

Intro to Reinforcement Learning Part I 11-777 Multimodal Machine Learning Fall 2020

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Admin

Matching for midterm presentation

 \rightarrow Give feedback to relevant teams

Due by Wednesday 10/28 8pm ET

https://forms.gle/fEQgmk4g6Yfop6Ct5

Peer-feedback preferences

This form is meant to help create a better matching for the next peer-feedback process. Please let us which teams you would like to give feedback to by specifying your top six teams.

The top ranked team should be the most relevant team for you (but not your own team) and the sixth most relevant team should have rank six.

This form is due by Wednesday 10/28 8pm ET.

Note: Specifying your preferences does not guarantee that you will be asked for feedback for these teams but an integer linear program will do its best to achieve that.

Your email address (twoertwe@andrew.cmu.edu) will be recorded when you submit this form. Not you? <u>Switch account</u>



Admin

Instructions for the **midterm presentations** are on piazza resources:

- https://piazza.com/class_profile/get_resource/kcnr11wq24q6z7/kg742kik5kv6tg
- Deadline for pre-recorded presentation: Friday, November 13th, 2020 at 8pm ET
- 7 minutes, mostly about error analysis and updated ideas, don't try to present everything...

Instructions for **midterm report** are also online:

- <u>https://piazza.com/class_profile/get_resource/kcnr11wq24q6z7/kgraer741fw3n4</u>
- Deadline: Sunday, November 15th, 2020
- 8 pages for teams of 3 and 9 pages for the other teams
- Multimodal baselines, error analysis, proposed ideas

Used Materials

Acknowledgement: Some of the material and slides for this lecture were borrowed from the Deep RL Bootcamp at UC Berkeley organized by Pieter Abbeel, Yan Duan, Xi Chen, and Andrej Karpathy, as well as Katerina Fragkiadaki and Ruslan Salakhutdinov's 10-703 course at CMU, who in turn borrowed much from Rich Sutton's class and David Silver's class on Reinforcement Learning.

Contents

- Introduction to RL
- Markov Decision Processes (MDPs)
- Solving known MDPs using value and policy iteration
- Solving unknown MDPs using function approximation and Q-learning



Reinforcement Learning





ALVINN, 1989







DQN, 2015





Reinforcement Learning



Markov Decision Process (MDPs)



Markov assumption + Fully observable

A state should summarize all past information and have the **Markov** property.

 $\mathbb{P}[R_{t+1} = r, S_{t+1} = s' | S_0, A_0, R_1, ..., S_{t-1}, A_{t-1}, R_t, S_t, A_t] = \mathbb{P}[R_{t+1} = r, S_{t+1} = s' | S_t, A_t]$ for all $s' \in S, r \in \mathcal{R}$, and all histories

We should be able to throw away the history once state is known

- If some information is only partially observable: Partially Observable MDP (POMDP)

Return

We aim to maximize *total discounted reward*:

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Discount factor

 γ close to 0 leads to "myopic" evaluation γ close to 1 leads to "far-sighted" evaluation **Definition**: A policy is a distribution over actions given states

 $\pi(a \mid s) = \mathbf{Pr}(A_t = a \mid S_t = s), \forall t$

- A policy fully defines the behavior of an agent
- The policy is stationary (time-independent)
- During learning, the agent changes its policy as a result of experience

Special case: deterministic policies





Learn the optimal policy to maximize return

1 START



Reinforcement Learning

• Sequential decision making

Supervised Learning

• One-step decision making

Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward

Supervised Learning

- One-step decision making
- Maximize immediate reward

Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward
- Sparse rewards

Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision



Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward
- Sparse rewards
- Environment maybe unknown

Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision
- Environment always known





Imitation learning!





Imitation learning!





Obtain expert trajectories (e.g. human driver/video demonstrations):

 $s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$

Imitation learning!







Obtain expert trajectories (e.g. human driver/video demonstrations):

 $s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$

Perform supervised learning by predicting expert action

D = {(s0, a*0), (s1, a*1), (s2, a*2), ...}

Imitation learning!







Obtain expert trajectories (e.g. human driver/video demonstrations):

 $s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$

Perform supervised learning by predicting expert action

D = {(s0, a*0), (s1, a*1), (s2, a*2), ...}

But: distribution mismatch between training and testing Hard to recover from sub-optimal states Sometimes not safe/possible to collect expert trajectories

Learn the optimal policy to maximize return

1 START



State and action value functions

- Definition: the state-value function $V^{\pi}(s)$ of an MDP is the expected return starting from state s, and following policy

$$V^{\pi}(s) = \mathbb{E}_{\pi}\left[G_t | S_t = s
ight]$$
 Captures long term reward

- Definition: the **action-value function** $Q^{\pi}(s, a)$ is the expected return starting from state s, taking action a, and then following policy

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi}\left[G_t|S_t=s, A_t=a
ight]$$
 Captures long term reward

Optimal state and action value functions

- Definition: the **optimal state-value function** $V^*(s)$ is the maximum value function over all policies

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

- Definition: the **optimal action-value function** $Q^*(s, a)$ is the maximum action-value function over all policies

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

Solving MDPs

• Prediction: Given an MDP (S, A, T, r, γ) and a policy $\pi(a|s) = \mathbb{P}[A_t = a|S_t = s]$ $V^{\pi}(s)$ $Q^{\pi}(s, a)$

find the state and action value functions.

Solving MDPs

• Prediction: Given an MDP (S, A, T, r, γ) and a policy $\pi(a|s) = \mathbb{P}[A_t = a|S_t = s]$ $V^{\pi}(s)$ $Q^{\pi}(s, a)$

find the state and action value functions.

• **Optimal control**: given an MDP (S, A, T, r, γ) , find the optimal policy (aka the planning problem). Compare with the learning problem with missing information about rewards/dynamics.

$$V^*(s) = Q^*(s,a)$$

Value functions

- Value functions measure the goodness of a particular state or state/action pair: how good is it for an agent to be in a particular state or execute a particular action at a particular state, for a given policy
- **Optimal value functions** measure the **best possible** states or state/action pairs under all possible policies

	state values	action values
prediction	v_{π}	q_{π}
control	V_*	q_*

State value functions

Action value functions

$$V^{\pi}(s) = \max_{\pi} V^{\pi}(s)$$
$$V^{*}(s)$$

$$Q^{\pi}(s,a)$$



State value functions

Action value functions

$$V^{\pi}(s) = \max_{\pi} V^{\pi}(s)$$

$$V^{*}(s)$$

$$Q^{\pi}(s,a)$$

 $Q^{*}(s,a) = \max_{\pi} Q^{\pi}(s,a)$
 $Q^{*}(s,a)$

State value functions

Action value functions

 $V^{\pi}(s) = \max_{\pi} V^{\pi}(s)$ $V^{\pi}(s)$ $Q^{\pi}(s, a) = \max_{\pi} Q^{\pi}(s, a)$ $Q^{*}(s, a)$ $V^{*}(s) = \max_{a} Q^{*}(s, a)$

State value functions

Action value functions

$$V^{\pi}(s) = \sum_{a} \pi(a|s)Q^{\pi}(s,a)$$

$$V^{\pi}(s) = \max_{\pi} V^{\pi}(s)$$

$$Q^{\pi}(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

$$V^{*}(s) = \max_{a} Q^{*}(s,a)$$

Obtaining the optimal policy

Optimal policy can be found by maximizing over Q*(s,a)

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \ Q^*(s,a) \\ 0, & \text{else} \end{cases}$$

Obtaining the optimal policy

Optimal policy can be found by maximizing over Q*(s,a)

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \ Q^*(s,a) \\ 0, & \text{else} \end{cases}$$

Optimal policy can also be found by maximizing over V*(s') with one-step look ahead

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \mathbb{E}_{s'} \left[r(s, a, s') + \gamma V^*(s') \right] \\ 0, & \text{else} \end{cases}$$
$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^*(s')) \right] \\ 0, & \text{else} \end{cases}$$

So, how do we find $Q^*(s,a)$ and $V^*(s)$?

Recursively: $G_{t} = r_{t+1} + \gamma r_{t+2} + \gamma^{2} r_{t+3} + \gamma^{3} r_{t+4} \dots$ $= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^{2} r_{t+4} \dots \right)$ $= r_{t+1} + \gamma G_{t+1}$

So, how do we find Q*(s,a) and V*(s)?

Recursively:
$$G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots$$

= $r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right)$
= $r_{t+1} + \gamma G_{t+1}$

By taking expectations: $V^{\pi}(s) = \mathbb{E}_{\pi} \left[G_t | S_t = s \right]$ $= \mathbb{E}_{\pi} \left[r_{t+1} + \gamma G_{t+1} | S_t = s \right]$ $= \mathbb{E}_{\pi} \left[r_{t+1} + \gamma V^{\pi}(S_{t+1}) | S_t = s \right]$

So, how do we find $Q^*(s,a)$ and $V^*(s)$?

Recursively:
$$\begin{aligned} G_t &= r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots \\ &= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right) \\ &= r_{t+1} + \gamma G_{t+1} \end{aligned}$$
By taking expectations:
$$V^{\pi}(s) = \mathbb{E}_{\pi} \left[G_t | S_t = s \right]$$

$$= \mathbb{E}_{\pi} [r_{t+1} + \gamma G_{t+1} | S_t = s]$$

$$= \mathbb{E}_{\pi} [r_{t+1} + \gamma V^{\pi} (S_{t+1}) | S_t = s]$$

$$= \sum_{a} \pi(a|s)$$

So, how do we find Q*(s,a) and V*(s)?

Recursively:
$$\begin{aligned} G_t &= r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots \\ &= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right) \\ &= r_{t+1} + \gamma G_{t+1} \end{aligned}$$
By taking expectations:
$$V^{\pi}(s) = \mathbb{E}_{\pi} \left[G_t | S_t = s \right]$$

$$= \mathbb{E}_{\pi} [r_{t+1} + \gamma G_{t+1} | S_t = s] = \mathbb{E}_{\pi} [r_{t+1} + \gamma V^{\pi} (S_{t+1}) | S_t = s] = \sum_{a} \pi(a|s) \mathbb{E}_{s'} [r(s, a, s') + \gamma V^{\pi}(s')]$$
Bellman expectation

So, how do we find Q*(s,a) and V*(s)?

$$\begin{array}{ll} \text{Recursively:} & G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} ... \\ & = r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} ... \right) \\ & = r_{t+1} + \gamma G_{t+1} \\ \text{By taking expectations:} & V^{\pi}(s) = \mathbb{E}_{\pi} \left[G_t | S_t = s \right] \\ & = \mathbb{E}_{\pi} \left[r_{t+1} + \gamma G_{t+1} | S_t = s \right] \\ & = \mathbb{E}_{\pi} \left[r_{t+1} + \gamma V^{\pi}(S_{t+1}) | S_t = s \right] \\ & = \sum_a \pi(a|s) \mathbb{E}_{s'} \left[r(s, a, s') + \gamma V^{\pi}(s') \right] \\ & = \sum_a \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right] \\ \end{array}$$



 $V^{\pi}(s) = \sum_{a} \pi(a|s)$



 $V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a)$



 $V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$



$$Q^{\pi}(s,a) = \sum_{s'} p(s'|s,a)$$







Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$

Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$

Solve the linear system

variables: $V^{\pi}(s)$ for all s constants: p(s'|s,a), r(s,a,s')

Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$

Solve the linear system

variables:
$$V^{\pi}(s)$$
 for all s
constants: p(s'|s,a), r(s,a,s')

Solve by iterative methods

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

Policy Evaluation

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$



1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

2. Policy Improvement

Find the best action according to one-step look ahead

$$\pi_{[k+1]}(a|s) = \arg\max_{a} \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V_{[k]}^{\pi}(s') \right]$$

Policy Iteration

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

2. Policy Improvement Find the best action according to one-step look ahead $\pi_{[k+1]}(a|s) = \arg\max_{a} \sum_{s} p(s'|s, a) \left[r(s, a, s') + \gamma V_{[k]}^{\pi}(s') \right]$

Policy Iteration

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

2. Policy Improvement Find the best action according to one-step look ahead $\pi_{[k+1]}(a|s) = \arg \max_{a} \sum_{s} p(s'|s, a) \left[r(s, a, s') + \gamma V_{[k]}^{\pi}(s') \right]$

Repeat until policy converges. Guaranteed to converge to optimal policy.

Bellman optimality for state value functions

The value of a state under an optimal policy must equal the expected return for the best action from that state



For the Bellman expectation equations we summed over all leaves, here we choose the **best** branch

Bellman optimality for state value functions

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For the Bellman expectation equations we summed over all leaves, here we choose the **best** branch

Bellman optimality for action value functions



For the Bellman expectation equations we summed over all leaves, here we choose the **best** branch

Bellman optimality for action value functions



Solving the Bellman optimality equations

$$V^{*}(s) = \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^{*}(s')) \right]$$

Solving the Bellman optimality equations

$$V^{*}(s) = \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^{*}(s')) \right]$$

Solve by iterative methods

$$V_{[k+1]}^*(s) = \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V_{[k]}^*(s')) \right]$$

Value Iteration

Algorithm:

Start with $V_0^*(s) = 0$ for all s. For k = 1, ... , H:

For all states s in S:

$$V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s') \right)$$

Value Iteration

Algorithm:

Start with $V_0^*(s) = 0$ for all s. For k = 1, ..., H: For all states s in S: $V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s') \right)$ $\pi_k^*(s) \leftarrow \arg\max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s') \right)$ Find the best action according to one-step look ahead This is called a value update or Bellman update/back-up

Value Iteration Repeat until policy converges. Guaranteed to converge to optimal policy.

Algorithm: Start with $V_0^*(s) = 0$ for all s. For k = 1, ..., H: For all states s in S: $V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s') \right)$ $\pi_k^*(s) \leftarrow \arg\max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s') \right)$ Find the best action according to one-step look ahead This is called a value update or Bellman update/back-up

Q-Value Iteration

 $Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q_{k+1}^*(s,a) \leftarrow \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q_k^*(s',a'))$$

Summary: Exact methods

Fully known MDP states transitions rewards



Repeat until policy converges. Guaranteed to converge to optimal policy.

Summary: Exact methods



Repeat until policy converges. Guaranteed to converge to optimal policy.

Summary: Exact methods



Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations: Iterate over and storage for all states and actions: requires small, discrete state and action space Update equations require fully observable MDP and known transitions

Solving unknown MDPs using function approximation

Recap: Q-value iteration

 $Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q_{k+1}^*(s,a) \leftarrow \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q_k^*(s',a'))$$

This is problematic when do not know the transitions

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$ Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$

- Q-value iteration: $Q_{k+1}(s, a) \leftarrow \sum_{s'} P(s'|s, a)(R(s, a, s') + \gamma \max_{a'} Q_k(s', a'))$ Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s, a)} \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$
- (Tabular) Q-Learning: replace expectation by samples
 - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$ simulation and exploration
 - Consider your old estimate: $Q_k(s, a)$
 - Consider your new sample estimate:

$$\operatorname{target}(s') = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

$$\operatorname{error}(s') = \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

Tabular Q-learning update

learning rate

$$\begin{array}{l} \downarrow \\ Q_{k+1}(s,a) = Q_k(s,a) + \alpha \operatorname{error}(s') \\ = Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right) \end{array}$$

Key idea: implicitly estimate the transitions via simulation

Algorithm:

Start with $\,Q_0(s,a)\,$ for all s, a. Get initial state s

For k = 1, 2, ... till convergence

Sample action a, get next state s'

If s' is terminal:

target = r(s, a, s')Sample new initial state s'

else:

$$\operatorname{target} = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$
$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$
$$s \leftarrow s'$$

Slides from Fragkiadaki

Bellman optimality

$$Q^{*}(s,a) = \mathbb{E}_{s'} \left[r(s,a,s') + \gamma \max_{a'} Q^{*}(s',a') \right]$$

Bellman optimality

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$

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$$s \leftarrow s'$$

- Choose random actions?
- Choose action that maximizes $Q_k(s,a)$ (i.e. greedily)?
 - ε-Greedy: choose random action with prob. ε, otherwise choose action greedily
Epsilon-greedy

Poor estimates of Q(s,a) at the start:

Bad initial estimates in the first few cases can drive policy into sub-optimal region, and never explore further.

$$\pi(s) = \begin{cases} \max_{a} \hat{Q}(s, a) & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{otherwise} \end{cases}$$

Gradually decrease epsilon as policy is learned.

Tabular Q-learning

Algorithm:

Start with
$$\,Q_0(s,a)$$
 for all s, a.

Get initial state s

For k = 1, 2, ... till convergence

Sample action a, get next state s'

If s' is terminal:

target =
$$r(s, a, s')$$

Sample new initial state s'

else:

$$\begin{aligned} & \operatorname{target} = r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \\ & Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right) \\ & s \leftarrow s' \end{aligned}$$

Slides from Fragkiadaki

ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

Convergence

- Amazing result: Q-learning converges to optimal policy -even if you're acting suboptimally!
- This is called off-policy learning
- Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate

small enough

... but not decrease it too quickly



Tabular Q-learning

Algorithm:

Start with $\,Q_0(s,a)\,$ for all s, a. Get initial state s

For k = 1, 2, ... till convergence

Sample action a, get next state s'

If s' is terminal:

target =
$$r(s, a, s')$$

Sample new initial state s'

else:

$$\operatorname{target} = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$
$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$
$$s \leftarrow s'$$

Tabular: keep a |S| x |A| table of Q(s,a) Still requires small and discrete state and action space How can we generalize to unseen states?

ε-Greedy: choose random action with prob. ε, otherwise choose action greedily







How can we generalize to unseen states?

Q-learning with function approximation to **extract informative features** from **high-dimensional** input states.



DQN, 2015

Represent value function by Q network with weights ${\bf w}$

$$Q(s,a,\mathbf{w})pprox Q^*(s,a)$$



+ high-dimensional, continuous states+ generalization to new states

- Optimal Q-values should obey Bellman equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s',a')^* \mid s,a
ight]$$

- Treat right-hand $r + \gamma \max_{a'} Q(s', a', \mathbf{w})$ as a target
- Minimize MSE loss by stochastic gradient descent

$$\mathcal{U} = \left(\textit{r} + \gamma \max_{\textit{a}} \textit{Q}(\textit{s}',\textit{a}',\textit{w}) - \textit{Q}(\textit{s},\textit{a},\textit{w})
ight)^2$$

- Minimize MSE loss by stochastic gradient descent

$$I = \left(r + \gamma \max_{a} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w})\right)^{2}$$

- Converges to Q* using table lookup representation
- But diverges using neural networks due to:
 - Correlations between samples
 - Non-stationary targets



Experience replay

- To remove correlations, build data-set from agent's own experience

$$\begin{array}{|c|c|c|c|}\hline s_1, a_1, r_2, s_2 \\\hline s_2, a_2, r_3, s_3 \\\hline s_3, a_3, r_4, s_4 \\\hline & \\ \hline & \\ \hline & \\ s_t, a_t, r_{t+1}, s_{t+1} \end{array} \rightarrow \begin{array}{|c|c|c|} s, a, r, s' \\\hline exploration, epsilon greedy is important! \\\hline \end{array}$$

- Sample random mini-batch of transitions (s,a,r,s') from **D**

Fixed Q-targets

- Sample random mini-batch of transitions (s,a,r,s') from D
- Compute Q-learning targets w.r.t. old fixed parameters w-
- Optimize MSE between Q-network and Q-learning targets

$$\begin{array}{c} s_1, a_1, r_2, s_2 \\ s_2, a_2, r_3, s_3 \\ s_3, a_3, r_4, s_4 \\ \dots \\ s_t, a_t, r_{t+1}, s_{t+1} \end{array}$$

S

$$\mathcal{L}_{i}(w_{i}) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_{i}} \begin{bmatrix} \left(r + \gamma \max_{a'} Q(s', a'; w_{i}^{-}) - Q(s, a; w_{i}) \right)^{2} \end{bmatrix} \overset{Q(s,a_{1},w) \cdots Q(s,a_{m},w)}{\mathsf{Q}_{i}} \overset{Q(s,a_{1},w) \cdots Q(s,a_{m},w)} \overset{Q(s,a_{1},w) \cdots Q(s,a_{m},w)}{\mathsf{Q}_{i}} \overset{Q(s,a_{1},w) \cdots$$

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- Use stochastic gradient descent
- Update w- with updated w every ~1000 iterations

Deep Q-learning for Atari



Deep Q-learning for Atari

- End-to-end learning of values Q(s,a) from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is Q(s,a) for 18 joystick/button positions
- Reward is change in score for that step



Network architecture and hyperparameters fixed across all games

Slides from Fragkiadaki

Mnih et.al., Nature, 2014

Deep Q-learning for Atari

- End-to-end learning of values Q(s,a) from pixels s
- Input state s is stack of raw pixels from last 4 frames

Encourage Markov property

- Output is Q(s,a) for 18 joystick/button positions
- Reward is change in score for that step



Network architecture and hyperparameters fixed across all games

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Mnih et.al., Nature, 2014



Superhuman results





Superhuman results on Montezuma's Revenge



Encourages agent to explore its environment by maximizing **curiosity.** I.e. how well can I predict my environment? 1. less training data 2. stochastic 3. unknown dynamics So I should explore more.

Burda et. al., ICLR 2019

Summary: Exact methods



Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations: Iterate over and storage for all states and actions: requires small, discrete state and action space Update equations require fully observable MDP and known transitions



How can we generalize to unseen states?

Summary: Deep Q-learning



Stochastic gradient descent + Experience replay + Fixed Q-targets

Works for high-dimensional state and action spaces Generalizes to unseen states

Applications: RL and Language

RL and Language

Task-independent



Task-dependent

Language-assisted

Key Opens a door of the same color as the key.

Skull They come in two varieties, rolling skulls and bouncing skulls ... you must jump over rolling skulls and walk under bouncing skulls.

Language-conditional

Go down the ladder and walk right immediately to avoid falling off the conveyor belt, jump to the yellow rope and again to the platform on the right.

Language-conditional RL

- Instruction following
- Rewards from instructions
- Language in the observation and action space

• Navigation via instruction following



Go to the green torch

Train

Go to the short red torch Go to the blue keycard Go to the largest yellow object Go to the green object



Go to the tall green torch Go to the red keycard Go to the smallest blue object

Chaplot et. al., AAAI 2018 Misra et. al., EMNLP 2017

• Navigation via instruction following



Go to the green torch

Train

Go to the short red torch Go to the blue keycard Go to the largest yellow object Go to the green object



Go to the tall green torch Go to the red keycard Go to the smallest blue object

Fusion Alignment

Ground language Recognize objects Navigate to objects Generalize to unseen objects

Chaplot et. al., AAAI 2018 Misra et. al., EMNLP 2017

• Interaction with the environment



Chaplot et. al., AAAI 2018

• Gated attention via element-wise product



Fusion Alignment Ground language Recognize objects

- Policy learning
 - Asynchronous Advantage Actor-Critic (A3C) (Mnih et al.)
 - uses a deep neural network to parametrize the policy and value functions and runs multiple parallel threads to update the network parameters.
 - use entropy regularization for improved exploration
 - use **Generalized Advantage Estimator** to reduce the variance of the policy gradient updates (Schulman et al.)





Chaplot et. al., AAAI 2018



Language-conditional RL: Rewards from instructions



Sparse, long-term reward problem General solution: reward shaping via auxiliary rewards

Montezuma's revenge

Language-conditional RL: Rewards from instructions



Montezuma's revenge

Sparse, long-term reward problem General solution: reward shaping via auxiliary rewards

Encourages agent to explore its environment by maximizing **curiosity**. How well can I **predict** my environment? 1. Less training data 2. Stochastic 3. Unknown dynamics So I should **explore more**.

Pathak et. al., ICML 2017 Burda et. al., ICLR 2019
Language-conditional RL: Rewards from instructions



Montezuma's revenge

Sparse, long-term reward problem General solution: reward shaping via auxiliary rewards

Natural language for reward shaping

- "Jump over the skull while going to the left"

from Amazon Mturk :-(asked annotators to play the game and describe entities

Intermediate rewards to speed up learning

Goyal et. al., IJCAI 2019

Language-conditional RL: Rewards from instructions



Montezuma's revenge

Natural language for reward shaping

Encourages agent to take actions related to the instructions



Goyal et. al., IJCAI 2019

Language-conditional RL: Rewards from instructions



Montezuma's revenge

Natural language for reward shaping

Encourages agent to take actions related to the instructions



Goyal et. al., IJCAI 2019

Language-conditional RL: Language in S and A

• Embodied QA: Navigation + QA





Most methods similar to instruction following

Das et. al., CVPR 2018

Language-assisted RL

- Language for communicating domain knowledge
- Language for structuring policies

• Properties of entities in the environment are annotated by language





is an enemy who chases you



is a stationary collectible

from Amazon Mturk :-(asked annotators to play the game and describe entities







is a stationary immovable wall

Narasimhan et. al., JAIR 2018

• Properties of entities in the environment are annotated by language



• Properties of entities in the environment are annotated by language



• Properties of entities in the environment are annotated by language



Grounded language learning

Helps to ground the meaning of text to the dynamics, transitions, and rewards Language helps in multi-task learning and transfer learning

Narasimhan et. al., JAIR 2018

• Learning to read instruction manuals



The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

Figure 1: An excerpt from the user manual of the game Civilization II.

• Learning to read instruction manuals



The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

- 1. Choose **relevant** sentences
- 2. Label words into action-description, state-description, or background

• Learning to read instruction manuals



The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

Map tile attributes:

- Terrain type (e.g. grassland, mountain, etc)
- Tile resources (e.g. wheat, coal, wildlife, etc)

City attributes:

- City population
- Amount of food produced
- Unit attributes:
 - Unit type (e.g., worker, explorer, archer, etc)
 - Is unit in a city ?

- 1. Choose relevant sentences
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• Learning to read instruction manuals



- Phalanxes are twice as effective at defending cities as warriors.
- ullet Build the city on plains or grassland with a river running through it. \checkmark
- You can rename the city if you like, but we'll refer to it as washington.
- There are many different strategies dictating the order in which advances are researched



Relevant sentences

A: action-description S: state-description

• Learning to read instruction manuals



Method	% Win	% Loss	Std. Err.
Random	0	100	
Built-in AI	0	0	—
Game only	17.3	5.3	± 2.7
Sentence relevance	46.7	2.8	± 3.5
Full model	53.7	5.9	\pm 3.5
Random text	40.3	4.3	± 3.4
Latent variable	26.1	3.7	± 3.1

Grounded language learning Ground the meaning of text to the dynamics, transitions, and rewards Language helps in learning

• Learning to read instruction manuals





Language is most important at the start when you don't have a good policy Afterwards, the model relies on game features

Language for structuring policies

• Composing modules for Embodied QA



Das et. al., CoRL 2018

Language for structuring policies

• Composing modules for Embodied QA





Das et. al., CoRL 2018