Motivation :-

$N >>>$ G_N on $\{1, \dots, N\}$ Vertices.

Sample :- n - vertices from N

Construct :- $G_n \hookrightarrow G_N$

distance :- close is G_n to G_N ?

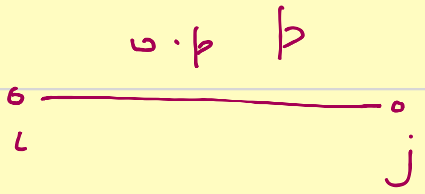
Examples :-

Erdös - Renyi Graphs

$\boxed{p = \frac{x}{n}} \longleftarrow$ ⓧ

$V = \{1, \dots, n\}$ Edge :- $i \sim j$ with Probability p

- independent of each other



[undirected] $\equiv G(n, p)$

- Vertex i — Expected # of edges.

  $\equiv O(n)$.

Expected number of edges for $G(n,p)$

$\equiv O(n^2)$.

Dense - graphs $V = \{1, \dots, n\}$

G_n - graph on n - vertices

\# edges of G_n $\equiv$ order n^2 $\equiv$ **Dense graphs**

Questions :-

(?) - Metric or distance function on dense Graphs. $\equiv d$.

- labelling
- variation in n

- Space of all graphs on finite vertices

$$\mathcal{G} = \{ G_n : \begin{matrix} n \text{ - vertices} \\ G_n \text{ - graph} \end{matrix} \}$$

$\boxed{d}$ - metric

— are there interesting limit points $\{G_n\}_{n \geq 1}$ as $n \to \infty$ in $(\mathcal{G}, d)$. ?

- L. Lovasz :- Graph limit Book.
- Bollabas - Riordan :- Survey

Example 2 :- $K : [0,1]^2 \to [0,1]$ Symmetric, measurable $\boxed{G(n,K)}$

$V_n = \{1, \dots, n\}$

$$\begin{matrix} 1 & 2 & \dots & n \\ \uparrow & \uparrow & & \uparrow \\ U_1 & U_2 & \dots & U_n \end{matrix}$$

Edges: $i \sim j$

w.p. $K(U_i, U_j)$

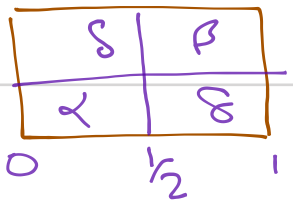
$U_i = \text{i.i.d } U[0,1]$

- $K \equiv p \longrightarrow$ Erdős Renyi Graph

Typical examples

Dense

- $K \equiv$



$0 < \alpha, \beta, \delta < 1$?

- labelling "uniform manner"

$\boxed{X_{ij}}$

  - made vertices Exchangeable

  - Aldous - Hoover Exchangeable random variables

Approach two.