**Due: Thursday, October 27th**

1. Let $\epsilon > 0$ be given. If $f$ is a function in $N$ arguments such that changing the $i$th coordinate will change the value of $f$ by at most $c_i$ and if $Y = (Y_1, \dots, Y_N)$ are independent random variables, then show that
$$P[\mid f(Y) - E[f(Y)] \mid \geq \epsilon] \leq 2 \exp(-\frac{2\epsilon^2}{\sum_{i=1}^{N} c_i^2}).$$
*You may consult books/literature and try to present as complete a proof as possible.*

2. If $G$ is a graph with $n$ vertices, then show that $s(F, G)$ changes by at most $\frac{k(k-1)}{n(n-1)}$ if one edge is changed.

3. Let $\epsilon > 0$ be given and $X_n$ be a sequence of random variables such that
$$P(\mid X_n - A \mid > \epsilon) \leq \exp(-\epsilon^2 n^2).$$
Show that $X_n \to A$ with probability 1.

4. Let $k \geq 1$. Denote by $C_k$ the set of all functions from $[0, 1]^k$ to $[0, 1]$. Let
$$P_k = \{h \in C_k \, : \, \exists \, h_1, \dots, h_k \in C_1 \text{ such that } h(x) = \prod_{i=1}^{k} h_i(x_i)$$
Fix $\epsilon > 0$ and let $h \in C_k$. Show there exist $s_1, s_2 \in \text{Span}(P_k)$ such that
   (a) $\mid h - s_1 \mid \leq s_2$ -almost everywhere,
   (b) $\int s_2 \prod_{i=1}^{k} dx_i \leq \epsilon$.
   Is the above approximation true if $h$ was just integrable ?