# BU CS 332 – Theory of Computation

## Lecture 17:

- Midterm II review

Reading for midterm:

Sipser Ch, 3, 4, 5

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# Format of the Exam Same as Midterm I:

- In Class, 70 minutes
- Via Gradescope
- Questions will test:
  - Knowledge of facts learned
  - Understanding of main concepts learned
  - Ability to use the tools learned
- Not meant to be hard!

# Study Tips

- Review problems from HW 5-7, discussion sections 6-9, solved exercises/problems in Sipser, suggested exercises on the homework, and the practice midterm.
  - We will ask you a question from the homework exercises (or a close variant), so make sure you understand these

- While you are not required to prepare a cheat-sheet, this is a great way to study!

# Study Tips

- Try to solve the practice midterm in same setting as the exam (70 minutes, one sitting). The exam will have a similar format.
- Make use of office hours
- Make use of Nathan the tutor
- If you need more practice, there are lots of problems in the book. We're happy to talk about any of these problems in office hours!

# For the exam itself

- You may cite without proof any result…
  - Stated in lecture  or discussion
  - Stated and proved in the main body of the text (Ch. 3-5)
  - These include worked-out solutions in the main text.
- Not included above: homework problems, (solved) exercise/problems in the book.

- Showing your work / explaining your answers will help us give you partial credit..

# Midterm II Topics

# Turing Machines (3.1, 3.3)

- Know the three different “levels of abstraction” for defining Turing machines and how to convert between them: Formal/state diagram, implementation-level, and high-level
- Know the definition of a configuration of a TM and the formal definition of how a TM computes
- Know how to “program” Turing machines by giving implementation-level descriptions

# TM Variants (3.2)

- Understand the following TM variants: Multi-tape TMs, Nondeterministic TMs, Enumerators
- Know how to give a simulation argument (implementation-level description) to compare the power of TM variants
- Understand the specific simulation arguments we've seen: multi-tape TM by basic TM, nondeterministic TM by basic TM, enumerator by basic TM and basic TM by enumerator.

# Universality of TMs (3.3, 4.2)

Universal TMs

- Understand how to use a TM to simulate another machine (DFA, another TM)
  - The actual construction (program it!)
  - The concept and implications

PDA

- Understand the Church-Turing Thesis

# Decidability (4.1)

- Know the specific decidable languages from language theory that we've discussed, and how to decide them: $A_{DFA}$, $E_{DFA}$, $EQ_{DFA}$, $A_{CFG}$, $E_{CFG}$, etc. $EQ_{CFG}$ (undecidable)

accept Empty

- Know how to use a reduction to one of these languages to show that a new language is decidable

# Undecidability (4.2)

- Know the definitions of countable and uncountable sets and how to prove countability and uncountability
- Understand how diagonalization is used to prove the undecidability of $A_{TM}$

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- Know that a language is decidable iff it is recognizable and co-recognizable, and understand the proof

$L \in coRE \Leftrightarrow \overline{L} \in RE$

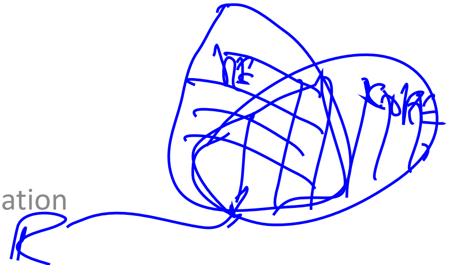
$\exists M$ s.t.

$x \notin L \Leftrightarrow M(x) = acc$

$R = RE \cap coRE$

$\exists M$ $\quad L \in RE$

$x \in L \Rightarrow M(x) = acc$



# Reducibility (5.1)

- Understand how to use a reduction (contradiction argument) to prove that a language is undecidable
- Know the reductions showing that $HALT_{TM}$, $E_{TM}$, $REGULAR_{TM}$, $CFL_{TM}$, $EQ_{TM}$ are undecidable

- You are not responsible for understanding the computation history method. However, you should know that the languages POST and $EQ_{CFG}$ are undecidable, and reducing from them might be useful.

# Mapping Reducibility (5.3)

- Understand the definition of a computable function
- Understand the definition of a mapping reduction
- Know how to use mapping reductions to prove decidability, undecidability, recognizability, and unrecognizability

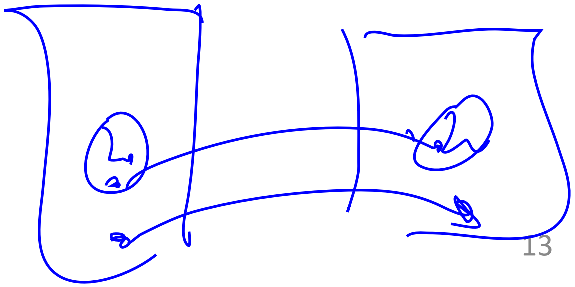
$L_1 \leq_m L_2$

"$L_1$ is no harder than $L_2$"

Formally: $L_1 \leq_m L_2$ if there exists a computable mapping $f$:

if $x \in L_1$ then $f(x) \in L_2$

$x \notin L_1$ then $f(x) \notin L_2$



# Tips for the Exam

# True or False

- It’s all about the justification!
- The logic of the argument has to be clear
- Restating the question is not justification; we’re looking for some additional insight

All regular languages are Turing recognizable.

# True or False

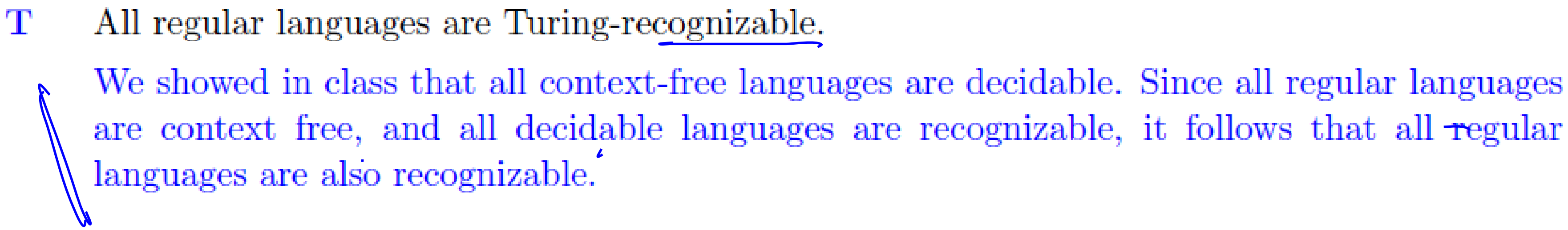
- It's all about the justification!
- The logic of the argument has to be clear
- Restating the question is not justification; we're looking for some additional insight

T All regular languages are Turing recognizable.

We showed in class that that non-regular languages are not always decidable.
We also showed that if both a language and its complement are recognizable then both are decidable, and regular languages are closed under complement.
It follows that regular languages are Turing recognizable.

# True or False

- It’s all about the justification!
- The logic of the argument has to be clear
- Restating the question is not justification; we’re looking for some additional insight



**T** All regular languages are Turing-recognizable.

We showed in class that all context-free languages are decidable. Since all regular languages are context free, and all decidable languages are recognizable, it follows that all regular languages are also recognizable.

# Undecidability proofs

Show that the language $Y$ is undecidable. (10 points)

We show that $Y$ is undecidable by giving a reduction from $A_{\mathsf{TM}}$. Suppose for the sake of contradiction that we had a decider $R$ for $Y$. We construct ~~a decider~~ der for $A_{\mathsf{TM}}$ as follows:

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"On input ⟨M, w⟩:
    1. Use M and w to construct the following TM M':
       M' = "On input x:
            1. If x has even length, accept
            2. Run M on w
            3. If M accepts, accept. If M rejects, reject."
    2. Run R on input ⟨M'⟩
    3. If R accepts, reject. If R rejects, accept."
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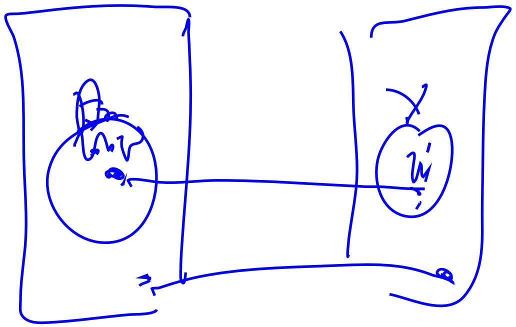
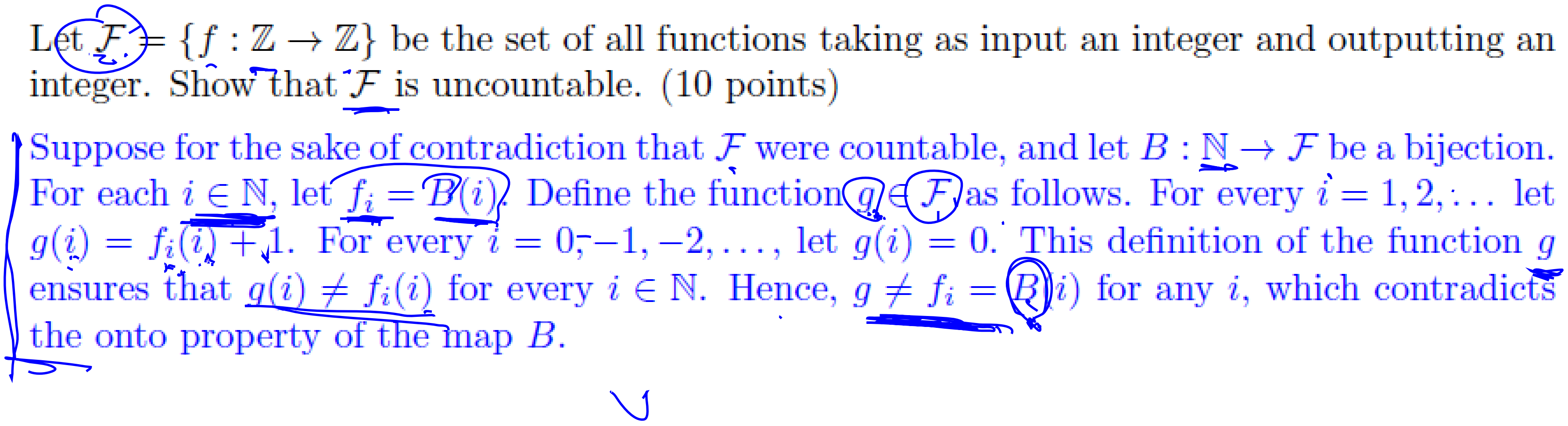


If $M$ accepts $w$, then the machine $M'$ accepts all strings. On the other hand, if $M$ does not accept $w$, then $M'$ only accepts strings of even length.

Hence this machine decides $A_{\mathsf{TM}}$ which is a contradiction, since $A_{\mathsf{TM}}$ is undecidable. Hence $Y$ must be undecidable as well.

# Uncountability proofs



Let $\mathcal{F} = \{f : \mathbb{Z} \to \mathbb{Z}\}$ be the set of all functions taking as input an integer and outputting an integer. Show that $\mathcal{F}$ is uncountable. (10 points)

Suppose for the sake of contradiction that $\mathcal{F}$ were countable, and let $B : \mathbb{N} \to \mathcal{F}$ be a bijection. For each $i \in \mathbb{N}$, let $f_i = B(i)$. Define the function $g \in \mathcal{F}$ as follows. For every $i = 1, 2, \ldots$ let $g(i) = f_i(i) + 1$. For every $i = 0, -1, -2, \ldots$, let $g(i) = 0$. This definition of the function $g$ ensures that $g(i) \neq f_i(i)$ for every $i \in \mathbb{N}$. Hence, $g \neq f_i = B(i)$ for any $i$, which contradicts the onto property of the map $B$.

- The 2-D table is useful for thinking about diagonalization, but is not essential to the argument
- The essential part of the proof is the construction of the "inverted diagonal" element, and the proof that it works

# Practice Problems

# Decidability and Recognizability

Let $A = \{\langle D \rangle \mid$ $D$ is a DFA that does not accept any string containing an odd number of $1'$s$\}$

Show that $A$ is decidable

# Prove that $\overline{E_{\mathrm{TM}}}$ is recognizable

# Prove that if $A$ and $B$ are decidable, then so is $A \setminus B$

# Countable and Uncountable Sets

# Show that the set of all valid (i.e., compile without errors) C++ programs is countable

A Celebrity Twitter Feed is an infinite sequence of ASCII strings, each with at most 140 characters. Show that the set of Celebrity Twitter Feeds is uncountable.

# Undecidability and Unrecognizability

# Prove or disprove: If $A$ and $B$ are recognizable, then so is $A \setminus B$

# Prove that the language $ALL_{\mathrm{TM}} = \{\langle M \rangle | M \text{ is a TM and } L(M) = \Sigma^*\}$ is undecidable

# Give a nonregular language $A$ such that $A \leq_m L(0^*1^*)$ or prove that none exists

# Give an undecidable language $A$ such that $A \leq_m \; L(0^*1^*)$ or prove that none exists

# Give an undecidable language $A$ such that $L(0^*1^*) \leq_m A$ or prove that none exists