Rereading Einstein on Radiation

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Citation: Physics Today **58**, 2, 30 (2005); doi: 10.1063/1.1897520
View online: https://doi.org/10.1063/1.1897520
View Table of Contents: https://physicstoday.scitation.org/toc/pto/58/2
Published by the American Institute of Physics

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The concepts of spontaneous and stimulated emission are well known from Einstein's 1917 paper on radiation, but his theory of radiation comprises many other concepts—the paper is a treasure trove of physics.

Daniel Kleppner

Albert Einstein had a genius for extracting revolutionary theory from simple considerations: From the postulate of a universal velocity he created special relativity; from the equivalence principle he created general relativity; from elementary arguments based on statistics he discovered energy quanta. His 1905 paper on quantization of the radiation field (often referred to, inaccurately, as the photoelectric-effect paper) was built on simple statistical arguments, and in subsequent years he returned repeatedly to questions centered on statistics and thermal fluctuations. In 1909, Einstein showed that statistical fluctuations in thermal radiation fields display both particlelike and wavelike behavior; his was the first demonstration of what would later become the principle of complementarity. In 1916, when he turned to the interplay of matter and radiation to create a quantum theory of radiation, he once again based his arguments on statistics and fluctuations.

Einstein's theory of radiation is a treasure trove of physics, for in it one can discern the seeds of quantum electrodynamics and quantum optics, the invention of masers and lasers, and later developments such as atom-cooling, Bose–Einstein condensation, and cavity quantum electrodynamics. Our understanding of the cosmos comes almost entirely from images brought to us by radiation across the electromagnetic spectrum. Einstein's theory of radiation describes the fundamental processes by which those images are created.

Einstein's 1905 paper on quantization endowed Max Planck's quantum hypothesis with physical reality. The oscillators for which Planck proposed energy quantization were fictitious, and his theory for blackbody radiation lacked obvious physical consequences. But the radiation field for which Einstein proposed energy quantization was real, and his theory had immediate physical consequences. His paper, published in March, was the first of his wonder year. In rapid succession he published papers on Brownian motion, special relativity, and his quantum theory of the specific heat of solids.

In 1907, his interest shifted to gravity, and he took the first tentative steps toward the theory of general relativity. His struggle with gravitational theory became all-consuming until November 1915, when he finally obtained satisfactory gravitational field equations. During those years of struggle, however, Einstein apparently had a simmering discontent with his understanding of thermal radiation, for in July 1916, he turned to the problem of how matter and radiation can achieve thermal equilibrium. One could argue that 1916 was too soon to deal with that problem because there were serious conceptual obstacles to the creation of a consistent theory. Einstein, in his Olympian fashion, simply ignored them. In the next eight months, he wrote three papers on the subject, publishing the third, and best known, in 1917.

In his theory of radiation, as the 1917 paper has come to be known,[1] Einstein introduced the concepts of spontaneous and stimulated emission, with rates characterized by the A and B coefficients. Einstein's A and B coefficients are the only remnants of this theory that appear in many textbooks on quantum theory, and the only part of the paper I encountered as a student. His achievements, however, were far broader.

In developing his theory of radiation, Einstein employed a crucial new concept: stationary states, introduced in Niels Bohr's 1913 paper on hydrogen. The idea of nonradiating states of an electron orbiting a nucleus could be described as deep nonsense, because, according to the well-established principles of electromagnetic theory, the electron would radiate intensely, emitting a broad spectrum as it crashed into the nucleus. One might argue that the time had come to set aside classical laws, but major features of Bohr's model rested squarely on those laws. The concept of stationary states was exactly the type of truth-in-nonsense that, in Einstein's hands, could be used to work magic.

Einstein's theory of radiation analyzes the processes by which the energy and momentum states of a gas of atoms achieve equilibrium with a thermal radiation field. His reasoning is transparent and novel. For example, he did not take as his starting point the known field for thermal radiation, given by the Planck radiation law. Instead, Einstein assumed that the atoms are in thermal equilibrium and then deduced the properties of the radiation field required to maintain the equilibrium. The field turned out to be given precisely by the Planck radiation law!

The raw ingredients of Einstein's 1917 paper are few, and all but one of them are classical: the Wien displacement law, the canonical Boltzmann distribution, Poynting's theorem, and microscopic reversibility. The sole quantum idea Einstein invoked was the concept of stationary states. From these elements he created a complete description of the basic radiation processes and a full description of the properties of the photon.

Einstein's 1917 paper is a pleasure to read because of its simplicity and modesty. For instance, his new and elegant derivations of the Planck radiation law and his proof of the Bohr frequency rule are presented with little

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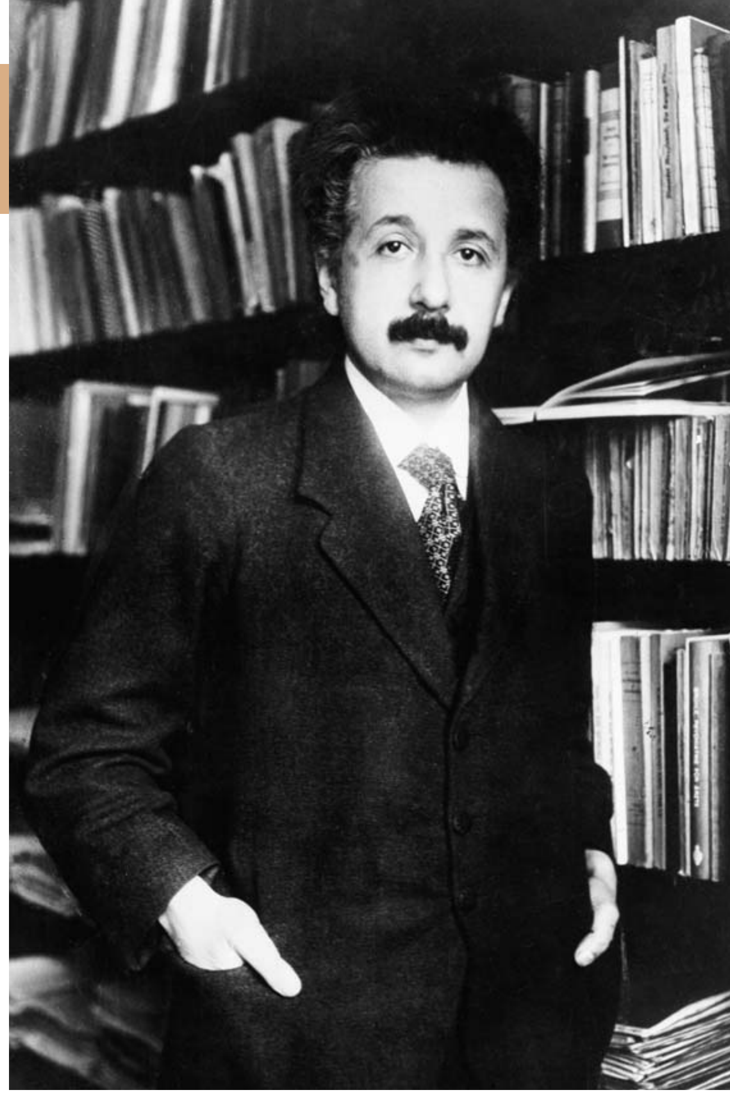

Einstein at home in Berlin, about 1915. (Courtesy of ullstein bild, Berlin.)

comment to call attention to those accomplishments. Here is a brief summary of the paper.

Part 1: Emission and absorption

How does a gas of atoms maintain the populations of its stationary states in equilibrium with a radiation field? Einstein considered atoms having two energy levels, a and b, with energies $E_b > E_a$. (For simplicity, we assume that there is only one quantum state for each energy.) The states are occupied according to the Boltzmann distribution with probabilities $P_a = C\exp(-E_a/kT)$ and $P_b = C\exp(-E_b/kT)$, where k is Boltzmann's constant, T is temperature, and C is a normalizing factor. The atoms are bathed by blackbody radiation with a yet-to-be determined spectral density $\rho(\nu, T)$, where ν is the circular frequency. He made use of the Wien displacement law, which was based on a combination of thermodynamics and electromagnetic theory: $\rho_{\rm Wien}(\nu, T) = \nu^3 f(\nu/T)$, in which f is an unknown function. In addition, Einstein introduced the following three processes by which atoms interact with radiation.

Spontaneous emission. Einstein proposed that an excited atom in empty space will make a transition to a lower state by a process he called spontaneous emission. The probability that this takes place in time dt is $dW = A_{ba}\,dt$, where A_{ba} is a constant. The novelty of this proposal may not be obvious because spontaneous emission is now familiar. However, a process that appears to happen without cause could only be called novel. Furthermore, spontaneous emission describes not a radiation rate but the rate of change of probability. Thus, the language of spontaneous emission has revolutionary implications.

Absorption. The rate at which an oscillator absorbs energy from a force field with a broad spectrum is proportional to the spectral density of the field. Thus, Einstein asserted that the probability that an atom in the lower energy state will make a transition $a \to b$ in time dt is $dW = \rho(\nu, T)\,B_{ab}\,dt$, where B_{ab} is a constant to be determined.

Stimulated emission. An oscillator absorbs energy or emits energy depending on its phase with respect to the driving force. Consequently, Einstein argued that a radiation field causes an atom in the upper energy state to make a transition to the lower state at a rate proportional to the radiation density, a process he named stimulated emission. The probability of making a transition $b \to a$ in time dt is $dW = \rho B_{ba}\,dt$, where B_{ba} is a constant to be determined.

By imposing the conditions for thermal equilibrium in the presence of those processes and invoking the Wien displacement law, Einstein showed that B_{ba} is proportional to A_{ba}; that $B_{ba} = B_{ab}$; that in making a transition, the atom emits monochromatic radiation with frequency given by the Bohr condition $\nu = (E_b - E_a)/h$, where h is Planck's constant; and that for the atoms to achieve thermal equilibrium, $\rho(\nu, T) = (8\pi h\nu^3/c^3)/(\exp[h\nu/kT] - 1)$. This last equation is the Planck radiation law.

Part 2: The photon

Einstein turned next to the question of how the kinetic energy of a gas of atoms maintains thermal equilibrium with a radiation field. Again, he assumed the canonical distribution for the energy (that is, the Maxwell–Boltzmann distribution of velocities), analyzed the mechanisms of interaction with the radiation, and then found the radiation field required to maintain thermal equilibrium. He used "a method which is well known from the theory of Brownian motion and which I have used already many times for numerical calculations of motion in radiation."

Thermal radiation damps the momentum of an atom by generating a retarding force Rv due to the atom's motion in the isotropic radiation field, where v is the velocity and R is to be evaluated. Opposing this force is the effect of fluctuations in the radiation field that tend to increase the momentum. Einstein considered a short interval τ in which fluctuations cause momentum Δ of varying sign and magnitude to be transferred to the atom. The effects of the damping force and momentum kicks balance each other on average, and he found that the momentum obeys a diffusion equation like that for Brownian motion of molecules in a liquid: $\overline{\Delta^2}/\tau = 2RkT$. (The bar indicates a time average.) The task is first to investigate how $\overline{\Delta^2}$ and R each depend on the radiation field, and second, to find the field that satisfies the diffusion equation.

The calculation proceeds along the following lines: The atom does a random walk in momentum space because of the random transfer of momentum in each elementary process. The atom takes a random step in momentum whenever it absorbs or emits a light quantum, so that the number of momentum steps an atom makes in time τ is twice the number of absorptions, $2 \times \rho(\nu, T)B_{nm}\tau$. The damping force Rv arises from the interplay of the Doppler effect and the frequency dependence of $\rho(\nu, T)$. The absorption rate of a moving atom differs from that of an atom at rest because the effect of the motion is to shift the spectrum.

When these results are put together, the field required for thermal equilibrium turns out once again to be described by the Planck radiation law. The argument seems transparent, but its implications are unexpected. For instance, atoms interacting with radiation at temperature T assume a Maxwell–Boltzmann velocity distribution due to the absorption and emission of radiation even in the absence of collisions. However, the analysis leads to a far more profound result.

Momentum transfer in each step of radiation is central to Einstein's analysis. That the light quantum carries momentum as well as energy is an innovation of the 1917 paper. (The term "photon" was not coined until 1928, and Einstein never cared for it.) In the 1905 paper on the photoelectric effect, for instance, momentum played no role. With the realization that the excitation of an electron is accompanied by momentum transfer, the full nature of the light quantum was finally revealed.

Zur Quantentheorie der Strahlung.

Von A. Einstein[1]).

Die formale Ähnlichkeit der Kurve der chromatischen Verteilung der Temperaturstrahlung mit Maxwellschen Geschwindigkeits-Verteilungsgesetz ist zu frappant, als daß sie lange hätte verborgen bleiben können. In der Tat wurde bereits W. Wien in der wichtigen theoretischen Arbeit, in welcher er sein Verschiebungsgesetz

$$\varrho = \nu^3 f\left(\frac{\nu}{T}\right) \qquad (1)$$

ableitete, durch diese Ähnlichkeit auf eine weitergehende Bestimmung der Strahlungsformel geführt. Er fand hierbei bekanntlich die Formel

$$\varrho = a\,\nu^3 e^{-\frac{h\nu}{kT}}, \qquad (2)$$

welche als Grenzgesetz für große Werte von

The opening paragraph of Einstein's 1917 paper, "On the Quantum Theory of Radiation."

On the art of ignoring conceptual obstacles

Einstein made numerous assumptions in the 1917 paper, often without comment. Absorption and stimulated emission, for example, are evidently introduced by analogy with the behavior of classical oscillators; it is not obvious why a two-state quantum system should have much in common with a classical oscillator. He applied Boltzmann statistics to his two-state atoms, although one might question why quantum systems should obey classical statistics. Within a decade, such assumptions would be justified from a rigorous theory, but in 1917 his approach bordered on the cavalier.

The most radical assumption was that one can describe the dynamics of atoms in terms of probabilities. A probabilistic point of view is inherent in the concept of the rate of spontaneous emission. Einstein recognized the implications of such language and justified his assumption of a spontaneous emission rate by stating, "The statistical law assumed here corresponds to the law of a radioactive reaction, and the elementary process assumed here corresponds to a relation in which only γ rays are emitted."

The "law of a radioactive reaction" is Ernest Rutherford's law for radioactive decay: $N(t) = N(0)\ \exp(-t/\tau)$, where τ is a time characteristic of the atom. This law aroused controversy when Rutherford announced it in 1900.[2] The problem could be put this way: How does any given radioactive atom "know" when its time has come—that is, when it should emit a gamma ray? That a process such as a chemical reaction can take place at a uniform rate—that is, a constant probability per unit time—is plausible because the number of particles is large and the reactions take place during random collisions. The random decay of an isolated atom is another matter entirely.

By 1916 the controversy had died down, but it had never been resolved. Einstein evidently still had misgivings because he commented, "The weakness of the theory lies . . . in the fact that it leaves moment and direction of the elementary processes to 'chance'; all the same, I have complete confidence in the reliability of the method used here." His instinct was sound because his method indeed turned out to be reliable. In a decade, however, the problem of chance would come to haunt him; it eventually isolated him from the mainstream of physics.

An equally daunting problem for Einstein was the lack of any theory for calculating the spontaneous emission rate. That—and a multitude of other problems on atomic structure and dynamics—would simply have to wait for the creation of a complete quantum theory. The problem of spontaneous emission was solved by Paul Dirac in 1927. Einstein was quite aware of the deficiency, for his 1917 paper is titled "*On* the Quantum Theory of Radiation" (emphasis added).

Finally, Einstein questioned whether it is reasonable to calculate the Doppler effect for a quantum radiation field in which the energy is absorbed as a particle, not a wave. He argued that whatever the form of the radiation theory, the first-order Doppler effect would be preserved. He calculated only to first order in v/c. In second order, the quantum radiation process causes a tiny energy shift due to recoil of the atom, which is absent in the wave picture. His analysis, based on momentum conservation, would have included that effect. The recoil shift was observed more than a half century after Einstein's paper was published.

Treasures

Einstein's paper is crammed with gems. The concept of spontaneous emission, which embodies the fundamental interaction of matter with the vacuum, is conspicuous among them. Spontaneous emission sets the scale for all radiative interactions. The rates of absorption and stimulated emission, for instance, are proportional to the rate for spontaneous emission. Spontaneous emission can be viewed as the ultimate irreversible process and the fundamental source of noise in nature. However, with the development of cavity quantum electrodynamics—the study of atomic systems in close-to-ideal cavities—in the 1980s, the physical situation was profoundly altered. In such cavities, spontaneous emission evolves into spontaneous atom–cavity oscillations. Although the dynamical behavior is totally altered, the atom–vacuum interaction that causes spontaneous emission sets the time scale for that evolution.

Among the achievements of the 1917 paper, the concept of the photon shines brilliantly. Einstein postulated energy quantization in 1905, but only with the publication of his radiation paper was the photon demonstrated to have all the properties of a fundamental excitation. Thus, the radiation paper played a seminal role in the eventual creation of quantum electrodynamics.

Another gem is the concept of stimulated emission. To claim that Einstein almost invented the laser would be an exaggeration, but the laser's underlying mechanism, stimulated emission of radiation, was a creation of his radiation theory. If Einstein had known about lasers, he would probably have invented laser cooling; his analysis of momentum transfer in a thermal radiation field can be immediately applied to atomic motion in a laser field. If the spectral width of a thermal field is replaced by the natural linewidth of the atom, Einstein's viscous damping force would give rise to the phenomenon known as optical molasses. This fundamental process of laser cooling was rediscovered by the atomic community in the 1980s.

Ironically, when the quantum theory, which was needed to fully realize the vision of his radiation theory, was developed between 1925 and 1928, Einstein turned his back on it. His supernatural intuition finally failed him.

A missed opportunity

Einstein's theory of radiation provided a complete characterization of the particlelike properties of the light quantum. It is curious that Einstein did not work out the statistical mechanics of these particles, particularly because his 1905 proposal for the energy quantization of radiation was based on the analogy between the entropies of thermal radiation and a system of particles. When he received a short paper from Satyendra Nath Bose in 1924 that derived the Planck law by treating photons as indistinguishable particles, Einstein immediately forwarded it for publication. I can picture him slapping his forehead and exclaiming, "How could I have missed that!" He immediately applied Bose's reasoning to a gas of indistinguishable atoms, creating Bose–Einstein statistics. Shortly after, he pointed out the possibility of Bose–Einstein condensation. Many physicists regard that as his last major contribution to physics.

A work of art?

Analogies between science and art are easily exaggerated, but consider the following: Just as a great work of art can inspire thoughts and feelings beyond the imagination of the artist, Einstein's radiation theory had consequences that even he could not have imagined. Great performances in music or dance seem to be executed effortlessly. Einstein presented his radiation theory so simply and naturally that one could believe he tossed it off casually. Works of art cause elation. Anyone fortunate enough to understand Einstein's radiation theory cannot fail to be elated.

I have drawn liberally on the scientific biography of Einstein by Abraham Pais: "Subtle is the Lord": The Science and the Life of Albert Einstein*, Oxford University Press, New York (1982). I thank Wolfgang Ketterle for a helpful comment.*

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