line integral (*Hint* : Use the coordinate method.)

Q6. Make use of the coordinate change method, as discussed in the class, to solve the equation

$$u_x + 2u_y + (2x - y)u = 2x^2 + 3xy - 2y^2.$$
