# EP2827: Thermodynamics
# Homework Set II*

February 22, 2020

1. A corollary which follows from the Carathéodory statement of the second law is,

   ***In the space of thermodynamic equilibrium states, no two reversible adiabatic (hyper)surfaces can ever intersect.***

   Prove this by contradiction by repeating the argument in the class, i.e. if the opposite holds then the Kelvin-Planck version of the second law is violated. 5 points

2. In the lecture(s) while proving that the integrating factor for the heat pfaffian is a function of the temperature, $\theta$, we set up a thought experiment where we had a composite system made up of a main system, $A$ and reference system $B$. We deduced from the equations,

$$\frac{\lambda}{\Lambda} = \frac{\partial \Sigma}{\partial \sigma}, \ \frac{\lambda'}{\Lambda} = \frac{\partial \Sigma}{\partial \sigma'}$$

   that the integrating factors for the main system, the reference system and the composite system must be of the forms:

$$\lambda = \varphi(\theta)\, f(\sigma).$$

$$\lambda' = \varphi(\theta)\, f'(\sigma')$$

$$\Lambda = \varphi(\theta)\, g(\sigma, \sigma')$$

   Prove that $g(\sigma, \sigma') = g(\Sigma(\sigma, \sigma'))$. Hint: Show that the Jacobian of the transformation from $\sigma, \sigma'$ to $g(\sigma, \sigma')$, $\Sigma(\sigma, \sigma')$ vanishes. 5 points

3. Show that the following Pfaffian form (of three variables $x; y; z$) does not admit an integrating factor:

$$đF = xdy + kdz,$$

   where $k$ is a constant. (Hint: Refer to the note on exact and inexact differentials posted on the website). 5 points

4. For an ideal gas with constant heat capacities, show that:
   **A.** The entropy can be given by the expression

$$S = C_V \ln P + C_P \ln V + \text{const.}$$

*Due in class on Wednesday, 28th Feb.

**B.** The adiabatic compressibility is

$$\kappa_S \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S = \frac{1}{\gamma P},$$

where $\gamma = C_P/C_V$.

5 + 2 = 7 points

5. For a paramagnetic ideal gas, obeying both the Curie's law and the ideal gas equation of state, show that the entropy is given by,

$$S = C_{V,M}\ln T + nR\ln V - \frac{\mu_0 M^2}{2C_C} + \text{const.},$$

where $C_{V,M}$ is the heat capacity at constant volume and magnetization, assumed constant, and $C_C$ is the Curie constant. 5 points

6. A hydrostatic system of constant heat capacity, $C_P$ and at a temperature $T_i$ is put in contact with a reservoir at a higher temperature $T_f$. The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to

$$C_P\left[x - \ln(1+x)\right]$$

where $x = -\left(T_f - T_i\right)/T_f$. Prove that this entropy change is positive. 5 + 2 = 7 points

7. Show that for a hydrostatic system,

$$\left(\frac{\partial H}{\partial T}\right)_P = C_P$$

and,

$$\left(\frac{\partial H}{\partial P}\right)_T = -\mu C_P,$$

where $\mu \equiv \left(\frac{\partial T}{\partial P}\right)_H$ is the Joule-Thomson coefficient and $H$ is the enthalpy of the system. For an ideal gas, $\mu = 0$. Show that for an ideal gas, $H = H(T)$, i.e. the enthalpy is purely a function of the temperature. 2 + 2 + 2 = 6 points